A Double Tube Method for Measuring Hydraulic Conductivity of Soil in Situ Above a Water Table

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ABSTRACT

A method for measuring hydraulic conductivity of soil in situ above a water table is proposed. The method consists essentially of saturating a limited soil region below an auger hole in which two concentric tubes are placed. Hydraulic conductivity is calculated from measurements of the rate of change of the water level in the inner tube. The procedure is based on separating the flow between the tubes due to different water levels in the tubes from the total flow, which includes continued intake of water by the soil. Dimensionless parameters describing the flow component due to different water levels in the tubes are used in the calculation of the conductivity. These parameters were determined with a resistance network analog for three soil conditions, a uniform soil, a soil underlain by material of much lower conductivity, and a soil underlain by material of much higher conductivity. The depth to the material of different conductivity was taken as a variable. The method is illustrated with an example.

In principle, it is possible to apply the various methods for determining hydraulic conductivity of soil below a water table (3, 6) to the above-the-water-table case by creating a high enough artificial water table. This will, however, not be practical in most instances and various research efforts have been directed to finding methods for determining saturated conductivity of soil in situ above a water table. Procedures whereby the conductivity is derived from intake rates in initially unsaturated soils, for instance the shallow-well pump-in method, have the disadvantage that the flow systems from which K must be calculated are influenced by boundary conditions and unsaturated conductivity characteristics, which are both difficult to determine. For cylinder infiltrometers, intake rates can be numerically smaller or larger than the saturated conductivity, depending on the internal drainage conditions (1, 4) and the unsaturated conductivity characteristics of the soil (1). Consequently, reliable conductivity data generally cannot be obtained from intake rate measurements alone. Tensiometers and piezometers were used by Winger3 to determine when gradients below a cylinder had reached unity for numerical equality of intake rate and conductivity.

This paper describes a method whereby the conductivity is determined from a flow system of known behavior, the auger hole being made up of a closed circular conduit through which water is added continuously to the tubes. When it is expected that the soil below the bottom of the hole is saturated to sufficient extent, the water level in the inner tube is raised or lowered. The water level in the outer tube is kept constant. The subsequent rate of fall or rise of the water surface in the inner tube is measured. From the simultaneous measurement of flow rates and water level differences thus obtained, the conductivity of the soil below the auger hole is evaluated by means of dimensionless flow factors. These factors express flow rate in relation to soil conductivity, system geometry and water level differences in the tubes. The flow factors were determined with a resistance network analog and are presented in the form of graphs.

The time required for sufficient saturation below the bottom of the auger hole is difficult to predict and largely a matter of judgment and experience. However, sufficient saturation can be verified after one or more K-measurements, for instance by consistency in the results of consecutive tests or by comparing calculated approximate depths of saturation with required depths (see Discussion).

NOMENCLATURE

d = depth of penetration of inner tube into bottom of auger hole or pit
D = depth of slowly permeable material below bottom of auger hole
Dn = depth of highly permeable material below bottom of auger hole
F = flow factor, symbol for the dimensionless parameter Qn/(wKHn)
H = distance of water level in inner tube below (positive) or above (negative) water level in outer tube
H = H at balanced flow

Figure 1—Geometry of double tube method.

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\[ H = \text{distance of water in inner tube below (positive) or above (negative) the balanced-flow level (} H' = H - H_0) \]

\[ h_r = \text{depth of water in tubes during period of saturating soil below auger hole} \]

\[ I = \text{average intake rate of soil at bottom of inner tube (} \frac{L}{T} \text{)} \]

\[ K = \text{saturated hydraulic conductivity of soil below bottom of auger hole (dimensions } \frac{L}{T} \text{)} \]

\[ R_t = \text{radius of inner tube} \]

\[ R_s = \text{radius of standpipe on inner tube in which water level measurements are taken} \]

\[ Q_n = \text{flow, or flow component, leaving or entering through bottom of inner tube due to a difference } H \text{ between the water levels in inner and outer tube (} \frac{L}{T} \text{)} \]

\[ Q_i = \text{flow or flow component, leaving through bottom of inner tube due to intake (} Q_i = \text{intake rate)} \]

\[ t = \text{elapsed time} \]

