Energy Balance Model of Spatially Variable Evaporation from Bare Soil

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ABSTRACT

Most models of evaporation (E) provide estimates at one rather than many locations and thus cannot be used to describe the spatial variability of evaporation. An energy balance model (EBM) that estimates E at many locations was tested, improved and validated, using daily evaporation measurements made with microlysimeters, giving an r^2 value of 0.82 for regression of actual vs. estimated evaporation. The model is based on the surface energy balances of dry and drying soil. Data needed include only wind speed and soil surface temperature measurements obtained on a suitably small time interval (e.g., 0.5 h) with an automated weather station and reference dry soil at one location, and measurements of pre-dawn and midday soil surface temperature made with a hand-held infrared thermometer at as many locations as desired for evaporation prediction. The reference dry soil was established in a plastic bucket buried in the soil and protected from rain and irrigations. Model improvements included an easy method of accurately estimating continuous soil surface temperature at many points in a field. Also, an empirically fitted transfer coefficient function for the sensible heat flux from the reference dry soil showed that sensible heat flux from the relatively hot reference dry soil was dominated by free convection. Soil heat flux and reflected shortwave radiation terms are omitted in the EBM and this was shown to reduce model accuracy by as much as 9.2% of the measured evaporation. The model may prove useful for prediction of spatial variability of evaporation based on soil surface temperatures.

Estimation of evaporation from bare soil surfaces in the field is a difficult problem that has recently been approached in two conceptually different ways: (i) models based on the energy balance at the soil surface (e.g., Lascano and van Bavel, 1986; Reynolds and Walker, 1984; Evett and Lascano, 1993), and (ii) measurements by microlysimetry (Boast and Robertson, 1982; Salehi, 1984; Boast, 1986; Evett et al., 1995). Microlysimetry has the advantage that the spatial variability of evaporation can be directly examined. A disadvantage is that measurements are difficult and time consuming.

Ben-Asher et al. (1983), building on work by Fox (1968), developed an EBM that used average daily wind speed and the difference between midday maximum soil surface temperatures of a reference dry soil and a drying soil to estimate daily evaporation, E_d (mm), from the drying soil. In simplified form the model is:

\[ E_d = S(T_{o,max} - T_{d,max})/L_e \]  

where \( L_e \) is the latent heat of vaporization (2.4 MJ kg\(^{-1}\)), \( T_{o,max} \) and \( T_{d,max} \) are the maximum midday temperatures (K) of the dry and drying soils, respectively; and S is a positive function of average daytime wind speed and of average daily soil surface temperature. The advantage of this model is that \( T_{o,max} \) may be measured at a single previously established dry soil site while \( T_{d,max} \) may be measured at as many drying soil locations as needed to study the spatial variability of \( E_d \). Ben-Asher et al. (1983) regressed evaporation measured in soil boxes against \( (T_{o,max} - T_{d,max}) \) but the \( r^2 \) value was only 0.61. However, solar insolation on the sides of the boxes and the relatively shallow depth of soil may have adversely affected their results. Previous to our study Eq. [1] had not been tested against measured evaporation and could not be considered well validated.

The overall goal of this study was to make several changes to the Ben-Asher EBM, in an effort to overcome some assumptions that we thought limiting, and to test the original and modified EBMs against a data set of daily bare soil evaporation measured using microlysimeters. Modifications were limited to those that would not require measurements beyond those that could be taken with a hand held infrared thermometer and a single automated weather station.

THEORY

To provide a basis for discussion of the needed modifications and to give details and discussion not previously published, we present the energy balance theory. Subtracting the energy balance equations for a dry
Long wave radiation from a surface at temperature \( H_o \) is described by the Stefan-Boltzmann law:

\[
L_oE = (G_o - G_d) + K_o(a_o - a_d) + (H_o - H_d) + (L_{o,\text{out}} - L_{d,\text{out}})
\]  \[2\]

where \( H \) is the sensible heat flux, \( G \) is the soil heat flux, \( L_oE \) is the latent heat flux, \( K_o \) is the solar (shortwave) radiation, \( L \) is the long wave radiation (all in \( \text{W m}^{-2} \)), and \( \alpha \) is the albedo. The subscript 'out' indicates outgoing long wave radiation. The sensible heat fluxes for dry and drying soils may be written as (Rosenberg et al., 1983, p.124):

\[
H_o = \rho C_p(T_o - T_d)D_{ho}
\]  \[3\]

\[
H_d = \rho C_p(T_d - T_o)D_{hd}
\]  \[4\]

where \( \rho \) is the air density (1.2 \( \text{kg m}^{-3} \)), \( C_p \) is the specific heat of air (1010 \( \text{J kg}^{-1} \text{K}^{-1} \)), \( D_{hi} \) is the exchange coefficient for sensible heat flux (\( \text{m s}^{-1} \)), \( T_o \) and \( T_d \) are the surface temperatures (\( \text{K} \)) of the dry and drying soils, respectively, and \( T_i \) is air temperature (\( \text{K} \)) at the 2-m reference height. Long wave radiation from a surface at temperature \( T (\text{K}) \) is described by the Stefan-Boltzmann law:

\[
L_{\text{out}} = \epsilon \sigma T^4
\]  \[5\]

where \( \epsilon \) is the Stefan-Boltzmann constant (5.67 \( \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4} \)) and \( \sigma \) is the emissivity (taken to be an average of 0.95). Re-writing the longwave radiation term in Eq. [2], we have:

\[
L_{o,\text{out}} - L_{d,\text{out}} = \epsilon \sigma(T_o^4 - T_d^4)
\]  \[6\]

