ANALYSIS OF COVARIANCE OF YIELD AND TIME TO FIRST SILKS IN MAIZE [1]

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INTRODUCTION

In the course of his investigations of heredity in numerous selfed lines from 14 varieties of maize (*Zea mays* L.), Jenkins (*5*)[3] examined the relation between yield and the time elapsing after June 30 to the first appearance of silks. The 14 varieties, developed at various latitudes in Iowa, were all planted at Ames, near the center of the State. Each inbred line was grown in three replications, the corresponding datum recorded in table 1 below being their mean. The present probem is to determine the relation between the two variates, yield and time to silking, in the inbred lines as well as in the varieties themselves.

The analysis of the covariance of yield and time to first silks was reported earlier by Snedecor (*6*). However, recently so much progress has been made in the technique of covariance, that a reexamination of the data seems desirable. This is especially appropriate because covariance is proving to be an extremely valuable statistical method in biological research. Since the introduction of the technique by Fisher, it has been expanded by him (*3*, *4*) and his co-workers (*1*, *9*). A number of studies have been reported both in field trials and in experiments with animals. This article is designed to place before its readers some of the recent advances in covariance methodology.

DATA

The data on yield and days to first silks are displayed in table 1. As is usual in such computations there are required the sums, sums of squares, and sums of products of the observed values in each variety. These are recorded in the last three columns of the table. As examples, consider the data for the variety Silver King. The sum for time to silking is

$$24+31+26+30=111.$$

The sum of squares for yields is

$$(7.7)^2+(5.4)^2+(5.2)^2+(4.0)^2=131.49.$$

The sum of products is

$$(24)(7.7)+(31)(5.4)+(26)(5.2)+(30)(4.0)=607.4.$$

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[3] Reference is made by number (italic) to Literature Cited, p. 459.

TABLE 1.—*Mean time to first silks together with mean yield of 3 15-hill plots of corn planted to 14 varities*

Variety	Number of inbred lines	Silks or yield	Period after June 30 to first silks, and shelled corn per plot												Sum	Mean	Sum of squares	Sum of products
Four County White	24	Silks ... days	28	26	29	31	26	29	26	28	25	27	25	28				
		Yield ... pounds	7.3	5.7	5.2	6.8	4.5	7.8	4.2	7.4	7.4	7.9	7.4	5.9				
