Comparison of Density-independent Expressions for Moisture Content Determination in Wheat at Microwave Frequencies

A. W. Kraszewski; S. Trabelsi; S. O. Nelson

US Department of Agriculture, Agricultural Research Service, Richard B. Russell Agricultural Research Center, Athens, GA 30604-5677, USA

(Received 30 July 1997; accepted in revised form 27 March 1998)

The accuracy of moisture content determination in grain from measurement of electromagnetic parameters is known to be dependent on the bulk density of the sample used for the measurement. Various means of limiting this effect have been considered in the past. This paper identifies various density-independent expressions developed for this purpose and compares their effectiveness in minimizing the density dependence of the predicted moisture content. The same procedure and data set from more than 180 measurements on hard red winter wheat, Triticum aestivum L., at frequencies of 11.3 and 16.8 GHz on samples in free space were used for the comparison. Several density-independent expressions were identified that predict wheat moisture content with a standard error of calibration of about 0.2% moisture for moisture contents ranging from 10.6 to 18.2% w.b. Although these results were obtained for static samples of grain, the principles can be extended to grain flowing through a tubular conduit, or moving on a conveyor, for continuous measurements.

© 1998 Silsoe Research Institute

Notation

- \( r \) correlation coefficient
- \( v \) volume, \( \text{m}^3 \)
- \( A, B, C \) numerical coefficients
- \( \alpha \) attenuation, decibels
- \( M \) moisture content, percent wet basis
- \( N \) number of samples
- \( T \) complex transmission coefficient
- \( X \) density-independent function or expression

Greek letters

- \( \pi \) attenuation constant
- \( \beta \) phase constant
- \( \gamma \) propagation constant
- \( \delta \) loss angle
- \( \varepsilon \) relative complex permittivity
- \( \varepsilon' \) dielectric constant
- \( \varepsilon'' \) loss factor,
- \( \varsigma, \eta \) permittivity functions
- \( \theta \) argument of transmission coefficient
- \( \lambda \) wavelength, m
- \( \mu \) relative permeability
- \( \rho \) material density, \( \text{kg/m}^3 \)
- \( \sigma \) d.c. conductivity, S/m
- \( \phi \) phase shift, degrees
- \( \omega \) angular frequency \( (\omega = 2\pi f) \), rad/s
- \( \Gamma \) complex reflection coefficient

\( \Phi, \Psi \) functions

Suffixes:

- \( d \) dry material
- \( f \) frequency
- \( i \) current number
- \( m \) mixture
- \( n \) serial number
- \( s \) solid
- \( w \) water
- \( 0 \) free space
1. Introduction

Moisture content is one of the most important factors affecting the quality of grain during storage, processing and transport. Grain of too high moisture content readily spoils, and, when needlessly overdried, loses nutritional value and can incur physical damage during handling. The moisture content, $M$, is defined as

$$ M = \frac{m_w}{m_w + m_d} \quad (1) $$

where $m_w$ is the mass of water, and $m_d$ is the mass of dry material. For a given volume of material, $v$, Eqn (1) may be rewritten in the form

$$ M = \frac{m_w/v}{(m_w/v) + m_d/v} = \frac{k}{k + g} = \frac{k}{\rho} \quad (2) $$

where $k$ and $g$ are partial densities of water and dry material, respectively, and $\rho$ is the density of wet material.

Currently, the standard methods for grain moisture measurement require weighing and oven drying of samples to determine both $m_w$ and $m_d$ according to Eqn (1). The methods require a long time for completion (up to 19 h for wheat kernels and 72 h for whole-kernel corn), and results are obtained for replicated samples of a few grams each. Rapid moisture tests are made with electrical meters that measure static samples of known weight or volume and use the correlation between the dielectric properties of a material and its water content to predict moisture content. Because of variation in moisture distribution in large grain lots in elevators, ships or mill storages, many samples must be measured for reliable determination of an average moisture content. A rapid moisture measuring system that could sense the moisture content of a flowing stream of grain and detect the variations in real time would be a valuable tool for grain quality control.

Electrical sensing of moisture content is based on changes in an electrical signal interacting with a moist material. At any operating frequency, this interaction is related to the amount of water in a given volume, and is affected to a lesser extent by the mass of the dry material. Thus, the partial density of water, $k$, can be determined from the electrical measurement. It is evident from Eqn (2) that determination of moisture content requires the density of the moist material, $\rho$, to be known. This information can be obtained from a separate density measurement, e.g. by weighing a sample of given volume or by using a $\gamma$-ray density gauge. Changes and fluctuations in the amount of material under test produce electrical responses similar to changes in the water content, and therefore create a measurement error. This error can be eliminated only if the mass or density of wet material in the measuring space is kept constant during the calibration procedure as well as during the measurement.