Substituting Eqs. [3] through [6] into [2] and integrating gives the total evaporative flux for any period \( t_1 \) to \( t_2 \):

\[
\int_{t_1}^{t_2} L_oE dt = \int_{t_1}^{t_2} \left\{ (G_o - G_d) + K_m(a_o - a_d) + \rho C_p[(T_o - T_d)D_{ho} - (T_d - T_o)D_{hd}] + \epsilon \sigma(T_o^4 - T_d^4) \right\} dt
\]  \[7\]

Measurement, at many locations, of the variables in Eq. [7] is onerous. Accordingly Ben-Asher et al. (1983) made simplifying assumptions that eliminated the soil heat flux and solar radiation terms and reduced the measurements of temperature to measurements of maximum daily soil surface temperature. The first assumption was that for any diurnal period the integrated soil heat flux and short wave radiation terms were negligible:

\[
\int_{t_1}^{t_2} ([G_o - G_d] + K_m(a_o - a_d)) dt \ll \int_{t_1}^{t_2} L_oE dt
\]  \[8\]

resulting in an EBM of the form:

\[
\int_{t_1}^{t_2} L_oE dt = \int_{t_1}^{t_2} ([H_o - H_d] + (L_{o,\text{out}} - L_{d,\text{out}})) dt
\]  \[9\]

Although Fox (1968) showed the plausibility of Eq. [8] the validity of this assumption will be examined below.

With the assumptions that (i) the air temperature at reference height is everywhere the same, and (ii) the aerodynamic resistance to heat flux is everywhere the same, the equations for sensible heat flux may be subtracted:

\[
H_o - H_d = \rho C_p(T_o - T_d)D_{hi}
\]  \[10\]

thus eliminating the need to measure air temperature. Furthermore, Ben-Asher et al. (1983) assumed that wind speed was constant over a day so that \( D_{hi} \) was constant and, using data from Rosenberg (1974, Fig. 3.3), developed the relationship:

\[
D_{hi} = 0.0079 U^{0.96}
\]  \[11\]

where \( U \) is average daytime wind speed (\( \text{m s}^{-1} \)) at a reference height of 2 m. We will address the assumption of constant wind speed below. Equation [11] was developed for a 0.5-m tall sugar beet (Beta vulgaris L. ssp. vulgaris) crop. For bare soil and neutral atmospheric conditions, a better equation for \( D_{hi} \) is (Kreith and Sellers, 1975):

\[
D_{hi} = k^2U[\ln(z/z_o)]^2
\]  \[12\]

where \( z \) is the reference height (m), \( z_o \) is the roughness length (m), \( k \) is the von Kármán constant = 0.41, and \( U \) is the horizontal wind speed (\( \text{m s}^{-1} \)) at the reference height.

Examining Eq. [6], [9], [10] and [11] (or [12]), we see that with these assumptions only \( T_o \), \( T_d \) and wind speed need be measured in order to calculate latent heat flux on an instantaneous basis. However, instantaneous measurement of even these 3 variables is laborious. Accordingly, Ben-Asher et al. (1983) assumed that soil surface temperature could be approximated by a periodic function in time, \( t \):

\[
T(t) = \bar{T} + 0.5(T_{\text{max}} - T_{\text{min}})\sin(\omega t)
\]  \[13\]

where \( \bar{T} = (T_{\text{max}} + T_{\text{min}})/2 \) is the average temperature, \( 0.5(T_{\text{max}} - T_{\text{min}}) \) is the amplitude, and \( \omega = 2\pi/\tau \) is the angular frequency (radians per unit time). Also, \( T_{\text{max}} \) is the maximum temperature and \( T_{\text{min}} \) the minimum temperature in the period, and \( \tau \) is the period (24 h from midnight to midnight for daily \( E \)). For Eq. [13], \( t \) is time in the same units as \( \tau \) with \( t = 0 \) corresponding to the time when \( T(0) = \bar{T} \) and \( T \) is increasing (i.e., start of sine wave).

