SOME FACTORS INFLUENCING THE YIELD AND MORTALITY OF PONDEROSA PINE IN THE SOUTHWEST [1]

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INTRODUCTION

A knowledge of those factors influencing the yield of timber stands that are more or less subject to control by man is of practical importance to the forester. The following analysis of the effect on yield of the factors of volume left after cutting and the average size of reserved trees was undertaken in an effort to add to that knowledge. Data for such a study were available from permanent sample plots established during the past 20 years by the Southwestern Forest and Range Experiment Station on cut-over areas in the ponderosa pine type for the purpose of determining the possible yield of saw timber at the end of a cutting cycle. Analyses of these plots were made by Krauch in 1926 [2] with particular emphasis on the individual tree. In the present study, unit of area (1 acre) has been substituted as the basis of analysis, a method that lends itself readily to statistical treatment.

Because analyses of the relation of increment to time based on periods of 20 years and less usually lead to dangerous extrapolation and are of little practical assistance in predicting yield, this relation was not considered except in a preliminary fashion, and the analysis was centered on the yield at the end of 15 years, in which sense time could be considered as held constant.

MATERIAL AND METHODS

Ninety-three 1-acre plots on which growth records had been taken over a period of at least 15 years were used as the basis of the study. All plots were established immediately after logging, and hence represented the total change subsequent to cutting. There is a rather wide diversity of soil conditions among the plots, but no claim is made of exact representativeness of the universe of cut-over ponderosa pine (*Pinus ponderosa* Lawson).

For each 1-acre plot the following values were determined: (1) Volume of reserved stand; (2) volume of the average tree; (3) increment at the end of 15 years; and (4) total mortality by 5-year periods. All quantities are given in Scribner board feet, the merchantability limits being 10 inches d. b. h.[3] and an 8-inch top diameter.

The reserved volume on each plot was determined by the use of volume tables. In order to eliminate so far as possible any biased

[1] Received for publication Jan. 29, 1935; issued July 1935.

[2] KRAUCH, H. THE DETERMINATION OF INCREMENT IN CUT-OVER STANDS OF WESTERN YELLOW PINE IN ARIZONA. Jour. Agr. Research 32: 501–541, illus. 1926.

——— THE DETERMINATION OF INCREMENT IN CUT-OVER STANDS OF WESTERN YELLOW PINE IN THE SOUTHWEST. Jour. Forestry 28: 978–986, illus. 1930.

[3] Diameter breast high, or 4½ feet above the ground.

errors and to keep the random errors as small as possible, separate volume tables were used for those plots representing two widely different soil types. Separate tables were used also for the two broad age classes, yellow pine and blackjack.[4] Trees that died during the period after cutting were removed from the reserved volume in order to prevent excessive variation in yield and are treated separately under Mortality Analysis (p. 783).

The volume of the average tree on each acre was determined by dividing the total reserved volume by the number of merchantable trees. This variable was selected because it gave some indication of the size, and roughly of the age, of the reserved stand left on each plot.

The increment was found by simply subtracting the original volume of the reserved stand from the volume at the end of 15 years. No allowance was made for a possible change in form of the reserved trees after cutting, although such a change creates a small biased error in the increment values.

In the following analysis advantage was taken of correlation methods and of some of the analytical systems developed by Fisher.[5] The influence of random errors in the independent and dependent variables on a few of the more important statistical constants was kept in mind. It has been demonstrated that random errors in the dependent variable lower the correlation coefficient but do not appreciably alter the regression coefficients, while random errors present only in the independent variables and those in both dependent and independent variables, tend to decrease the magnitude of both the correlation and regression coefficients.[6] It is obvious that in analyses of this kind any tests of significance between regression and correlation coefficients are of questionable value, since it cannot be definitely stated whether the differences observed are due to some definite assignable cause or to the inability to measure each variable accurately. This should not, however, vitiate the test of whether a correlation or regression coefficient is significant from zero, because higher values for both could reasonably be expected if it were possible to measure the variables exactly. In other words, if either coefficient is found to be significant when random errors are present, enough evidence must be accumulated within the data to indicate it over and above the influence produced by random errors.