The bulk density of grain depends upon kernel shape, dimensions, moisture content and surface structure and condition. Hence, maintaining a constant grain density during continuous moisture content measurements under grain elevator or mill conditions is impractical. Various means for limiting the density variation during moisture content measurement by dielectric sensing have been considered in the past, and it was concluded that the most reliable solution is the use of some density-independent expression, i.e. a relationship between the electrical properties of grain and its moisture content that does not vary with changes in bulk density of the grain. Work on finding and testing such functions started many years ago with instruments operating at microwave frequencies. Recently, some successful expressions using frequencies below the microwave range have been reported.

The purpose of this paper is to present various density-independent expressions developed through the years, to compare their effectiveness in limiting the density effect in wheat moisture sensing and to discuss their features and limitations. To provide an objective comparison, each expression is used for predicting the moisture content from the same set of experimental data. Physical aspects of the expressions are discussed, and directions for further research are considered.

2. General considerations

There are two common situations in industry where an on-line continuous moisture content monitoring system might be applied: (1) where the material layer thickness cannot be controlled and (2) where elements of a transporting system can be used for controlling the material layer thickness. For example, these situations correspond to a belt conveyor and to transporting the material through a dielectric or metal pipe, chute, duct or auger conveyor. They are schematically presented in Fig. 1 where radiating elements (R) of a measuring system are also shown. For free-space microwave measurements there is no need for physical contact between the instrument and the material, and the moisture content determination can be provided in a continuous and non-destructive manner. This feature is especially important in the food industry where requirements for cleaning and sanitation of the product transport system are very stringent.

2.1. Wave propagation in a layer of material

When an electromagnetic plane wave is transmitted through a layer of homogeneous dielectric material of
thickness $d$ surrounded by air, the complex transmission coefficient $T$ for perpendicular wave incidence can be expressed as

$$T = \frac{(1 - \Gamma^2)e^{-j\gamma d}}{1 - \Gamma^2 e^{-2j\gamma d}}$$  \hspace{1cm} (3)$$

where $\Gamma$ is the complex reflection coefficient at the air–material interface, $\gamma$ is the complex propagation constant of the material and $d$ is the thickness of the material layer. The complex reflection coefficient at the interface is

$$\Gamma = \frac{1 - \sqrt{\epsilon}}{1 + \sqrt{\epsilon}}$$  \hspace{1cm} (4)$$

where $\epsilon$ is the relative complex permittivity. The propagation constant $\gamma$ for non-magnetic materials ($\mu = 1$) is given as,

$$\gamma = \alpha + j\beta = \frac{2\pi}{\lambda_0} \sqrt{\epsilon}$$  \hspace{1cm} (5)$$

where $\alpha$ is the attenuation constant, $\beta$ is the phase constant and $\lambda_0$ is the free-space wavelength. The material permittivity, $\epsilon = \epsilon' - j\epsilon''$, consists of a real part known as the dielectric constant and an imaginary part called the loss factor. The ratio of the two components is known as the loss tangent and denoted as $\tan\delta = \epsilon''/\epsilon'$. The loss factor accounts for all energy dissipation in the material, including that related to dielectric relaxation, $\epsilon''_{\text{die}}$, and to the d.c. conductivity of the material. Thus, the total loss factor can be expressed as a sum

$$\epsilon'' = \epsilon''_{\text{die}} + \frac{\sigma}{\omega \epsilon_0}$$  \hspace{1cm} (6)$$

where $\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$ is the permittivity of free space, $\omega = 2\pi f$ is the angular frequency, where $f$ is the operating frequency, and $\sigma$ is the d.c. conductivity. At microwave frequencies ($f > 10^3$ Hz), the second term in Eqn (6) is negligible and can be omitted. There is a direct relationship between the properties of the material and components of its propagation constant. By solving Eqn (5) for both components of the propagation constant, and assuming that energy loss is not very significant, i.e. $(\epsilon')^2 \gg (\epsilon'')^2$, which is true in most practical cases, the following approximate expressions are obtained:

$$\alpha \approx \frac{\pi \epsilon''}{\lambda_0 \sqrt{\epsilon'}} \text{ (Np/m)}, \quad \beta \approx \frac{2\pi}{\lambda_0} \sqrt{\epsilon'} \text{ (rad/m)}$$

and

$$|\Gamma| \approx \frac{\sqrt{\epsilon'} - 1}{\sqrt{\epsilon'} + 1}$$  \hspace{1cm} (7)$$