Using Eq. [13] to describe the instantaneous temperatures in Eq. [6] results in fourth-power sine functions. To avoid integration of these sine functions, it
is necessary to reduce the fourth-order temperature terms in Eq. [6] to first-order terms. Letting \( \Delta T = T_o - T_d \) and \( T_m = (T_o + T_d)/2 \), we have:

\[
T_o - T_d = (T_m + \Delta T/2)^4 - (T_m - \Delta T/2)^4
\]

or

\[
T_o - T_d = 4T_m^3 \Delta T + T_m(\Delta T)^3
\]

Thus the outgoing longwave radiation balance is:

\[
L_{o,\text{out}} - L_{d,\text{out}} = \sigma 4T_m^3 \Delta T[1 + (\Delta T)^2/(4T_m^2)]
\]

and the approximation:

\[
L_{o,\text{out}} - L_{d,\text{out}} = 4 \sigma 3T_m^3(T_o - T_d)
\]

has an error of \( \epsilon(\Delta T)^3T_m^3 \). For typical soil temperature maxima and minima from our study the difference between Eq. [16] and [17], summed over one half day with 15 minute time steps, is only \( \approx 0.01\% \) (Evett, 1989, Appendix B).

Since Eq. [17] is still 3rd order in \( T_m \), Ben-Asher et al. (1983) assumed the quantity \( T_m^3 \) to be essentially constant across the range of \( T_m \) and redefined \( T_m = (T_o + T_d)/2 \) where \( T_o \) and \( T_d \) are the diurnal average surface temperatures of dry and drying soils, respectively. However, this latter assumption, when used with Eq. [17], results in \( \approx 15\% \) underestimation of the diurnal value of Eq. [16] (Evett, 1989, Appendix B).

Ben-Asher et al. (1983) assumed that the minimum temperatures \( T_{o,\text{min}} \) and \( T_{d,\text{min}} \) were equal. Introducing Eq. [10] and [17] into [9] and using Eq. [13] to represent \( T_o \) and \( T_d \), this assumption leads to:

\[
\int L_c Edt = \int \left[ \rho C_p D_H + 4 \sigma T_m^3 \right]
\times \left[ 0.5(T_{o,max} - T_{d,max})[1 + \sin(\omega t)] \right] dt
\]

Assuming that the latent heat of vaporization is essentially constant at 2.4 MJ kg\(^{-1}\), we can divide both sides of Eq. [18] to convert to depth of water equivalent in millimeter. Integrating gives:

\[
E_d = \int Edt = 6 \rho C_p D_H + 4 \sigma T_m^3
\times (1 + \pi 2^{0.5}(T_{o,max} - T_{d,max})/L_c
\]

Equations [19] and [1] are identical. The limits of integration were chosen, (i) by assuming that all energy flux terms would be in phase, (ii) by noting that the soil heat flux is positive (flow away from soil surface is positive) from \(-3\) h to \(9\) h given that Eq. [13] correctly describes the soil surface temperature over time, and (iii) by assuming that \( E \) is positive only when \( G \) is positive, and that negative values of \( E \) could be ignored. Note that the terms in Eq. [18] are constant except for the sine term. For the sine term, the zero hour is the time at which soil temperature is increasing and equal to the average diurnal temperature. For example, this might occur at about 0900 h making the time period of integration from 0600 h to 1800 h. However, the limits of integration are still -3 h to 9 h. Use of Eq. [19] requires only 3 measurements: daily average daytime wind speed, maximum reference dry soil surface temperature, and maximum drying soil surface temperature.

Since Eq. [19] performed poorly for Ben-Asher et al. (1983), we evaluated the assumptions leading up to Eq. [19]. Wind speed is generally not constant during a 24 h period and thus should be averaged on a smaller time scale such as 0.5 h. This is no extra measurement burden since a weather station is needed to measure wind speed for the average daily value. A smaller time scale requires numerical integration which is easily accomplished on a portable computer. Moreover, if we use the EBM represented by Eq. [9], numerical integration obviates the 15% underestimation of the long wave radiation term caused by the assumptions leading to Eq. [17] and eliminates the need to assume that \( L_c \) is constant.

Equation [12] (or [11]) is valid only within the internal boundary layer, a layer extending from the ground upward within which the momentum flux should be independent of height and within which a logarithmic wind profile, characteristic of the underlying surface, should develop. However, the reference dry soil was in a small, isolated container (0.3 m diameter in this study). The thickness of the fully adjusted layer over the reference dry soil was only \( \approx 0.5 \) cm (Rosenberg et al., 1983, Eq. 4.7) so the air temperature relative to the reference soil would have to be measured at \(< 0.5\)-cm height in order to be useful in Eq. [3], whereas weather stations commonly measure wind speed at 2- or 3-m height. Also, during the day, the reference dry soil was a small, relatively hot area with relatively cold air flow above. These circumstances suggest that buoyancy (air density) effects would dominate in sensible heat transfer from the reference soil to the atmosphere, in which case the effect of wind speed on the transfer coefficient might be reduced.

If different transport coefficients apply to the drying soil and the reference dry soil, then Eq. [10] cannot be used to combine the sensible heat flux terms in Eq. [9] and air temperature cannot be excluded from the data needed for the model. However, since a weather station is already required for wind speed measurements it is easy to acquire air temperature as well. An empirical transfer coefficient function for the reference dry soil, \( D_{H,0} \), can be defined in terms of the wind speed, \( U \) (m s\(^{-1}\)), at reference height, \( z \):
where $c_0$ and $c_1$ are fitted parameters (unitless). We could not determine the values of these parameters a priori although we expected the value of $c_1$ to be much lower than unity.