		Silks ... days	35	26	30	29	31	31	30	30	29	28	26	28	681	28.38	19,451	
		Yield ... pounds	5.0	5.9	6.8	6.4	4.8	6.3	5.0	6.1	7.9	5.7	6.4	6.4	150.2	6.26	968.06	4,251.7
Silver King	4	Silks ... days	24	31	26	30									111	27.75	3,113	
		Yield ... pounds	7.7	5.4	5.2	4.0									22.3	5.58	131.49	607.4
U. S. Selection 133	7	Silks ... days	30	25	29	39	27	32	23						205	29.29	6,169	
		Yield ... pounds	2.8	4.7	3.7	2.6	8.4	1.8	5.0						29.0	4.14	149.18	809.6
Iodent	34	Silks ... days	38	36	36	35	35	37	35	31	33	34	37	33				
		Yield ... pounds	6.9	7.4	5.6	6.8	9.5	5.5	8.6	7.6	9.5	7.6	8.1	9.9				
		Silks ... days	33	33	29	32	30	35	30	34	36	30	33					
		Yield ... pounds	5.8	9.4	6.7	8.4	12.0	9.4	15.9	7.4	6.6	5.4	8.4					
		Silks ... days	30	30	34	32	35	36	38	31	32	30	31		1,134	33.35	38,044	
		Yield ... pounds	7.3	7.1	8.9	10.0	9.0	5.2	10.8	3.9	9.2	8.6	12.2		280.6	8.25	2,494.72	9,316.9
U. S. Selection 204	5	Silks ... days	36	36	38	38	39								187	37.40	7,001	
		Yield ... pounds	7.0	6.5	2.3	4.5	5.7								26.0	5.20	149.28	966.7
Lancaster Surecrop	11	Silks ... days	33	33	32	36	33	38	30	38	31	32	32		368	33.45	12 384	
		Yield ... pounds	9.6	7.8	9.6	7.7	8.2	7.3	11.3	9.5	8.8	8.4	6.8		95.0	8.64	837.16	3,165.8
Black Yellow Dent	8	Silks ... days	34	38	33	32	36	35	34	34					276	34.50	9,546	
		Yield ... pounds	11.6	7.4	11.6	9.6	9.3	4.8	4.2	4.3					62.8	7.85	561.70	2,157.4
Proudfit Yellow Dent	6	Silks ... days	34	36	34	36	36	40							216	36.00	7,800	
		Yield ... pounds	6.8	10.4	8.4	9.6	9.0	8.6							52.8	8.80	472.08	1,904.8
Krizer Bros. Yellow Dent	7	Silks ... days	35	30	43	35	35	32	30						240	34.29	8,348	
		Yield ... pounds	7.6	7.9	6.8	8.6	6.0	5.3	8.4						50.6	7.23	375.02	1,728.0
McCulloch Yellow Dent	7	Silks ... days	36	29	39	39	38	38	34						253	36.14	9,223	
		Yield ... pounds	7.7	6.5	8.5	6.4	6.4	9.0	5.9						50.4	7.20	371.52	1,832.6
Osterland Yellow Dent	7	Silks ... days	31	33	33	33	27	32	36						225	32.14	7,277	
		Yield ... pounds	4.8	9.2	8.5	8.8	9.2	7.9	5.9						54.3	7.76	439.23	1,736.9
Clark Yellow Dent	7	Silks ... days	27	33	25	35	27	27	26						200	28.57	5,802	
		Yield ... pounds	4.3	8.4	6.6	4.9	5.8	7.6	3.7						41.3	5.90	261.71	1,187.8
Walden Dent	7	Silks ... days	36	37	34	41	33	33	29						243	34.71	8 521	
		Yield ... pounds	5.8	6.5	10.8	2.0	5.6	8.5	9.6						48.8	6.97	392.30	1,642.2
Argentine Flint	8	Silks ... days	42	40	43	34	29	30	33	27					278	34.75	9,928	
		Yield ... pounds	2.3	4.9	2.9	5.3	5.3	5.3	2.8	2.6					31.4	3.92	136.58	1,072.8
Total (on 14 varieties)	142	Silks ... days													4,617	32.51	152,607	
		Yield ... pounds													995.5	7.01	7,740.03	32,380.6