The complex transmission coefficient, Eqn (3), can be expressed in polar form as

$$T = |T| e^{j\theta}$$  \hspace{1cm} (8)$$

where $|T|$ is the modulus of the complex transmission coefficient and $\theta$ is its argument. When measuring moisture content by microwave techniques, the attenuation $\alpha$ and the phase shift $\phi$, are usually measured and directly related to the complex transmission coefficient.
through the following expressions:
\[ \mathcal{A} = -20 \log|T| \text{ (dB)} \quad \text{and} \quad \phi = -\theta + 360n \text{ (deg)} \quad (9) \]

where \( n \) is zero or a positive integer. The phase-shift ambiguity (finding the proper value of \( n \)) can be resolved by another measurement at either another frequency\(^7\) or with samples of different thickness. Both attenuation and phase shift are taken to be positive numbers. It is a common practice to determine the material complex permittivity from a transmission measurement in free space. From Eqns (7) and (9) the two components of the complex permittivity can be expressed as
\[ \varepsilon' \approx \left( 1 + \frac{\phi \lambda_0}{360d} \right)^2 \]

and
\[ \varepsilon'' \approx \frac{\mathcal{A} \lambda_0 \sqrt{\varepsilon}}{8 \cdot 686 \pi d} \quad (10) \]

where \( \phi \) is the phase shift in degrees, \( \phi = \phi_1 - \phi_0 \), with \( \phi_0 = 360/\lambda_0 \) being the phase shift in the empty measuring space in degrees, and \( \mathcal{A} = \mathcal{A}_1 - \mathcal{A}_0 \) is the attenuation introduced by the material, expressed in decibels.

### 2.2. Ratio of attenuation and phase shift

It has been observed experimentally\(^8\) that the ratio of the attenuation and phase shift measured in free space, when correlated with the material moisture content, behaves as a density-independent function. From Eqns (10), the two variables can be expressed as
\[ \mathcal{A} \approx d \frac{8 \cdot 686 \pi}{\lambda_0 \sqrt{\varepsilon}} \varepsilon'' \text{ (dB)} \quad \text{and} \quad \phi \approx \frac{360}{\lambda_0} (\sqrt{\varepsilon} - 1) \text{ (deg)} \quad (11) \]

Their ratio can be expressed as a density-independent function,
\[ (X_1 = \frac{\mathcal{A}}{\phi} = \frac{8 \cdot 686 \pi \varepsilon''}{360 \sqrt{\varepsilon}(\sqrt{\varepsilon} - 1)} = c \zeta X_2 = c \eta \tan \delta) \quad 1 \]

where constant \( c = 0.0758 \), and permittivity functions
\[ \eta = \sqrt{\varepsilon}/(\sqrt{\varepsilon} - 1), \zeta = (\sqrt{\varepsilon} + 1)/\sqrt{\varepsilon} \text{ and} \]

\[ X_2 = \frac{\varepsilon''}{\varepsilon' - 1} \quad (13) \]

This is the reciprocal of the density-independent function reported earlier\(^9,10\) and proposed as a density-independent function in moisture content measuring systems.\(^11-14\)

It is interesting that the same reciprocal was also found to be a size-independent function in resonant cavity measurements.\(^16\) Because both measured variables of Eqn (11) are directly proportional to layer thickness, the density-independent function \( X_1 \) can be correlated with the material moisture content without regard to fluctuations in the material layer thickness \( d \) where \( d < \lambda \). This is often a valuable feature.

### 2.3. Density-normalized permittivity

A new density-independent function has been recently developed by Trabelsi.\(^16,17\) This function is based on the observation that in the complex permittivity plane, the normalized variables \( \varepsilon'/\rho \) and \( \varepsilon''/\rho \) for all temperatures and moisture contents can be expressed by the linear equation
\[ \frac{\varepsilon''}{\rho} = a_f \left( \frac{\varepsilon'}{\rho} - b_0 \right) \quad (14) \]

where \( a_f \) is the slope of the line, which depends only upon the operating frequency, and \( b_0 \) is the intercept constant, which, for a given material, has the same value at all frequencies and corresponds to the density-normalized zero-moisture material permittivity or the density normalized permittivity of the material at very low temperature. Because of the special meaning of the loss tangent in describing energy balance and losses in dielectric material, the density-independent expression was created by normalizing the loss tangent to the bulk density. When the density is extracted from Eqn (14), the following can be written:
\[ X_3 = \frac{\tan \delta}{\rho} = \frac{a_f b_0 \tan \delta}{a_f \varepsilon' - \varepsilon''} \quad (15) \]