Substituting Eq. [3], [4] and [6] into Eq. [9], one obtains an EBM including separate terms for sensible heat flux from dry and drying soil and the fourth-order terms for longwave radiation:

$$ E_d = \int_{t_1}^{t_2} E dt = \int_{t_1}^{t_2} [\rho C_p (T_a - T_d) D_{Ho} dt - \rho C_p (T_d - T_a) D_{Hd} dt + \epsilon_a (T_a^4 - T_d^4)]/L_{et} dt $$

We took $t_1$ and $t_2$ to be the times of weighing of microlysimeters on successive days. This model requires instantaneous values of $T_a$, $T_d$, $T_s$ and $U$ but if it is numerically integrated on a suitable time step, e.g., 0.5 h, average $T_s$ and $U$ values for each time step may be interpolated from weather station data. The sine wave approximation of Eq. [13] combined with the assumption that $T_{a,min} = T_{d,min}$ provides an easy way to model the diurnal variation of $T_a$ and $T_d$ but this model has not been tested in this context.

Finally, the assumption described by [8], that the integrated short wave radiation and soil heat flux terms were negligible relative to evaporation, has not been well tested. Ben-Asher et al. (1983) presented data showing that the left-hand side of [8] could be as large as 15% of $E_d$ when evaporation rates were high and become an even larger percentage of $E_d$ as the soil dried.

Our study had several objectives. First, we tested the sine wave approximation for soil surface temperature (Eq. [13]) and developed an alternative method for modeling $T_a$ and $T_d$. Second, we determined the values of the parameters $c_0$ and $c_1$ in Eq. [20] describing $D_{Ho}$. Third, we compared the evaporation estimates from Eq. [19] (model of Ben-Asher et al., 1983) and [21]. Fourth, we simulated values of the neglected short wave radiation and soil heat flux terms and evaluated the impact of omitting them from the model.

**MATERIALS AND METHODS**

Field experiments were conducted at the University of Arizona’s Marana Agricultural Center (626-m elevation above mean sea level, 32.5° N lat) = 50 km northwest of Tucson. A 1-ha area was used in Field E-2 under the second span of a lateral-move sprinkler with low-pressure circular spray nozzles. The soil is a Pima clay loam in the fine-silty, mixed, thermic family of Typic Torrifuvents (Post et al., 1978). Experiment 1 was conducted in March and April 1985 and Experiment 2 in November and December 1986. The field was clean tilled and the surface nearly level and flat in the area under consideration.

For both experiments, two reference dry soils were established by packing plastic buckets (29-cm i. d., 34-cm deep) with air-dry soil (sieved to 2 mm) to a bulk density of 1.6 Mg m⁻³ and burying them in the field so that the soil surfaces in the buckets were at the same elevation as the field surface. Burial occurred 2 weeks before the experiment began, to allow the reference soils to equilibrate thermally. The buckets were sealed during irrigation to prevent wetting.

Microlysimeters were also used in both experiments. Microlysimeters are tubes inserted into the soil, removed with the soil inside intact, and then capped at the bottom. They are replaced in holes in the soil such that the surface of the soil in the tube, the top of the tube, and the surrounding soil surface are all at the same elevation. They are periodically removed and weighed in order to estimate evaporation. We have shown elsewhere that ML walls should be made of material with low thermal conductivity to prevent unwanted heat transport between the soil surface and ML bottom, and that the bottom cap should be of a material with high thermal conductivity to ensure that the ML is thermally coupled with the underlying soil (Evett et al., 1995). For the conditions reported here, the 8.15-cm i. d., 30-cm deep white polyvinylchloride (PVC) plastic MLs we used are adequate for continuous measurement of evaporation over at least 9 d after an irrigation without replacement (Evett et al., 1995).

**Experiment 1**

In March and April 1985, surface temperature measurements of the soil inside MLs, the reference dry soil, and two adjacent field soil locations were used to test the sine wave approximation and assumption that $T_{a,min} = T_{d,min}$. Microlysimeters were pushed into the soil before an irrigation as described by Evett et al. (1995).

Soil surface temperatures were measured in 4 MLs, with infrared thermometry as described below, and using thermistors as described by Evett et al. (1995). The surface thermistor was pushed through the soil from 1 cm below the surface until the tip of the thermistor had just begun to disturb the surface. Our intent was to measure as close to the surface as possible without exposing the thermistor to direct solar radiation. In addition, one thermistor was installed just beneath the surface in one reference dry soil and thermistors were similarly installed just beneath the surface at 2 adjacent locations in the field.
The Mls were separated by = 0.2 m and the adjacent field locations were withing 2 m of the microlysimeters. The reference dry soil was = 2 m away from the MLs. Thermistors were installed on Day of the Year 92, 1985, the day after irrigation. The thermistors were scanned every 15 minutes by two data loggers (Model 21X, Campbell Scientific, Inc., Logan, UT)\(^1\), which recorded the averages of six readings taken at 10-s intervals.