In many cases it is advantageous to "code" the data in order to simplify the computation. The number 20 or 30 might well have been subtracted from each number of days to first silks. With a calculating machine available, however, there would have been no net gain of time in the present computations.

Since it is sums of squares and products of deviations from mean that are required, it is necessary to "correct" the varietal sums of squares and sum of products of the two variates. For each variety three correction terms are necessary, those for Silver King being—

For time to silks, $(111)^2/4=3,080.25$,
For yield, $(22.3)^2/4=124.32$,
For products, $(111)(22.3)/4=618.82$.

The divisor 4 is the number of inbred lines in the variety; that is, the number of items added to form the sum. The sums of squares and products of deviations from means in this variety are calculated in this way—

For time to silks, $3,113.00-3,080.25=32.75$,
For yield, $131.49-124.32=7.17$,
For products, $607.40-618.82=-11.42$.

These results for Silver King, together with the corresponding sums for the other varieties, are entered in table 2. From these statistics may be computed most of the desired information about the relation between yield and time to first silks.

TABLE 2.—*Sums of squares and products, correlation and regression coefficients, reduction in sum of squares, and sum of squares of errors of estimate in time to first silks and yield of 14 varieties of maize*

Variety	Degrees of freedom	Sum of squares and products			Correlation coefficient $\frac{Sxy}{\sqrt{(Sx^2)(Sy^2)}}$	Regression of yield on time to silks Sxy/Sx^2	Reduction in sum of squares $(Sxy)^2/Sx^2$	Errors of estimate	
		Time to silks Sx^2	Products Sxy	Yield Sy^2				Sum of squares $Sy^2-[(Sxy)^2/Sx^2]$	Degrees of freedom
Four County White	23	127.62	−10.22	28.06	−0.1708	−0.0801	0.82	27.24	22
Silver King	3	32.75	−11.42	7.17	−.7454	−.3487	3.98	3.19	2
U. S. Selection 133	6	165.43	−39.69	29.04	−.5726	−.2399	9.52	19.52	5
Iodent	33	221.76	−41.94	178.94	−.2105	−.1891	7.93	171.01	32
U. S. Selection 204	4	7.20	−5.70	14.08	−.5660	−.7917	4.51	9.57	3
Lancaster Surecrop	10	72.73	−12.38	16.71	−.3551	−.1702	2.11	14.60	9
Black Yellow Dent	7	24.00	−9.20	68.72	−.2265	−.3833	3.53	65.19	6
Proudfit Yellow Dent	5	24.00	4.00	7.44	.2994	.1667	.67	6.77	4
Krizer Bros. Yellow Dent	6	119.43	−6.86	9.25	−.2064	−.0574	.39	8.86	5
McCulloch Yellow Dent	6	78.86	11.00	8.64	.4215	.1395	1.53	7.11	5
Osterland Yellow Dent	6	44.86	−8.46	18.02	−.2976	−.1886	1.60	16.42	5
Clark Yellow Dent	6	87.71	7.80	18.04	.1961	.0889	.69	17.35	5
Walden Dent	6	85.43	−51.86	52.09	−.7774	−.6070	31.48	20.61	5
Argentine Flint	7	267.50	−18.35	13.34	−.3072	−.0686	1.26	12.08	6
Sum	128	1,359.28	−193.28	469.54				399.52	114

First, there are calculated in the usual manner the correlation and regression coefficients (*8*) recorded in the table. Turning again to

Silver King as an example, the correlation between yield and time to silking in this variety is

$$\frac{-11.42}{\sqrt{(32.75)\ (7.17)}}=-0.7454.$$

The regression of yield on time to first silks is

$$(-11.42)/(32.75)=-0.3487.$$

The regression coefficients in the several varieties vary from 0.1667 to −0.7917, 11 of them being negative. With the small number of degrees of freedom in most of the varieties, a good deal of sampling variation may be expected. One of the problems to be solved below is whether this observed variation among the regression coefficients

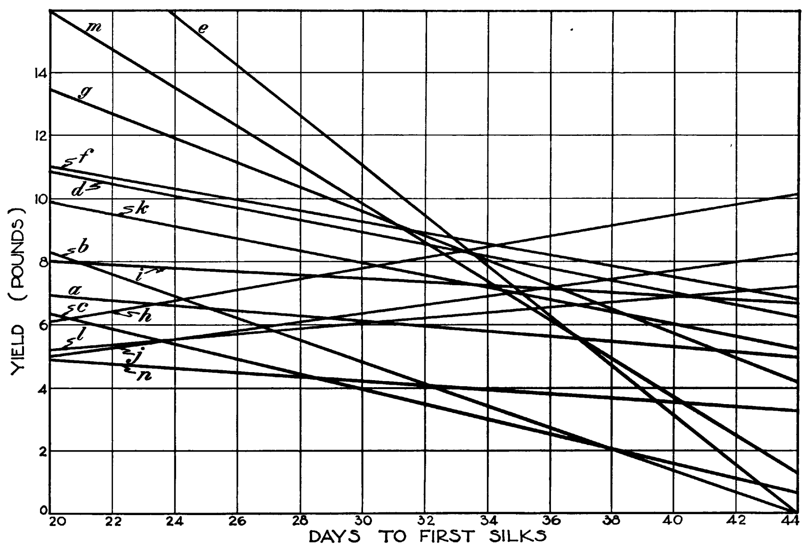

FIGURE 1.—Regressions of yield on days to first silks in 14 varieties of maize: *a*, Four County White; *b*, Silver King; *c*, U. S. Selection 133; *d*, Iodent; *e*, U. S. Selection 204; *f*, Lancaster Surecrop; *g*, Black Yellow Dent; *h*, Proudfit Yellow Dent; *i*, Krizer Bros. Yellow Dent; *j*, McCulloch Yellow Dent; *k*, Osterland Yellow Dent; *l*, Clark Yellow Dent; *m*, Walden Dent; *n*, Argentine Flint.

is significant. Since significant regressions imply significant correlations when there are only two variables, the same statement as above may be made about correlations.