As Eqn (15) is non-linear with moisture content, further observations lead to the density-independent expression, \( X_4 = \sqrt{X_3} \), which will be discussed later. As the loss tangent is included in several of these expressions, it is also interesting to test the density-independent properties of the simple function.
\[ X_2 = \sqrt{\tan \delta} \quad (16) \]

### 2.4. Dielectric mixture equations

Several authors have noted that the permittivity of many materials is a quadratic function of the material density.\(^18-20\) In general form it can be expressed by the mixture permittivity equation
\[ \varepsilon = (v_1 \sqrt{\varepsilon_1} + v_2 \sqrt{\varepsilon_2})^2 \quad (17) \]
where $\varepsilon$ is the complex permittivity of the mixture of two components with volume fractions $v_1 + v_2 = 1$. For a two-phase mixture of solid particles and air ($\varepsilon_1 = 1 - j0$), $v_2 = \rho_m/\rho_s$, where subscripts $m$ and $s$ refer to the mixture and the solid phase, respectively. It follows from Eqn (17) that equations for the permittivities normalized to the densities, have the following form:21

$$\sqrt{\frac{\varepsilon'_m - 1}{\rho_m}} = \sqrt{\frac{\varepsilon'_s - 1}{\rho_s}} \quad \text{and} \quad \sqrt{\frac{\varepsilon''_m}{\rho_m}} = \sqrt{\frac{\varepsilon''_s}{\rho_s}} \quad (18)$$

Thus, a density-independent function can be written as

$$X_6 = \frac{\sqrt{\varepsilon'}}{\sqrt{\varepsilon'} - 1} \quad (19)$$

Similar experimental observations20 lead to the conclusion that for some materials the density is a linear function of the cube root of the material dielectric constant. This observation provides another density-independent expression in the form22

$$X_7 = \frac{1}{3} \frac{\sqrt{\varepsilon'}}{\sqrt{\varepsilon'} - 1} \quad (20)$$

Another expression can be developed for materials showing a more linear relationship between the loss factor and the density. These materials might obey the following empirical density-independent expression:

$$X_8 = \frac{\varepsilon''}{\sqrt{\varepsilon'} - 1} \quad (21)$$

2.5. Multiple regression analysis

2.5.1. Single frequency measurements

This approach is based on an assumption that in the interaction of the electromagnetic wave with moist material, the difference between the effect of water (usually a strong effect but a small amount) and that of dry material (usually a weak effect but a larger quantity) can be distinguished.22-25 Thus, two measured material properties can be correlated with the partial density of water, $k$, and the partial density of dry material, $g$. The magnitude and phase of the reflection coefficient or the transmission coefficient, measured at one frequency, leads to functional relationships in the form

$$\mathcal{A} = \Phi_1(k, g) \quad \text{and} \quad \phi = \Phi_2(k, g) \quad (22)$$

Numerical coefficients of Eqns (22) will be characteristic for a particular material and given measurement conditions (temperature, frequency, layer thickness, sensor configuration, etc.). Solving these two equations by separation of variables, the following two relationships, expressing $k$ and $g$, can be obtained:

$$k = \Psi_1(\mathcal{A}, \phi) \quad \text{and} \quad g = \Psi_2(\mathcal{A}, \phi) \quad (23)$$

Substituting these values into the definition expressed in Eqn (1), equations are obtained for moisture content and density of moist material in terms of measured quantities $\mathcal{A}$ and $\phi$ in the form

$$M = \frac{\Psi_1(\mathcal{A}, \phi)}{\Psi_1(\mathcal{A}, \phi) + \Psi_2(\mathcal{A}, \phi)} \quad \rho = \frac{\Psi_1(\mathcal{A}, \phi)}{\Psi_1(\mathcal{A}, \phi) + \Psi_2(\mathcal{A}, \phi)} \quad (24)$$

A second set of relationships similar to Eqns (22) and containing two other measured variables (e.g. magnitude and phase of the reflection coefficient), could permit the determination of two more parameters of the material under test. The material temperature could be a third predicted quantity, and similar expressions could be developed that compensate for thermal effects in the range of practical interest.26,27