Thermistors were modified to be water resistant and were calibrated ensemble by placing in ice water and letting the bath warm to room temperature and then placing in boiling water and letting the bath cool to room temperature. All thermistors read to within 0.25 °C of the mean at all temperatures.

Infrared thermometer (Model 110 with 3° field of view, Everest Interscience, Inc., Fullerton, CA) soil surface temperature measurements were taken, just before dawn and between 1300 and 1330 h, on the surfaces of all MLs, the reference dry soils and the two field locations. The small ML diameter forced readings to be taken vertically and so vertical readings were taken on all surfaces. Measurements were recorded (Model 516-32, Omnidata Polycorder, Logan, UT) and the average and standard deviation of 10 readings were calculated automatically. If the standard deviation was more than 0.1 °C, the measurement was repeated. The infrared thermometer was calibrated against a blackbody.

**Experiment 2**

Data needed for finding the parameters in Eq. [20] and for comparing evaporation estimates from Eq's. [19] and [21] were collected for 10 days after an irrigation of 0.024 m on 24 November 1986 (Day of the Year 328). Evaporation was measured using 57 MLs at locations scattered over the 1-ha field under the same sprinkler system as was used in Exp. 1. The MLs were driven into the ground the day before irrigation. Extraction, capping, and weighing was finished by 0914 h on the day after irrigation. The ML bottoms were closed with thin non-stretching plastic tape in order to minimize interference with soil heat flux. On subsequent days, MLs were weighed during the first hour after sunrise (between 0700 and 0800 h).

Microlysimeters were weighed to a precision of 0.001 kg (equivalent to 0.00019-m depth of water) with a portable electronic scale (Model LZ-5000, Yamato Scientific Co., Tokyo). The balance was fit into the bottom of a modified 20 L bucket which served both to transport the balance around the field and as a wind shield during weighing. With this system, all MLs could be extracted, weighed and returned to their holes in a 1-hr period.

Soil surface temperatures of MLs and the reference dry soils were taken daily before dawn and between 1300 and 1330 h by infrared thermometry as discussed above. Soil temperatures were measured by a Model 21X data logger at the surface, as described above, and at 15- and 30-cm depths by thermistors at two mid-field locations and recorded every 15 minutes on cassette tape. Two weather stations were set up, one each at the southeast and northwest corners of the field. Each station measured wind speed (at 3 m); and relative humidity, air temperature, and solar radiation (all at 2 m). Wind speeds were corrected to the air temperature reference height of 2 m, assuming a logarithmic wind profile. Data were recorded on magnetic tape at 15-min intervals around the clock. The field was flat tilled and irrigated and rained on several times before measurements began, further flattening the surface. The roughness length, \(z_0\), was taken as 0.0003 m (Kreith and Sellers, 1975).

**Simulation of Neglected Terms**

Because it provides a complete physical description of soil surface energy and water balances, the mechanistic energy and water balance model ENWATBAL (Evett and Lascano, 1993) was used to simulate the short wave radiation and soil heat flux terms in the left-hand side of [8] for Days of the Year 329 through 338, Exp. 2. The model was parameterized with soil hydraulic property data gathered at Marana by Stockton (1971) and Coelho (1974). The data were fit to Mualem's (1976) equation for hydraulic conductivity as a function of soil water potential, and to van Genuchten's (1980) equation relating soil water content to potential, using the RETC program (RETexture Curve, van Genuchten et al., 1991). The relationship between soil water content of the top layer (finite difference layer) and soil albedo was parameterized with data from Idso et al. (1974) collected on a similar clay loam soil at Phoenix, Arizona. For the drying soil, the initial profiles of soil water content and temperature as well as the half-hourly input data for wind speed, solar radiation, air temperature and dew point temperature were calculated from our measured data. For simulation of dry soil energy balance, the initial soil water contents throughout the profile were set to an air-dry value of 0.01 m\(^3\) m\(^{-3}\) and initial soil temperatures were set equal to those for the drying soil.

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\(^1\)The mention of trade or manufacturer names is made for information only and does not imply any endorsement, recommendation, or exclusion by USDA-Agricultural Research Service or University of Arizona.
RESULTS

Experiment 1: Estimating Temperature Depression.

Microlysimeter surface temperatures measured by thermistor were subtracted from the corresponding reference dry soil temperatures in order to examine the actual temperature depression, \((T_o - T_d)\). There were large differences between measured temperature depression and that predicted using the sine wave approximation of Eq. [13] to calculate \(T_o\) and \(T_d\) even on days when the diurnal plot of measured \((T_o - T_d)\) resembled half a sine curve (Fig. 1, top) (The zero hour in Eq. [13] was set to occur at 0600 h, i.e., approximately sunrise). On days when afternoon clouds obscured the sun, the differences in shape were more dramatic (Fig. 1, bottom). The actual value of \((T_{o,max} - T_{d,max})\) as well as the diurnal trend of \((T_o - T_d)\), could be much different from that calculated using Eq. [13] and the surface temperatures measured at 1330 h. Although predawn ML and reference dry soil temperatures were generally within 1.5 °C of each other, the timing of the corresponding minimum in \((T_o - T_d)\) was not well reproduced using Eq. [13]. Considering that the functions shown in Fig. 1 will be integrated over time, it is obvious that the function due to the sine wave approximation will sum to a quite different and usually larger value than that based on measured values.