**PRINCIPLES OF METHOD**

A dimensionless term, \( Q_{n,i} / (\pi K R_t^2) \), is introduced. This term, called the flow factor \( F_i \), is obtained as follows. The flow \( Q_n \) leaving or entering the inner tube through the soil below the bottom of the auger hole due to a difference \( H \) between the water levels in the tubes, is divided by \( \pi K R_t^2 \). The term \( Q_{n,i} / (\pi K R_t^2) \) thus obtained, expresses the flow between the tubes as an average vertical velocity in the inner tube per unit K. The term is then divided by \( H/R \), which is a geometry factor containing the driving force \( H \). The resulting dimensionless factor, \( F_i = Q_{n,i} / (\pi K R_t^2) \), was determined with a resistance network for various conditions of soil nonuniformity and system geometry. The results were expressed in graphs which are presented in the next section.

The determination of \( K \) consists in principle of obtaining a simultaneous measurement of \( Q_{n,i} \) and \( H \) in the field and computing \( K \) as

\[ K = \frac{Q_{n,i}}{2HR_t F_i} \tag{1} \]

where \( F_i \) is the flow factor \( Q_{n,i} / (\pi K R_t^2) \) evaluated from the appropriate graph in the next section.

In addition to the flow component \( Q_{n,i} \), which is caused by a water level difference \( H \) between the two concentric tubes, water will be leaving both tubes at the bottom of the auger hole due to water intake by the soil. Therefore, the net flow leaving or entering the inner tube at the bottom of the auger hole is the resultant of the flow component due to different water levels in the tubes and the component due to intake by the soil. These components were called "leakage" and "seepage," respectively, in a previous paper on determining canal seepage losses with cylindrical devices (2).

The problem now is to isolate, for a certain \( H \), the leakage flow component \( Q_l \) from the measured net flow \( Q \) to or from the inner tube. The assumption is made that the intake flow components during the period of the measurements are constant with respect to time and unaffected by changes in water level in the inner tube. This assumption will be valid if the measurements are carried out in a short period of time and if \( H \) is kept relatively small (7). If \( H = 0 \), the flow from the inner tube is entirely due to the intake at the bottom of the hole. With no external supply of water to the inner tube, the water surface in the inner tube will drop and establish itself at a level whereby the inflow component due to "leakage" equals the outflow component due to intake. The distance of this balanced-flow level below the constant water level in the outer tube is called the balanced-flow differential head, symbol \( H_0 \) (2).

The flow component due to intake at the bottom of the inner tube can, according to previous work with seepage meters in canal bottoms [equation (9) in (2)], be expressed as

\[ Q_l = \frac{2.3 R_t^2}{HR_t F_i} \log \frac{H_0 - H_0}{H_0 - H} \tag{2} \]

where \( R_t \) is the radius of the standpipe where the water level changes are measured, \( H_0 \) is \( H \) at \( t = 0 \), and \( H_0 \) is \( H \) at time \( t \). At balanced-flow conditions, the flow component due to different water levels equals \( Q_n \). Therefore, \( H_0 \) and \( Q_l \) can be used in equation (1) as an appropriate combination of \( H \) and \( Q_n \) for calculation of \( K \). In formula:

\[ K = \frac{Q_l}{\pi H_0 R_t F_i} \tag{3} \]

Substitution of equation (2) into (3) gives

\[ K = \frac{2.3 R_t^2}{HR_t F_i} \log \frac{H_0 - H_0}{H_0 - H} \tag{4} \]

Equation (4) can be applied to various combinations of \( H, t \), and \( H_0 \) obtained by field measurements of the falling and/or rising water level in the standpipe on the inner tube. Each combination of \( H \) and \( t \) should yield the same \( K \). It is also possible to plot \((H_0 - H_0)/(H_0 - H_0)\) on the logarithmic scale of semilogarithmic paper against \( t \). The slope of the best fitting straight line through the origin, i.e., 0, is then substituted for the term \((L/T) \log(H_0 - H_0)/(H_0 - H_0)\) in equation (4).