YIELD ANALYSIS

As a preliminary step, the gross correlation coefficients for increment and reserved volume, for increment and volume of the average tree, and for reserved volume and volume of the average tree were determined and are given in table 1. The symbols used in the discussion are defined as follows:

X_1=increment at the end of 15 years.
X_2=reserved volume.
X_3=volume of average tree on plot.

[4] "Yellow pines" are ponderosa pines 200 years old and older; they have a deep-yellowish or cinnamon-brown bark. "Blackjacks" are 150 years old or less and have dark, nearly black bark.

[5] FISHER, R. A. STATISTICAL METHODS FOR RESEARCH WORKERS. Ed. 3, 283 pp., illus. Edinburgh and London. 1930.

[6] EZEKIEL, M. METHODS OF CORRELATION ANALYSIS. 427 pp., illus. New York. 1930. (Ch. 19.)

TABLE 1.—*Correlation coefficients for increment and reserved volume, increment and volume of average tree, and reserved volume and volume of average tree*

Variables	Gross coefficient of correlation	Gross coefficient of determination	Significance [1]
X_1 and X_2	0.7982	0.6371	Significant.
X_1 and X_3	−.3546	.1258	Do.
X_2 and X_3	−.0067		Insignificant.

[1] Determined by the aid of Fisher's table V. A., p. 176. (See footnote 5.)

The coefficients of determination, which are equal to the coefficient of correlation squared, are included because they express much more clearly the relationship between variables.

From table 1 it is evident that both reserved volume and the volume of the average tree are important factors influencing increment. Lack of intercorrelation between these factors indicates that a heavy reserved stand is not necessarily composed of large trees nor is a light reserved stand necessarily composed of small trees. As a matter of fact, the lack of significant correlation implies that the exact composition of each plot is due to chance. This lack of intercorrelation further suggested the possibility of improving the coefficient of total determination to approximately the sum of both gross determinations given in table 1, by combining the two variables X_2 and X_3 into a multiple correlation.

When this suggestion was followed out, the resulting coefficient of total determination was 0.7591 and the corresponding correlation coefficient 0.8712. Other important points in this analysis are given in table 2.

TABLE 2.—*Contribution of reserved volume and volume of average tree to volume growth*

Variables	Net regression coefficient	$\frac{\text{Standard error}}{\text{regression coefficient}}\times 100$	Coefficient of separate determination
X_1 and X_2	0.2486±0.0162	6.5	0.6352
X_1 and X_3	.9377± .1389	14.8	.1239
Total			.7591

The resulting regression equation is

$$X_1=647.43+0.2486X_2-0.9377X_3.$$

From table 2, column 4, it can be seen that the two coefficients of separate determination are approximately equal to the corresponding gross determination coefficients in table 1. The standard errors of the two regression coefficients are included in column 2, and in column 3 they are expressed as a percentage of the corresponding regression coefficients. It is evident that reserved volume measures its influence on increment rather accurately and that the

volume of the average tree also measures its influence on increment, but to a lesser degree of reliability.

The net regression coefficient of X_1 on X_3 is negative, which means that for a given reserved volume the yield per acre tends to become