2.5.2. Multifrequency measurements.

With the development of fast microprocessors and digital signal processors, carrying out measurements at several frequencies and analyzing their results in real time becomes easier and less expensive. This provides incentives for multifrequency measurements, and multiparameter measurements in general. Using the results of measurements carried out at two or more microwave frequencies provides a solution to the phase-shift ambiguity problem [see Eqn (9)] and also provides redundant information that can be used for repeated calculations of $M$ and $\rho$ according to Eqns (24) or for potential determination of other material variables. In preliminary studies, the material layer thickness and its temperature have been indicated as target variables.27 Further studies of criteria for frequency selection are needed before this approach can be applied in practical situations, but it seems certain now that such solutions are quite feasible and should materialize in the future. Results of multifrequency measurements might serve also as a basis for implementation of modern measuring techniques in microwave aquametry, such as neural network techniques, and modern methods of data analysis, such as principal component analysis.

3. Materials and methods

Two cultivars of hard red winter wheat, “Karl” and “Arapahoe”, Triticum aestivum L., grown in Nebraska in 1992 and 1994, respectively, were used for measurements at 24°C. Sample moisture content ranged from 10.6 to
18.2%, wet basis. The sample holder, a polyethylene container of 12×15 cm vertical cross section with 0.2 cm wall thickness, providing a layer of grain 10.4 cm thick, was filled with grain and placed between two horn antennas in free space. The horns were equipped with waveguide-to-coaxial adapters that permitted them to be connected via coaxial cables to the two ports of a vector network analyzer. The analyzer was calibrated in the transmission mode with an empty sample holder inserted between the two antennas. Measurements of the transmission coefficient were automated by a computer program written for this purpose and were performed at eight frequencies ranging from 10.3 to 18 GHz. Average error in attenuation measurement was less than ±0.25 dB, and the error in phase shift measurement was about ±3°.

After calibration of the measurement system with the empty sample holder between the two horn antennas, the sample holder was filled with grain and located in the same position between the horns. Starting from a loosely filled sample holder, the measurements of the attenuation and phase shift were repeated for samples of gradually increased density obtained by settling the sample and adding more grain. The bulk densities ranged from 720 to 879 kg/m³. The measurements were repeated for grain of various moisture contents at 24°C, providing a total of 181 data points at each of eight frequencies.

4. Results and analysis

Attenuation and phase shift data for the two hard red winter wheat cultivars were indistinguishable, so the data from measurements on both cultivars were pooled for the analysis. The results of these measurements at 16.8 GHz are shown in Fig. 2, where attenuation and phase shift are presented as functions of moisture content. Spread of the data results from density variation and demonstrates that there is practically no way to establish an accurate relationship between the wave parameters and the material moisture content that can be used for prediction of moisture content on the basis of a one-parameter measurement. For a given value of a measured parameter, e' or e", the corresponding value of moisture content varies up to 2–8%. The results of the measurements were then used to predict moisture content with the density-independent functions discussed above to verify the effectiveness of these expressions.

Permittivities of bulk wheat at two frequencies, 11.3 and 16.8 GHz, were calculated with Eqns (10) from the experimental data for e' and e", similar to that illustrated for 16.8 GHz only in Fig. 2. Next, by using the respective values for the dielectric constant, e', and loss factor, e", numerical values of expressions $X_n$, for $n = 1$ to 8, were calculated for each of the 181 data points. Results as functions of moisture content are presented in Figs 3–6. Data from each expression were fitted to a non-linear regression model in the form

$$X_n = a_n M_n^2 + b_n M_n + c_n$$

Then, each expression was solved for $M_n$ to provide a calibration equation for moisture content determination:

$$M_n = \sqrt{A_n X_n + C_n + B_n}$$

where $A_n = 1/a_n$, $B_n = -b_n/2a_n$ and $C_n = (b_n^2/4a_n^2) - c_n/a_n$. Table 1 lists numerical values for all expressions identified in Section 2 and calculated for two frequencies. The standard error of calibration (SEC) was also calculated from all 181 data points, where

$$SEC = \sqrt{\frac{\sum_{i=1}^{N} r_i^2}{N - p - 1}}$$
N is number of points, \( p \) is the number of independent variables in a multiple regression model and 
\( h_i = M_{oven} - M_i \) is the difference between the standard oven determination and the predicted moisture content in percentage, wet basis. The SEC values together with the correlation coefficients, \( r \), related to Eqn (25), are also listed in Table 1. Equation (26) can be called the density-independent calibration equation because it permits moisture content to be determined regardless of the material density.