The terms \( H_0 - H_0 \) and \( H_0 - H_0 \) in equation (4) represent the distance of the water level in the inner tube from the equilibrium level at balanced flow conditions. A simplification in processing the field data can therefore be obtained if the falling and/or rising water level measurements are taken with respect to the equilibrium level in the inner tube. Equation (4) then reduces to

\[ K = \frac{2.3 R_t^2}{HR_t F_i} \log \frac{H_0}{H_0} \tag{5} \]

where \( H_0 \) represents the water level elevation in the inner tube with respect to the level at balanced flow (\( H = H - H_0 \)). Equation (5) can again be directly applied to the field measurements, or \( H_0 \); \( H_0 \) can be plotted against \( t \) on semilogarithmic paper to determine the slope of the best fitting line for use in the calculation of \( K \).

The above principle of equating the "leakage" flow component due to a difference \( H_0 \) between the water levels in the two concentric tubes to the intake flow from the inner tube, is applicable regardless of nonuniformity or direction of the local intake rates at the bottom of the auger hole (2). For the special case of zero intake flow during the measurements, \( H_0 = 0 \) and \( H = H_0 \).

**FLOW FACTORS, AREA OF SAMPLING, AND RELATIVE TUBE DIAMETERS**

Values of \( F_i \) were determined from flow rates between the inner and the outer tube through the soil for various conditions of system geometry and soil conductivity. An electrical resistance network was used in these determinations.

To eliminate, for a given diameter of the inner tube, the effect of the actual diameter of the outer tube, an infinite radius was selected for the latter. The radial infinity was simulated on the network analog by terminating the network with a conductor at a suitable radial distance from the symmetry axis. After the evaluation of \( F_i \), the value to which the radius of the outer tube could be reduced for \(<10\%\) difference from the flow rates at infinity conditions, was determined (section 3, Area of sampling and relative tube diameters).

The subdivided radial network arrangement shown on the examples of complete flow systems, was calculated similar to a procedure presented in a previous paper (1).

Two conditions of soil nonuniformity were considered in this study. The soil below the bottom of the auger hole may be underlain by material of much lower conductivity, or by material of much higher conductivity. The depth below the auger hole of the material of different con-
ductivity, i.e., D for the less permeable material and \( D_p \) for the more permeable material, was varied. For large values of \( D \) or \( D_p \), the medium reduces to a uniform soil.

1. Soil underlain by material of much lower conductivity.—The stratum of lower conductivity in this case may consist of less porous soil with a lower saturated conductivity, or of nonwetted coarser material. Flow rates per unit \( K \) and \( H \) were determined in relation to \( d \) and the distance \( D \) to the less permeable material, which was taken as impermeable in the analyses. The results are expressed in dimensionless form in figure 2. If \( d/R_c = D^2/R_c \), the inner tube reaches the impermeable layer and \( Q_H = 0 \).

Examples of flow systems are presented for a medium that is uniform to great depth (figure 3) and a medium with an impermeable layer at a small distance below the bottom of the auger hole (figure 4). The systems in figures 3 and 4 show only the flow pattern due to different water levels in the tubes. For examples of flow patterns under the combined effect of different water levels in the tubes and of continued intake by the soil, reference is made to (2).

The top of the flow systems in figures 3 and 4 represents the bottom of the auger hole. The inner tube is shown as a heavy line at 21 cm. from the center line. Both systems have a termination strip at 81 cm. from the center representing infinite horizontal extent. Thus, the outer tube or the walls of the auger hole or pit are not shown. The actual length dimensions in the flow systems are relatively large and are applicable to seepage meters in canals or large (inner) tubes in open pits. Dividing all lengths by a factor three yields dimensions that would be more realistic for double tube installations in auger holes. An exception must be made for \( d \), which will generally become smaller with respect to \( R_c \) when \( R_c \) is reduced.