Collection of actual \((T_o - T_d)\) values on a useful interval (e.g., 1 h) for all MLs was not done since it is expensive either in labor or equipment. Therefore it was desirable to have some method of estimating \((T_o - T_d)\) from intensive automated measurements at one or two locations coupled with extensive measurements at all field locations only once or twice a day (i.e., our predawn and midday infrared thermometer readings).

Thermistor data showed that the surface soil temperature at the two field locations closely matched surface temperatures of the MLs, differing mainly in maximum, minimum and a slight phase shift. Regressions of ML temperatures vs. field soil temperatures showed very good correlation for all cases \((r^2 > 0.99)\). However, surface temperature maxima and minima change with field position so, as expected, the slopes and intercepts from the regressions were not usually unity and zero, respectively.

A scaling procedure was used to convert field soil temperatures \((FT)\) to estimates of ML temperatures, \(T_d\). Scaling was based on a linear relationship between field soil temperature and ML temperature defined such that maximum and minimum estimated ML temperatures equaled the maximum and minimum ML temperatures as measured by infrared thermometer \((MLIR_{\text{max}}\) and \(MLIR_{\text{min}}\), respectively). The relationship was:

\[
T_d = b_0 + b_1(FT) \tag{22a}
\]

where

\[
b_1 = (MLIR_{\text{max}} - MLIR_{\text{min}})/(FT_{\text{max}} - FT_{\text{min}}) \tag{22b}
\]

\[
b_0 = MLIR_{\text{max}} - b_1(FT_{\text{max}}) \tag{22c}
\]

and where \(FT_{\text{max}}\) and \(FT_{\text{min}}\) were the field soil temperatures measured by thermistor at the time of infrared thermometer measurement of maximum and minimum temperatures, respectively. For the period from 0700 to 1330 h MST, the value of \(FT_{\text{max}}\) was taken to be the field soil temperature at 0700 h MST that day. For the period from 1330 to 0700 h MST on the next day the values of \(FT_{\text{min}}\) and \(MLIR_{\text{min}}\) were defined as the corresponding temperatures measured on the next day.
Equations [22a], [22b], and [22c] were used to estimate temperatures for MLs that had been instrumented with thermistors. Regression of estimated vs. measured temperature showed very good correlation ($r^2 > 0.99$) for all cases and the slopes and intercepts of the regression lines were close to unity and zero, respectively. Deviations from slopes of unity and intercepts of zero were due only to the fact that the infrared temperatures measured on the MLs were usually not exactly the same as the temperatures measured by thermistors (due, for example, to changing cloud cover and different averaging intervals). The shape of the temperature curve was very well reproduced (Fig. 2) and for this reason, and since the estimated maximum and minimum temperatures were equal to the extremes as measured by infrared thermometer, the procedure was considered to predict accurately ML surface temperatures as they would be measured by infrared thermometry.

An analogous scaling procedure was used to estimate reference dry soil temperatures, $T_o$:

$$T_o = b_0 + b_1(FT)$$  \[23a\]

where

$$b_1 = (RDSIR_{max} - RDSIR_{min})/(FT_{max} - FT_{min})$$  \[23b\]

$$b_0 = RDSIR_{max} - b_1(FT_{max})$$  \[23c\]

Figure 2. Example of actual microlysimeter surface temperatures measured by thermistor (crosses) vs. those estimated using Eq. [22] (solid line), Day of year 93, 1985.

**Experiment 2: Fitting of Empirical Transfer Function Parameters**

A search was conducted for the best-fit parameters in Eq. [20] describing the sensible heat flux transfer coefficient, $D_{Ho}$, for the reference dry soil. Equation [21] was numerically integrated by the Euler method with a quarter-hour time step using data from Exp. 2. Data from Day 329 were omitted from this and subsequent analyses since drainage from some microlysimeters was observed during this first day after the irrigation. Half-hourly averages of wind speed were used and interpolated to the quarter hour. For each ML and the reference dry soil, soil surface temperatures, $T_d$ and $T_o$, were scaled from quarter-hourly mean temperatures, $FT$, measured at a mid-field location using Eq. [22a], [22b], and [22c]; and [23a], [23b], and [23c]. Integration began at the time of first weighing and was started and stopped at the midpoint of the weighing period on every day thereafter to give daily estimates of evaporation. Negative values of evaporation were not summed since the dew point was never reached during either experimental period. Equation [12] was used to describe $D_{Ho}$. Note that our numerical integration scheme eliminates the assumptions of Ben-Asher et al. (1983) that (i) all energy flux terms were in phase and, (ii) integration should only be done over the period for which soil heat flux was positive.