The regression equations for the 14 varieties are plotted in figure 1. The equation for Silver King is

$$\begin{aligned}\overline{Y}&=5.58-0.3487(X-27.75),\\&=-0.3487\ X+15.26,\end{aligned}$$

in which 5.58 pounds per plot is the mean yield for Silver King, and 27.75 days is the mean time to silking. In most of the varieties the tendency is for yield to increase with decreased time to the appearance of first silks. This seems reasonable, since both yield and earliness in these inbred lines may be considered evidences of vigor.

Certainly part of the variation in the yields of the inbred lines is associated with the regression of yield on time to silking. How great a part? The next step is directed toward the isolation of this portion from the total variance of the yields. The method is familiar to users of correlation analysis (*8, p. 17*), where r^2Sy^2 and $(1-r^2)Sy^2$ are the two parts of the sum of squares attributable respectively to regression and other causes. In the actual calculation, the value of r^2 is unnecessary. Since

$$r^2Sy^2=[(Sxy)^2/(Sx^2)(Sy^2)]\ (Sy^2)=(Sxy)^2/Sx^2,$$

it is sufficient to have the latter two quantities, $(Sxy)^2$ and Sx^2. Thus for Silver King, the reduction in sum of squares due to regression is

$$(Sxy)^2/Sx^2=(-11.42)^2/(32.75)=3.98,$$

the result being entered in the table. This is the part of the variation in strain yields associated with or caused by variation in earliness.

The remainder, $Sy^2-[(Sxy)^2/Sx^2]$, is the portion of the variation in yield not explained by regression; it is the sum of the squares of the errors of estimate. In the case of Silver King, this portion,

$$7.17-3.98=3.19,$$

is almost equal to the reduction ascribable to regression. This part of the variation in yield is independent of time to first silks. Presumably, this amount would be present even if all the strains silked at the same date. In the remainder of the table, comparison of the two portions of the sums of squares of variety yields, reduction, and remainder, gives an idea of the differing effects of regression in these groups of inbred lines. In Walden Dent more than half of the sum of the squares is associated with earliness, but in most of the varieties the fraction is decidedly smaller.

The degrees of freedom in the second column of table 2 are those applying to the sums of squares of yield and time to first silks. In each case, the degrees of freedom are less by one than the number of inbred lines. The deducted degree of freedom is associated with the mean which has been used in the computation. In the last column are the degrees of freedom for the sum of squares in the next preceding column. The number, in each case, is less by 2 than the number of inbred lines in the variety. The second degree of freedom to be deducted corresponds to the regression (or correlation) coefficient used in computing the errors of estimate.

One can now see clearly that the reduction in sum of squares owing to regression cannot be directly interpreted. The corresponding change in degrees of freedom must be taken into account. In a sample like Silver King the remaining sum of squares, less than half the original, is accompanied by only two degrees of freedom. The mean square error of estimate is therefore $3.19/2=1.60$. In testing the significance of the regression, this figure is directly comparable with the reduction in sum of squares, 3.98, with only one degree of freedom. The ratio, $F=3.98/1.60=2.5$, is not significant. From a different viewpoint, that of assessing the effect of regression, the mean square, 1.60, may be compared with the original mean square for yield, $7.17/3=2.39$, indicating a reduction in variance of only $(2.39-1.60)/2.39=33$ percent.

The sums of the last two columns of table 2 contributed valuable information. These are the pooled sums of squares of errors of estimate and degrees of freedom for all varieties, each error of estimate being measured from the individual variety regression. These sums are entered in the second line of table 3 and the mean square calculated,

$$399.52/114=3.50.$$

TABLE 3.—*Analysis of errors of estimate within varieties*

Source of variation	Degrees of freedom	Sum of squares	Mean square
Within varieties, from average regression	127	442.05	3.48
Within varieties, from individual regressions	114	399.52	3.50
Remainder, between variety regression coefficients	13	42.53	3.27

This mean square is an average of that variation in yields not explained either by variety differences or by regression. It is therefore attributed to experimental error, and is the appropriate datum with which the magnitude of other mean squares may be compared.