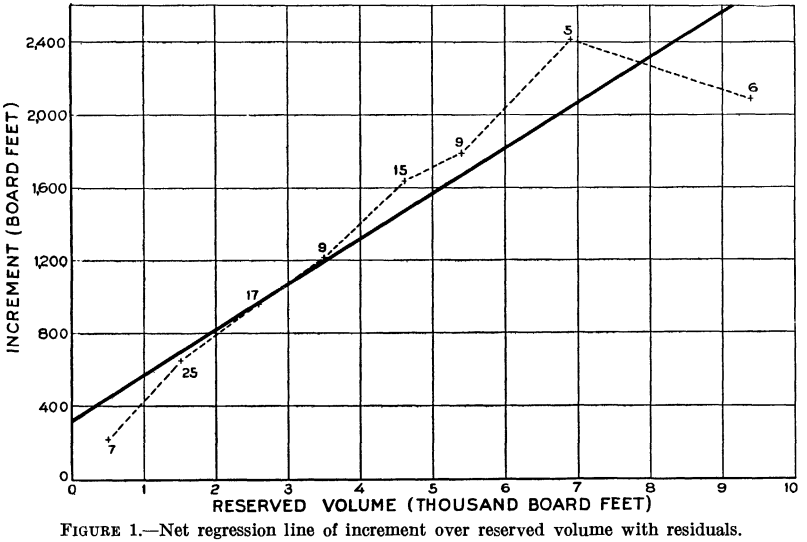

FIGURE 1.—Net regression line of increment over reserved volume with residuals.

less as the volume of the average tree increases. In other words, so far as increment is concerned it would be much more desirable to have the reserved volume divided among a relatively large number of small trees than among a few large ones.

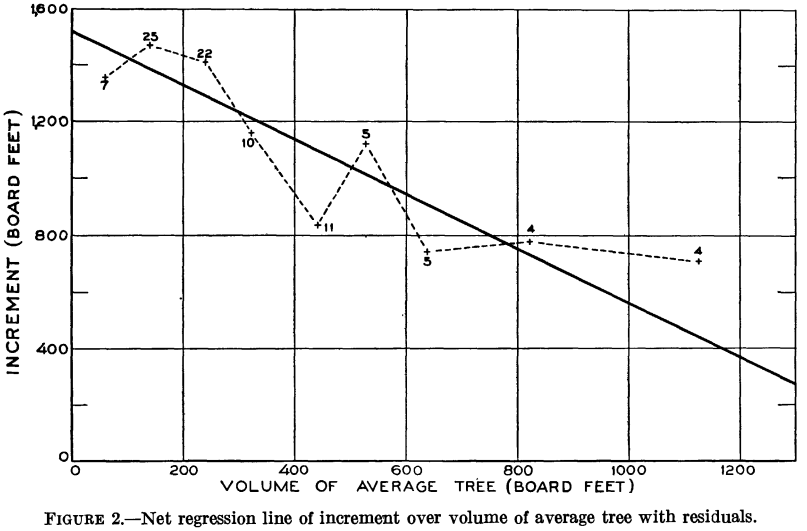

FIGURE 2.—Net regression line of increment over volume of average tree with residuals.

Up to this point the analysis has assumed that the influence of X_2 and X_3 on increment was linear. The truth of this assumption can be tested by plotting the residuals over the net regression lines

oi increment over reserved volume, and increment over the volume of the average tree. The results of these calculations are shown graphically in figures 1 and 2. There is a marked indication of

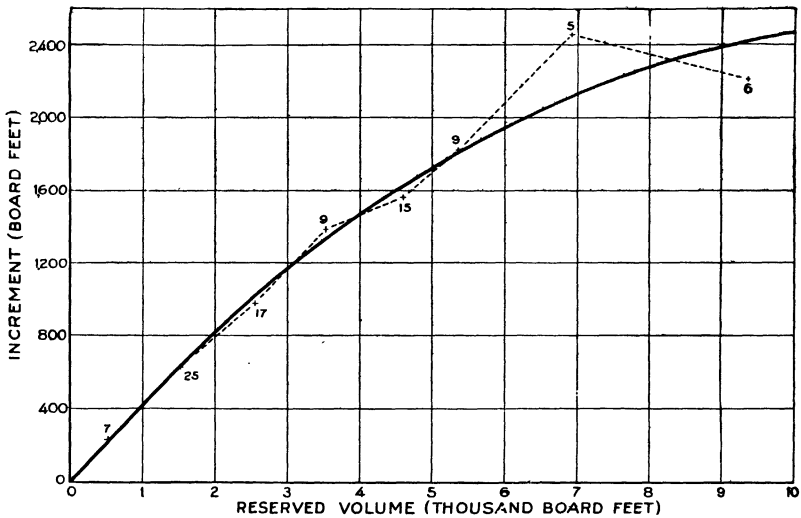

FIGURE 3.—Final net regression curve of increment over reserved volume with residuals.

curvature in figure 1 which is evident also, though it is not so pronounced, in figure 2.