The values of $c_0$ and $c_1$ were varied from 0.010 to 0.001 and from 1 to 0, respectively. For every combination of $c_0$ and $c_1$, values of daily evaporation were estimated for the 57 MLs for Days 330 through 338 and the sum of squared error (SSE) was calculated for measured vs. estimated evaporation. The lowest SSE resulted from values of 0.0038 for $c_0$ and 0.17 for $c_1$ and the best fit transfer coefficient function was thus:

$$D_{Ho} = 0.0038 U^{0.17}$$  \[24\]

The $r^2$ value was 0.82 for regression of estimated vs. measured evaporation (Table 1).

The low value of the exponent in Eq. [24] indicates that wind speed had little effect on sensible heat flux from the dry soil. This result supports the idea that buoyancy effects were of much greater importance for the reference dry soil than for the field as a whole. Equation [24] can be considered the dry soil transfer coefficient function for unstable conditions since only positive half hourly values of evaporation were summed while finding
the best-fit coefficients. For the most part, positive values occurred when the air was unstable.

Test of Original Energy Balance Model

Equation [19] was used to estimate evaporation for the 57 MLs of Exp. 2 using the values of $T_{\text{a,max}}$ and $T_{\text{d,max}}$ measured by infrared thermometry and the mean daily wind speeds for the 9 d after irrigation. Equation [12] was used to describe the exchange coefficient for sensible heat flux, $D_H$, in Eq. [19]. Regression of measured vs. estimated evaporation resulted in an $r^2$ value of 0.81 and a slope closer to 1 but a more negative intercept than for Eq. [21] (Table 1). Using Eq. [11] for $D_H$ resulted in a correlation coefficient of 0.79 and slope of 0.57 for regression of measured vs. estimated evaporation.

Table 1. Equations for regression of actual evaporation ($E_a$) vs. that estimated by the models ($E_{\text{est}}$), Exp. 2.

<table>
<thead>
<tr>
<th>Equation Details</th>
<th>$E_a$</th>
<th>$E_{\text{est}}$</th>
<th>$r^2$</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equations [21], [12] and [24]:</td>
<td>$E_a = -0.84 + 1.88 E_{\text{est}}$</td>
<td>$r^2 = 0.82$</td>
<td>3B</td>
<td></td>
</tr>
<tr>
<td>Equations [19] and [12]:</td>
<td>$E_a = -1.10 + 1.23 E_{\text{est}}$</td>
<td>$r^2 = 0.81$</td>
<td>3A</td>
<td></td>
</tr>
<tr>
<td>Equations [19] and [11]:</td>
<td>$E_a = -0.76 + 0.45 E_{\text{est}}$</td>
<td>$r^2 = 0.78$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENWATBAL model:</td>
<td>$E_a = 0.35 + 0.84 E_{\text{est}}$</td>
<td>$r^2 = 0.96$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eq’s [21], [12] and [24], corrected:</td>
<td>$E_a = -0.59 + 1.19 E_{\text{est}}$</td>
<td>$r^2 = 0.81$</td>
<td>3D</td>
<td></td>
</tr>
<tr>
<td>Eq’s [19] and [12], corrected:</td>
<td>$E_a = -1.25 + 0.89 E_{\text{est}}$</td>
<td>$r^2 = 0.79$</td>
<td>3C</td>
<td></td>
</tr>
</tbody>
</table>

Effect of Neglected Terms

The ENWATBAL model provided excellent estimates of daily evaporation (Table 1). Daily values of terms on the left-hand side of Eq. [8] resulting from ENWATBAL simulations of the energy and water balances of the dry and drying soils are shown in Table 2. The short wave radiation term, $K_{\text{sw}}(\alpha - \alpha_d)$, was positive on all days due to lower albedos for the drying soil, and was larger on the first few days after irrigation due to the much lower albedo of the drying soil then. The values of the soil heat flux term, $(G_o - G_d)$, on all but Day 337 were negative indicating greater heat flux toward the soil surface in the drying soil. The greater flux was due to the greater thermal conductance of the wet soil. Net daily heat flux was toward the soil surface for both the dry and drying soils on most days, but magnitudes were much lower for the dry soil, especially in the first few days after irrigation. In this experiment the short wave radiation and soil heat flux terms nearly canceled and their sum (left hand side of Eq. [8]) was always < 0.3 mm water equivalent. As a percentage of measured evaporation the left-hand side of Eq. [8] ranged from 0 to 9.2%.

Despite the relatively low values of the summed neglected terms, correction of the modeled evaporation by adding the correction for each day to each evaporation estimate caused important shifts in the regression intercepts and slopes (Table 1 and Fig. 3). The slight decline in coefficients of determination can be explained by noting that the average correction for a particular day is really not applicable to the data that is far from the mean evaporation for that day. That is, a ML that shows evaporation much lower than the daily mean is probably drier and has lower soil heat flux and higher albedo than one that shows evaporation much higher than the mean. The fact that $E$ estimates from the modified model (Eq. [21]) are brought closer to the 1:1 line, while those from the original model are moved farther away, reflects the more physically complete nature of the modified model.