The sums of the columns headed Sx^2, Sxy, and Sy^2 in table 2 are the pooled sums of squares and products for all varieties, each deviation being measured from the variety mean. These sums specify an average "within-variety" regression,

$$Sxy/Sx^2=(-193.28)/(1{,}359.28)=-0.1422,$$

and an average correlation,

$$\frac{Sxy}{\sqrt{(Sx^2)(Sy^2)}}=\frac{-193.28}{\sqrt{(1{,}359.28)(469.54)}}=-0.2419.$$

The corresponding regression equation is,

$$\begin{aligned}\overline{Y}&=7.01-0.1422\ (X-32.51),\\&=-0.1422\ X+11.63,\end{aligned}$$

where the means from the entire experiment are taken from table 1. The graph of this equation is plotted in figure 2. The corresponding within-variety correlation, -0.2419, is the appropriate average of the 14 variety correlation coefficients of table 2.

The yields of the inbred lines deviate from the within-variety regression more, in the aggregate, than from their individual variety regressions. The magnitude of the difference between the two sets of errors of estimate is easily computed. The sum of squares of the errors of estimate from the average regression is

$$469.54-[(193.28)^2/(1{,}359.28)]=442.05.$$

This within-varieties sum of squares of errors of estimate which is adjusted for the average within-variety regression is entered in table 3. The required difference is, then,

$$442.05 - 399.52 = 42.53.$$

This remainder in sum of squares is due to the failure of the individual variety regressions to coincide with the average within-variety regression. It is entered in table 3 along with the associated degrees of freedom. The resulting mean square (3.27) differs only slightly from the error mean square (3.50). The conclusion to be drawn is that the 14 variety regressions are not significantly different; that is, they differ from their average about as one would expect if they were drawn at random from a homogeneous population. In such a postulated population the variety regressions are all equal. The best available estimate of this common population regression is that within varieties. The degrees of freedom for this average regression are less by one than the sum in the second column of table 2. The one deducted corresponds to the regression coefficient, −0.1422, used in calculating the errors of estimate.

In the preceding paragraph a statement was made about the deviations of the yields of the inbred lines from the average regression. To be directly applicable the statement needs a slight qualification which can best be explained by references to the graphs in figures 1 and 2. In the latter figure is plotted the average within-variety regression equation. This graph passes through the point, mean days to silking=32.51 and mean yield=7.01, and has the slope −0.1422. Now, imagine the regression lines of figure 1 all passing through this same point. Each would retain its slope, but would be moved up or down till it contained the point (32.51, 7.01). Any differences among the varietal means would thus be eliminated from consideration. This is exactly what has been done in the foregoing calculations. The mechanism used was this: In each variety the sums of squares and products were those of deviations from the varietal means. The effect is the same as if each observed time and yield had been "corrected" by addition or subtraction so as to make the varietal means equal to the common experimental means, 32.51 and 7.01. It was noted above that even when this has been done, the pooled sum of squares of deviations from the variety regressions is less than the sum of the squares of all deviations from the average within-variety regression.

The next problem is to determine the behavior of the means of the 14 varieties. Mean earliness may be correlated with mean yield. The regression of mean yield on mean time to silks may or may not be significant. This regression may be the same as the average within-variety regression or it may be significantly different. These points and others are now to be considered.

The sums of squares and products appropriate for means may be calculated in the familiar manner (*6, example 2*), using the sums in table 1. For example, the sum of products is

$$\frac{(681)(150.2)}{24} + \frac{(111)(22.3)}{4} + \cdots$$

$$+ \frac{(278)(31.4)}{8} - \frac{(4617)(995.5)}{142} = 206.11.$$

But a simpler method is made available by the computations already completed. From the totals in table 1 it is easy to derive the two sums of squares and the sum of products for the whole experiment. These are—

For time to silks, $152{,}607-[(4{,}617)^2/142]=2{,}489.47.$
For yield, $7{,}740.03-[(995.5)^2/142]=761.01.$
For products, $32{,}380.6-[(4{,}617)(995.5)/142]=12.83.$

The results are entered in table 4. Below them are the "within-variety" totals copied from the last line of table 2. The differences are the sums of squares and products between means of varieties, the sum of products being the same as that got by the longer method shown above. The indicated coefficients and sums of squares of errors of estimate are computed as before. The appropriate results are transferred to the summary table 5. Also, for reference, the sums of squares relating to yield are entered in the same table.