2. Soil underlain by material of much higher conductivity.—Figure 5 shows the results of the flow analyses when the soil is underlain by much more permeable material, in this case material of infinite hydraulic conductivity. When \( d/R_c = D_p/R_c \), the inner tube functions as a permeameter. Equation (5) then reduces to the variable-head permeameter equation and \( F_2 = R_c/D_p \).

An example of a flow system for \( D_p = 16 \) cm. is shown in figure 6. The flow direction in this example is opposite the flow direction in figures 3 and 4. Otherwise, the remarks in the previous section also apply to the flow system of figure 6.

3. Area of sampling and relative tube diameters.—Figures 2 and 5 show that the effect of increasing \( D \) or \( D_p \) on \( Q_H \) becomes very small after \( D \) or \( D_p \) has reached a value of approximately 2 \( R_c \). Beyond \( D = D_p = 3 \) \( R_c \), the curves are practically identical and \( Q_H \) is almost the same as for a uniform medium of infinite depth. For practical purposes, therefore, saturation to a depth of approximately 2 \( R_c \) below the bottom of the hole will generally be sufficient to carry out the necessary measurements. Also, the height of the soil sample involved will not exceed 2 \( R_c \).

An idea of the horizontal size of the sample and the direction in which the conductivity is determined can be obtained by examining the area between the 0 and 80 equipotentials in figures 3 and 4 and between the 100 and 200 equipotentials in figure 6. In these areas, 80% of the total head difference is dissipated. Except for conditions of less permeable material at small distance below the bottom of the inner tube, i.e., small \( D-d \), the area
PROPOSED PROCEDURE

In theory, lining the auger hole with the outer tube is not required and the inner tube could be placed as a single tube in an open pit. From a practical standpoint, however, an outer tube penetrating the bottom of the hole may be preferred to minimize the volume of water needed for saturation, or to prevent the walls of the hole from caving in. After installation of the outer tube, which may be done similarly to the technique presented in (5), the inner tube is placed on the bottom of the hole. The inner tube should be equipped with guides, fingers, or other devices to obtain centering and stability. A layer of gravel or sand is now placed in both tubes to reduce soil puddling and water application is started to saturate the soil below the bottom of the hole. During this saturation period, the water surfaces in both tubes are initially held constant at the same level. This may be achieved by using a shorter pipe section for the inner tube than for the outer tube, so that the water stands above the inner tube. After saturation has progressed for some time, a standpipe with radius R, for faster rates of water level changes, is connected to the inner tube. The inner tube is then pushed down to the desired depth below the bottom of the auger hole. This depth should be minimized to reduce disturbance of the soil. The standpipe must extend above the constant water level in the outer tube to prevent water from entering the inner tube. The water surface in the inner tube will now establish itself at the balanced-flow level.

When sufficient saturation is expected, the balanced-flow water level in the standpipe is determined as reference level for the falling or rising water level measurements. A quantity of water is added to the standpipe and the subsequent rate of fall of the water level in the standpipe is measured. It is also possible to remove a quantity of water and to measure the rate of rise in the standpipe. During these operations, the water level in the outer tube is kept constant. In addition to the Ht and t measurements thus obtained, the geometry factors d and D or Dp must be recorded. Knowing R, the factor F can be determined from the appropriate graph and K can be calculated according to equation (5). The depth D or Dp below the auger hole of the material of much lower or much higher conductivity may be determined by visual examination of the soil profile or by carrying out K-measurements at various depths.

The techniques for maintaining constant water level in the outer tube and measuring rate of water level change in the standpipe on the inner tube can be selected from a number of more or less standard techniques that are available (overflow tanks, float valves, Mariotte syphons, and others for constant levels; electric probes, graduated standpipes, and others for rate of water level changes).

At one location, conductivity measurements can be carried out at various depths by starting with a shallow hole and increasing the depth of the hole to the next desired value after each completed measurement. When the depth of the auger hole is zero for measuring the K of the surface soil, the double-tube installation reduces to a buffered cylinder infiltrometer.