By introducing curvature into the regression surface, it should be possible to further improve the coefficient of total determina-

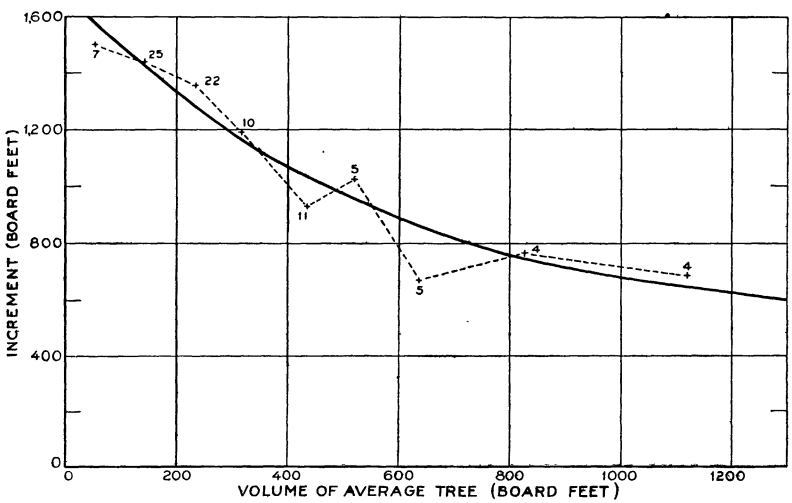

FIGURE 4.—Final net regression curve of increment over volume of average tree with residuals.

tion. This was done by the method of successive approximation.[7] The final net regression curves are given in figures 3 and 4. They represent graphically the influence of reserved volume of increment

[7] EZEKIEL, M. See ch. 14 of citation mentioned in footnote 6.

with the volume of the average tree held constant at its mean (351 board feet) and the influence of the volume of the average tree on increment with the reserved volume held constant at its mean (3,490 board feet). From figure 3 it is apparent that increment increases constantly with an increase in reserved volume but at a constantly decreasing rate. In figure 4 this relationship is reversed, that is, the increment becomes constantly less with an increase in the volume of the average tree, but the decrease comes at a constantly decreasing rate.

A test for joint correlation of the type where $X_1=\theta[a+f(X_2)-f(X_3)]$ was made. The results are shown graphically in figure 5. Since