TABLE 4.—*Sums of squares and products, correlation and regression coefficients, and sum of squares of errors of estimate in time to first silks and yield in maize*

Source of variation	Degrees of freedom	Sums of squares and products			Correlation coefficient	Regression of yield on time to silks	Errors of estimate	
		Time to silks Sx^2	Products Sxy	Yield Sy^2			Sum of squares $Sy^2-[(Sxy)^2/Sx^2]$	Degrees of freedom
Total	141	2, 489. 47	12. 83	761. 01	0. 0093	0. 0052	760. 94	140
Within varieties	128	1, 359. 28	−193. 28	469. 54	−. 2419	−. 1422	442. 05	127
Between means of varieties	13	1, 130. 19	206. 11	291. 47	. 3591	. 1824	253. 88	12

TABLE 5.—*Analysis of variance of yield and of errors of estimate in 14 varieties of maize*

Source of variation	Yield			Errors of estimate		
	Degrees of freedom	Sum of squares	Mean square	Degrees of freedom	Sum of squares	Mean square
Within variety, from individual regressions				114	399. 52	3. 50
Between variety regressions				13	42. 53	3. 27
Within variety (table 4)	128	469. 54	3. 67			
Between means of varieties (table 4)	13	291. 47	[1] 22. 42	12	253. 88	[1] 21. 16
Within vs. between regressions				1	65. 00	[1] 65. 00
Total	141	761. 01		140	760. 94	

[1] Highly significant.

The equation for the regression of mean yield on mean days to first silks is

$$\bar{Y}=7.01+.1824\ (X-32.51),$$
$$=0.1824\ X+1.08.$$

This equation is plotted in figure 2. The upward trend of the graph of this equation of means contrasts with the downward sloping graph of the within-variety equation. While, with only 12 degrees of freedom, the regression of means is not significant (*3, p. 129; or 6, p. 67*), it nevertheless has a reasonable interpretation. The tested varieties were selected from both northern and central Iowa, but grown in the latter section only. Those from the north tended to early maturity, thus failing to enjoy the advantages of the longer growing season available. The varieties more closely adapted to the climate, while starting slowly, still had ample time to produce a good crop before frost.

This reasoning, however, is not so strongly supported by the experimental evidence as it would be if the regression coefficient were significant. The variety means are plotted in figure 2. The large

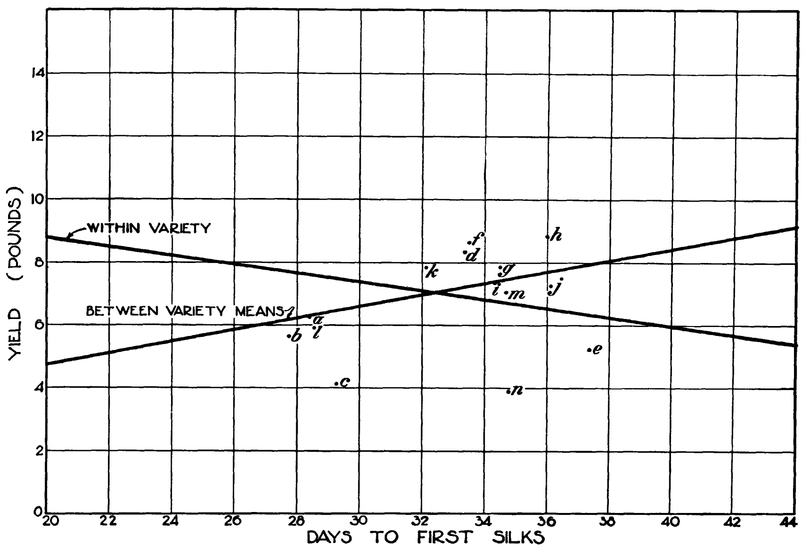

FIGURE 2.—Regressions of yield on days to first silks, within variety and between variety means, for 14 varieties of maize: *a*, Four County White; *b*, Silver King; *c*, U. S. Selection 133; *d*, Iodent; *e*, U. S. Selection 204; *f*, Lancaster Surecrop; *g*, Black Yellow Dent; *h*, Proudfit Yellow Dent; *i*, Krizer Bros. Yellow Dent; *j*, McCulloch Yellow Dent; *k*, Osterland Yellow Dent; *l*, Clark Yellow Dent; *m*, Walden Dent; *n*, Argentine Flint.