EXAMPLE

Field data for both a rising and a falling head in the standpipe are presented in Table 1. An electric probe was used in obtaining the data. The ratios H'/O'/H/F were computed (third column in table 1) and plotted against t on semilogarithmic paper (Figure 7). The slope of the best-fitting straight line, which was drawn by eye, is 0.121. The flow factor is determined from Figure 2 as 1.95, after which K is calculated with equation [5] as 0.036 cm. per min.

DISCUSSION

Sufficient depth of saturation below the bottom of the auger hole may be ascertained by periodically repeating K-measurements until consistent values are obtained. It
is also possible to estimate the depth of saturation after a K-determination has been completed. Assuming that the intake flow below the inner tube was in a vertical direction only, the depth to atmospheric pressure in the soil water below the tube can be calculated with Darcy's equation. Taking atmospheric pressure as the lower boundary of the saturated zone, the calculated depth should equal or exceed $2R_c$ (see Area of sampling and relative tube diameters), or

$$\frac{h_w}{L - 1} \geq \frac{2R_c}{K}$$  \[6\]

In this equation, $I$ is calculated with equation [3]. Since $I = Q_i / \pi R_c^2$, equation [3] can be written for this purpose as

$$I = \frac{K H_b F}{R_c}$$  \[7\]

This equation contains the factor $H_b$, which must be included in the field measurements if estimating the depth of saturation is desired. Equation [6] is only applicable if $I \gg K$. If $I \ll K$, more than sufficient depth of saturation can be expected.

In view of the limited depth of saturation required and the possibility of using large water depths in the concentric tubes, sufficient depth of saturation should not be difficult to obtain.

The basic assumption in the derivation of equations [4] and [5] is that the intake flow component is not appreciably affected by water level changes in the inner tube during the measurements. This will only be true if $H_b$ is small compared to $h_w$. According to equation [7] $H_b$ can be expressed as

$$H_b = \frac{1}{K} \frac{R_c}{F}$$  \[8\]

For a given installation, $H_b$ depends only on the intake gradient $1/K$. Where restricting layers in the soil limit infiltration ($d (I/K) < 1$ and $H_b$ will be small. For unrestricted intake (1) or for soils underlain by well-drained pervious material at shallow depth, ($I/K) > 1$ and $H_b$ may not be small relative to $h_w$. In this case, $Q_i$ at $H_b$ may be considerably less than $Q_i$ at $H = 0$, which invalidates equations [4] and [5] and requires use of a

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**Figure 7**—Plot of $H'/H'_i$ vs. elapsed time on semilog paper for data in table 1.

**Figure 8**—Equipment for double-tube method. From left to right: outer tube, inner tube, top plate for outer tube with graduated inner-tube standpipe and section of flexible tubing to connect inner tube to graduated standpipe at bottom of plate.
Substituting \( Q_H = \pi KHR_c F_i \) in this equation and solving for \( K \) gives

\[
K = \frac{\Delta H}{I R_c} \frac{1}{\int_0^t H dt} \tag{9}
\]

The integral in equation (9) is determined as the area under curve 1 between \( t = 0 \) and \( t \) where \( \Delta H \) is measured. The factors \( R_c \) and \( F_i \) are known for the particular installation and \( F_i \) is determined from figure 2 or 3, so that \( K \) can be calculated.

Field equipment especially designed for this procedure is shown in figure 3. The inner tube is connected with a short section of tubing to the graduated standpipe on the plate, which is placed on top of the outer tube. The plate is equipped with an O-ring to reduce leakage from the outer tube. The second standpipe connects to the water in the outer tube. At the bottom of this standpipe is an outlet with valve. Starting with both standpipes filled to the top, the water level drop in the inner-tube standpipe is measured while the outer-tube standpipe is kept full. The inner-tube standpipe is filled again and the drop of the water level in this standpipe is then measured while continuously adjusting the level in the outer-tube standpipe to that in the inner-tube standpipe by means of the valve. These measurements are then repeated until consistent results are obtained. Further details regarding field techniques, including comparisons with results from core samples, will be presented in a forthcoming paper.

**LITERATURE CITED**