When the relationship between the density-independent expression and moisture content in a limited range is quasilinear, instead of using Eqn (25), it is often more convenient to use linear regression of the data in the form

\[
X_n = a_n M_n + b_n
\]

which can be converted into a linear density-independent calibration equation of the following form:

\[
M_n = A_n X_n - B_n
\]

where \( A_n = 1/a_n \) and \( B_n = b_n/a_n \). The numerical coefficients for Eqn (29) for the two frequencies are listed in Table 2, together with the coefficients of correlation and SEC values calculated as before from Eqn (27). It may be noted that for expressions \( X_6 \) and \( X_7 \), which are almost linear functions of moisture content, the difference between the SEC values listed in Tables 1 and 2 is negligible, and, hence, the linear Eqn (29) can be applied.
The loss tangent in Eqn (12). The loss tangent is not a linear function of moisture content, but when multiplied by $\eta$ becomes the effective expression $X_1$. Similar relationships between the moisture content and loss tangent and expression $X_3$ are presented in Fig. 4. Those for expressions

### Table 1
Numerical coefficients for the calibration equations in the form $M_n = A_nX_n+B_n$, for $n = 1$ to 8 and their statistics at two frequencies

<table>
<thead>
<tr>
<th>Function</th>
<th>$f = 11.3$ GHz</th>
<th></th>
<th></th>
<th>$f = 16.8$ GHz</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = \frac{A}{\phi}$</td>
<td>12820.5</td>
<td>0.026</td>
<td>-94.633</td>
<td>0.9964</td>
<td>0.173</td>
<td>8333.4</td>
</tr>
<tr>
<td>$X_2 = \frac{\tan \delta}{\varepsilon - 1}$</td>
<td>1453.5</td>
<td>0.439</td>
<td>-84.178</td>
<td>0.9960</td>
<td>0.180</td>
<td>9813</td>
</tr>
<tr>
<td>$X_3 = \frac{\tan \delta}{\rho}$</td>
<td>834.03</td>
<td>6.559</td>
<td>-65.383</td>
<td>0.9958</td>
<td>0.188</td>
<td>700.3</td>
</tr>
<tr>
<td>$X_4 = \frac{\tan \delta}{\varepsilon - 1}$</td>
<td>126.42</td>
<td>-0.592</td>
<td>-268.31</td>
<td>0.9961</td>
<td>0.182</td>
<td>925.9</td>
</tr>
<tr>
<td>$X_5 = \frac{\sqrt{\varepsilon}}{\varepsilon - 1}$</td>
<td>1996.0</td>
<td>-21.07</td>
<td>-469.68</td>
<td>0.9757</td>
<td>0.460</td>
<td>808.4</td>
</tr>
<tr>
<td>$X_6 = \frac{\sqrt{\varepsilon}}{\varepsilon - 1}$</td>
<td>1196.2</td>
<td>-22.18</td>
<td>-376.61</td>
<td>0.9855</td>
<td>0.354</td>
<td>495.0</td>
</tr>
<tr>
<td>$X_7 = \frac{\varepsilon}{\varepsilon - 1}$</td>
<td>482.16</td>
<td>1.182</td>
<td>-74.29</td>
<td>0.9919</td>
<td>0.253</td>
<td>347.3</td>
</tr>
</tbody>
</table>

### Table 2
Numerical coefficients for the calibration equations in the form $M_n = A_nX_n+B_n$, for $n = 1$ to 8 and their statistics at two frequencies