![Figure 3](image-url) Comparisons of estimated vs. measured evaporation (dotted line is 1:1 line): (A) Ben-Asher et al. (1983) model using Eq. [12] for sensible heat flux ($D_H$); (B) modified model (Eq. [21] with Eq. [22] and [23] for the reference dry soil temperature ($T_o$) and the drying soil tempearture ($T_d$) and Eq. [12] and [24] for $D_H$ for the drying soil ($D_{\text{d}}$) and the reference dry soil ($D_{\text{r}}$); (C) neglected terms (see left-hand side of Eq. [8]) added to (A); and (D) neglected terms added to (B).
Table 2. Midnight-to-midnight integrated values of shortwave radiation, $K_o(\alpha_o - \alpha_d)$, and soil heat flux, $G_o - G_d$, at the soil surface from the ENWATBAL model for Days of the Year 329 - 338, 1986; and correction factors, Sum, for the energy balance models. Subscripts o and d refer to the reference dry soil and the drying soil, respectively, and $E_d$ is measured evaporation.

<table>
<thead>
<tr>
<th>Day of the year</th>
<th>$K_o\alpha_o$</th>
<th>$K_o\alpha_d$</th>
<th>$K_o(\alpha_o - \alpha_d)$</th>
<th>$G_o$</th>
<th>$G_d$</th>
<th>$G_o - G_d$</th>
<th>$K_o(\alpha_o - \alpha_d)$</th>
<th>$G_o - G_d$</th>
<th>Sum</th>
<th>$E_d$</th>
<th>Proportion of $E_d$</th>
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</thead>
<tbody>
<tr>
<td>329</td>
<td>3.19</td>
<td>1.89</td>
<td>1.30</td>
<td>0.14</td>
<td>2.16</td>
<td>-2.02</td>
<td>0.53</td>
<td>-0.82</td>
<td>-0.29</td>
<td>3.4</td>
<td>1.8</td>
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<td>330</td>
<td>3.09</td>
<td>1.86</td>
<td>1.23</td>
<td>0.21</td>
<td>1.58</td>
<td>-1.37</td>
<td>0.50</td>
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<td>1.11</td>
<td>0.02</td>
<td>0.71</td>
<td>-0.69</td>
<td>0.45</td>
<td>-0.28</td>
<td>0.17</td>
<td>2.2</td>
<td>7.7</td>
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<tr>
<td>332</td>
<td>2.97</td>
<td>2.54</td>
<td>0.43</td>
<td>-0.22</td>
<td>0.46</td>
<td>-0.68</td>
<td>0.18</td>
<td>-0.28</td>
<td>-0.10</td>
<td>2.7</td>
<td>3.7</td>
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<tr>
<td>333</td>
<td>2.70</td>
<td>2.58</td>
<td>0.12</td>
<td>0.22</td>
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<tr>
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<td>2.97</td>
<td>2.85</td>
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<td>1.03</td>
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<td>-0.25</td>
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<tr>
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<td>-0.17</td>
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<td>-0.02</td>
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<tr>
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<td>0.01</td>
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<td>-0.11</td>
<td>1.2</td>
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</tbody>
</table>

**SUMMARY**

Several changes were made to the model of Ben-Asher et al. (1983) in an effort to improve performance. In addition to wind speed required by the original model, automated data collection of air and soil surface temperatures was instituted. A scaling procedure produced diurnal curves of temperature at all locations using only the predawn and midday manual infrared thermometer measurements at these locations and automated measurements of surface temperature at one point in the field. The resulting diurnal curves were considerably more accurate than those from a sinusoidal diurnal soil temperature equation used in the original model. A best-fit function, for the transfer coefficient for sensible heat flux from the reference dry soil, was relatively insensitive to wind speed, thus supporting the idea that sensible heat flux from the reference soil was dominated by free convection and should be modeled differently than sensible heat flux from the field soil.

Despite the model changes, and numerical integration with 0.25-h time steps rather than 24-h time steps, there was little improvement in the model’s ability to predict the variability of evaporation. Both the original and modified energy balance models were reasonably good estimators of evaporation. However, addition of the neglected solar radiation and soil heat flux terms brought the evaporation estimates of the modified model closer to a 1:1 relationship with measured evaporation while the original model’s estimates deviated further from a 1:1 relationship when corrected. In this study the summed neglected terms were always < 10% of daily $E$, but under other conditions their sum could be a much larger percentage of $E$. Tests of the models represented by Eq’s. [19] and [21] should be conducted under other conditions to verify our results. For instance, our Exp. 2 was conducted in the fall while the soil was cooling, and the neglected terms (short wave radiation and soil heat flux) had opposite signs and so nearly canceled on some days. In the spring and early summer, the neglected terms could sum to much larger values. Also, we have some evidence (not presented here) that warmer and more advective conditions than reported here favor the modified model over the original.

Work on estimation of soil heat flux and soil albedo in both the reference dry soil and drying soils is necessary for further model improvement. It may be that a more complete mechanistic model such as ENWATBAL can be used to estimate the field average evaporation while the modified EBM is used to add the spatial variability component, but further research will be needed to investigate this possibility. The BASIC source code for numerical integration of Eq. 21 is available from the first author.

**REFERENCES**