errors of estimate are as conspicuous as the trend. The same tenuity of relation is exhibited again in table 5. The mean square error of estimate between means of varieties is almost as large as the mean square for yield, showing that little of the variability of variety yields is associated with differences in earliness. The final source of evidence is the test of the significance of the mean square error of estimate. Since

$$F = 21.16/3.50 = 6.05$$

far surpasses the 1 percent level (*6, table XXXV*), the mean variety yields deviate from their regression much more than can be explained by sampling variation. Under such circumstances, one cannot lightly assume that the regression of means corresponds to a biological entity. However, as indicated before, a positive regression has a reasonable interpretation in this experiment, and receives some support in the remaining effect to be discussed below.

It is of interest to observe the contrast between the total correlation coefficient of table 4, 0.0093, and that within variety, −0.2419. The latter is the correct average of the 14 intravariety coefficients. Now if all the data of table 1 had been thrown together, without consideration of the existing differences among the variety means, the significant within-variety correlation between yield and earliness would have been undetected, being automatically averaged with the between-variety coefficient, 0.3591, to make the total nonsignificant coefficient, 0.0093. This emphasizes the importance of isolating and studying the statistics of homogeneous groups in a large experiment. If it is desirable, these groups may be merged later, but with full knowledge of the homogeneity or heterogeneity of the entire mass of data.

In the right-hand part of table 5 there remains a degree of freedom which has not been discussed. It is attributed to the difference between the two regressions within and between varieties—that is, between the slopes of the two regression lines of figure 2. The mean square, 65.00, furnishes the test ot significance of this difference:

$$F=65.00/3.50=18.6,$$

a value far beyond the 1-percent level. It has been demonstrated before that one of these regressions differs significantly from zero while the other does not. It is now proved that the difference between them is highly significant. While the interpretation is clouded somewhat by the lack of a clear-cut linear relation among the variety means, discussed above, the indication is that the results of the genetic factors influencing the relation between earliness and yield in the varieties examined may be distinguished from those predominantly physiological and environmental governing this relation among the inbred lines of the varieties.

Some experiments are planned with the knowledge that neither the individual regressions nor the regression of means will differ significantly from the pooled within-group regression. In the earliness-yield problem which has been discussed, the former condition was met, but not the latter. When field-plot experiments are laid out in the randomized block or Latin square designs, there is every reason to expect that the regressions will be uniform. In such data, most of the computations described above may be omitted. The only ones necessary are those extracted from table 4 and set out in table 6. Merely the total regression and that within variety are used, with consequent reductions in the degree of freedom for errors of estimate. The sum of squares of errors of estimate of the variety means is got by subtraction. These errors are measured from the average within-variety regression. Since the regression of means is not used in the computation, no reduction is suffered in degrees of freedom for the errors of estimate of these means. The appropriate test of significance is

$$F=24.5/3.48=7.1,$$

highly significant as in table 5. If this particular information is the extent of that required, the foregoing computations may be still further abbreviated in the manner described by Fisher (*3, p. 257*) as well as by Snedecor (*7*) and Cox and Snedecor (*2*).

TABLE 6.—*Errors of estimate of variety means measured from the pooled "within-variety" regression*

[Data taken from table 4]

Source of variation	Degrees of freedom	Sums of squares and products: Time to silks Sx^2	Products Sxy	Yield Sy^2	Errors of estimate: Sum of squares $Sy^2-[(Sxy)^2/Sx^2]$	Degrees of freedom	Mean square
Total	141	2,489.47	12.83	761.01	760.94	140	--------
Pooled within variety	128	1,359.28	−193.28	469.54	442.05	127	3.48
Between-variety means	13	----------	----------	--------	318.89	13	24.5

SUMMARY

There is presented an analysis of the covariance between yield and time to first silks among inbred lines and 14 varieties of maize.

The 14 variety regressions of yield on time to first silks do not differ significantly among themselves, presenting no evidence against the hypothesis that there is a common regression in all the inbred lines.

The average within-variety regression differs significantly from that between-variety means, showing that variety mean yields do not respond to variety earliness in the same manner as the yields of inbred lines respond to their earliness. The variety means depart significantly from their regression, emphasizing this finding. There is included a convenient method for averaging the correlation coefficients of the 14 varieties.

An easy test appropriate for field plot trials is given.

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