<table>
<thead>
<tr>
<th>Function</th>
<th>$f = 11.3$ GHz</th>
<th></th>
<th></th>
<th>$f = 16.8$ GHz</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = \frac{A}{\phi}$</td>
<td>439.31</td>
<td>3.947</td>
<td>0.9943</td>
<td>0.220</td>
<td>408.59</td>
<td>4.122</td>
</tr>
<tr>
<td>$X_2 = \frac{\varepsilon}{\varepsilon - 1}$</td>
<td>51.13</td>
<td>4.435</td>
<td>0.9938</td>
<td>0.229</td>
<td>48.23</td>
<td>4.473</td>
</tr>
<tr>
<td>$X_3 = \frac{\tan \delta}{\rho}$</td>
<td>41.21</td>
<td>-1.901</td>
<td>0.9941</td>
<td>0.223</td>
<td>41.81</td>
<td>-2.201</td>
</tr>
<tr>
<td>$X_4 = \frac{\tan \delta}{\varepsilon - 1}$</td>
<td>50.19</td>
<td>-3.359</td>
<td>0.9879</td>
<td>0.320</td>
<td>50.14</td>
<td>-3.680</td>
</tr>
<tr>
<td>$X_5 = \frac{\sqrt{\varepsilon}}{\varepsilon - 1}$</td>
<td>26.71</td>
<td>-8.770</td>
<td>0.9753</td>
<td>0.456</td>
<td>24.71</td>
<td>-7.949</td>
</tr>
<tr>
<td>$X_6 = \frac{\sqrt{\varepsilon}}{\varepsilon - 1}$</td>
<td>15.83</td>
<td>-8.233</td>
<td>0.9851</td>
<td>0.355</td>
<td>14.81</td>
<td>-7.612</td>
</tr>
<tr>
<td>$X_7 = \frac{\varepsilon}{\varepsilon - 1}$</td>
<td>17.75</td>
<td>5.081</td>
<td>0.9894</td>
<td>0.300</td>
<td>17.15</td>
<td>4.982</td>
</tr>
</tbody>
</table>
$X_2$, $X_4$, $X_5$ and $X_6$, $X_7$ and $X_8$, are presented in Fig. 5 and Fig. 6, respectively. Even though some functions show relatively low slopes when plotted with the scales shown in these graphs, sensitivity of the measurement is adequate for accurate moisture content sensing in all instances.

Examination of data collected in Table 1, reveals that, although the SEC values for 11 GHz are generally a little smaller, there is no decisive superiority of one frequency over the other. Based on the SEC as a measure of effectiveness for density independence, $X_1 \ldots X_4$ provide more accurate predictions of moisture content than $X_6 \ldots X_8$. It must be remembered, however, that the above data are related to wheat only. For other materials the order of merit of the expressions could be different.

It should be pointed out that the expression $X_1$ is simply a ratio of two directly measured quantities, and therefore it does not require computation of $\varepsilon'$ and $\varepsilon''$. The ratio can be used even when the material layer thickness fluctuates, and this might have some importance in industrial applications. The expression $X_4$, on the other hand, during an off-line measurement, can also provide the value of bulk density of the material. It was also observed that the expressions which are quadratic with moisture content [fulfilling the nature of Eqn (25)] are generally more effective in alleviating the density effect.

Comparing the SEC figures, it can be concluded that with improved fitting to the experimental data, the quality of the moisture prediction is also improved. When the spread of the data resulting from the density fluctuations is comparable, expressions of higher slope versus moisture content provide better predictions.

To complete the review of density-independent microwave moisture determination, it might be of interest to mention that the procedure using multiple regression analysis with the same set of data, provides the following expressions:

$$
\varepsilon = 6.10 + 0.4979k - 0.04127g, \quad r = 0.9866
$$
$$
\phi = -167.27 + 6.4354k + 1.1742g, \quad r = 0.9907
$$

Solving Eqns (30) for $k$ and $g$, provides equations for the moisture content and the density of the moist material, according to Eqns (24):

$$
M = \frac{138.1 \varepsilon - 30.5 + 4.854\phi}{0.63414\phi - 6.188\varepsilon + 143.82} \quad \text{(\%)}
$$
$$
\rho = 0.63414\phi - 6.188\varepsilon + 143.82 \quad \text{(kg/m}^3\text{)}
$$

For the frequency of 16.8 GHz, the standard error of calibration for the moisture content is 0.351% moisture and that for density 5.42 kg/m$^3$.

5. Discussion

Comparison of the expressions for density-independent determination of moisture content was carried out with the same set of microwave experimental data. A linear regression model can be used with these expressions, especially for a limited moisture content range. In general, however, these expression should be used with a non-linear regression model by fitting them to a quadratic equation, which in turn may be inverted to provide a density-independent calibration equation for moisture content. The final measure of the effectiveness of the calibration equation in predicting the moisture content of the material is the standard error of calibration (SEC) calculated from a sum of differences between the moisture content determined by the standard oven method and the estimated values predicted by the calibration equation. The better the fit of the density-independent expression to the model, expressed in terms of the correlation coefficient, the more effective is the calibration equation in nullifying the material density effect, and the higher the slope of the expression versus moisture content, the more effective is the calibration equation. With these criteria, no essential difference was found between the effectiveness of expressions derived from the ratio of attenuation and phase shift ($X_1$ and $X_2$) and those developed from the density-normalized permittivity ($X_3$ and $X_4$). This observation is valid for wheat, but should be tested carefully for other particulate materials. The validity of this observation also remains to be proven for other temperature ranges.

The calculations presented for two hard red winter wheat cultivars at 24°C yielded calibration equations capable of estimating wheat moisture content with an SEC of about 0.2% moisture content in grain ranging from 10 to 18%. For an average bulk density of 810 kg/m$^3$, the SEC for bulk density prediction was less than 10 kg/m$^3$.

By using a microwave system for measurements, there is no need for physical contact between the grain and the measuring equipment, and it appears that the same calibration should suffice for different varieties of the same type of grain. The results, although obtained for static samples of grain, are well within the specifications required for commercial moisture meters. The principles developed for static measurements can most likely be extended to flowing grain with similar results.

The existing technology of microwave integrated circuits, microwave radiating elements (patch antennas) and digital signal processing, should allow the construction of robust and reliable equipment for operation under severe field conditions. Such instruments should be useful in yield monitoring in terms of moisture content, bulk density and dry mass of grain in real time. Availability of
these data on a continuous basis, in real time and in-situ, could provide an important step forward in realizing advantages of precision farming based on global positioning systems and computerized collection of data.

6. Conclusions

Moisture content of wheat can be determined independently of bulk density by measuring the relative complex permittivity (dielectric constant and loss factor) or wave parameters such as attenuation and phase shift at microwave frequencies. Several density-independent functions of the permittivity or wave parameters can be used to obtain linear or second-order calibration equations for grain moisture content in terms of the permittivity components or the wave parameters.

Comparison of eight such density-independent expressions by using measured data on hard red winter wheat at 11.3 and 16.8 GHz over moisture contents ranging from 10.6 to 18.2% at bulk densities from 720 to 879 kg/m³ at 24°C revealed standard errors of calibration of 0.2% moisture content or less for four of the expressions with non-linear calibrations. With linear calibration equations, five of the expressions yielded standard errors of calibration less than 0.3% moisture content.

The performance of the density-independent expressions for grain, established by this research on wheat samples, needs to be tested also over suitable ranges of temperature, with additional kinds of grain, and with flowing grain to determine how well the principles established with static samples apply to dynamic measurements.

Acknowledgments

The authors gratefully acknowledge financial support from the US Grain Inspection, Packers and Stockyards Administration and from Campbell Scientific, Inc., which was helpful in the conduct of this research.

References

1 Kraszewski A; Kulinski S An improved microwave method of moisture content measurement and control. IEEE Transactions on Industrial and Control Instrumentation, 1976, IECI-23(4), 364–370
5 Berbert P A; Stenning B C Analysis of density-independent equations for determination of moisture content of wheat in the radiofrequency range. Journal of Agricultural Engineering Research, 1996, 65, 275–286
8 Jacobsen R; Meyer W; Schrage B Density independent moisture meter in X-band. Proc. 10th European Microwave Conf., Warsaw, Poland, 1980: 216–220
9 Hoppe W; Meyer W; Schilz W Density-independent moisture metering in fibrous materials using a double-cut-off Gunn oscillator. IEEE Transactions on Microwave Theory & Techniques 1980, 28(12), 1449–1452
11 Meyer W; Schilz W Feasibility study of density-independent moisture measurement with microwaves. IEEE Transactions on Microwave Theory & Techniques, 1981, 29(7), 732–739
14 Kress-Rogers E; Kent M Microwave measurement of powder moisture and density. Journal of Food Engineering, 1987, 6, 345–376
15 Kraszewski A W; Nelson S O; You T S Use of a microwave resonant cavity for sensing dielectric properties of arbitrarily shaped biological objects. IEEE Transactions on Microwave Theory & Techniques, 1990, 38(7), 858–863
22 Powell S D; McLendon B D; Nelson S O; Kraszewski A; Allison J M Use of a density-independent function and microwave measurement system for grain moisture measurement. Transactions of the ASAE, 1988, 31(6), 1875–1881


24 Kraszewski A W; Nelson S O Wheat moisture content and bulk density determination by microwave parameter measurement. Canadian Agricultural Engineering 1992, 34, 327–335


27 Kraszewski A W; Trabelsi S; Nelson S O. Moisture content determination in grain by measuring microwave parameters. Measurement Science and Technology, 1997, 8(8), 857–863