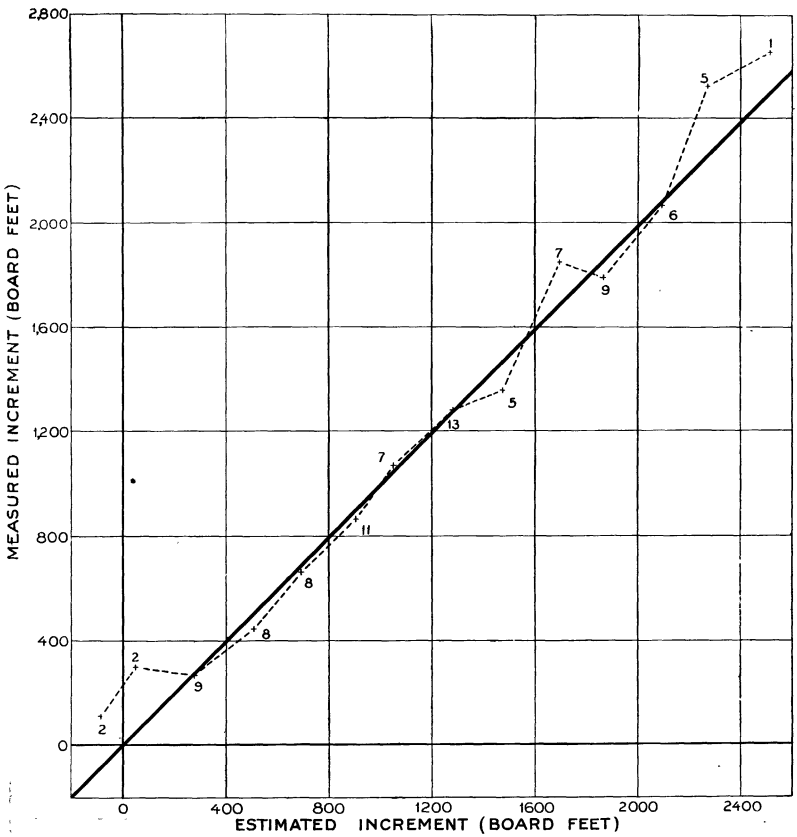

FIGURE 5.—Relation of measured to estimated increment.

there was no marked indication of curvature present, it was concluded that this type of joint relationship did not exist. Further tests for joint correlation could not be made because of insufficiency of data.

Through the introduction of curvature in the regression surface it was possible to improve the correlation coefficient from 0.8712 to 0.8974 and the corresponding coefficient of determination from 0.7591 to 0.8054. From this it was concluded that approximately 80 percent of the variance of increment at the end of 15 years is associated with reserved volume and the volume of the average tree left after cutting.

MORTALITY ANALYSIS

Any determination of yield for selectively cut stands is more or less affected by loss through mortality. The amount to be deducted from gross yields due to this factor alone is subject to considerable error because of its extreme variability. Available data now indicate that if mortality continues at its present rate the annual loss from this source will be between 10 and 30 percent of the average annual growth. It has, however, been assumed that this loss will decrease with time after cutting, and table 3, which gives the mortality for each plot by 5-year periods and the mean mortality for all plots in these periods, appears to lend credence to this hypothesis.

TABLE 3.—*Loss in board feet through mortality, by 5-year periods*

Plot no [1]	0–5 years	6–10 years	11–15 years	Total loss	Mean period loss
	Board feet	*Board feet*	*Board feet*	*Board feet*	*Board feet*
1	821	0	0	821	273.67
2	580	28	0	608	202.67
5	0	29	0	29	9.67
7	1,380	1,196	655	3,231	1,077.00
8	57	50	70	177	59.00
9	0	0	39	39	13.00
11	0	900	0	900	300.00
12	855	290	1,933	3,078	1,026.00
13	0	1,180	0	1,180	393.33
15	1,260	0	0	1,260	420.00
17	0	1,175	0	1,175	391.67
19	32	0	0	32	10.67
20	29	389	31	449	149.67
23	0	42	0	42	14.00
25	0	31	0	31	10.33
26	0	0	37	37	12.33
28	632	28	0	660	220.00
29	840	0	0	840	280.00
31	34	902	0	936	312.00
32	0	277	0	277	92.33
35	0	37	0	37	12.33
37	0	0	1,370	1,370	456.67
40	1,395	0	0	1,395	465.00
42	755	0	0	755	251.67
46	778	0	0	778	259.33
47	615	0	47	662	220.67
50	0	42	0	42	14.00
52	0	0	692	692	230.67
54	0	0	645	645	215.00
55	0	1,280	0	1,280	426.67
58	459	0	0	459	153.00
59	106	0	0	106	35.33
61	603	985	0	1,588	529.33
62	0	0	1,520	1,520	506.67
64	0	0	25	25	8.33
67	21	0	0	21	7.00
68	0	0	512	512	170.67
70	66	157	0	223	74.33
71	0	0	88	88	29.33
74	29	0	132	161	53.67
76	0	194	305	499	166.33
78	0	227	0	227	75.67
79	58	0	0	58	19.33
82	0	27	0	27	9.00
84	0	0	160	160	53.33
86	94	0	0	94	31.33
87	0	0	52	52	17.33
89	0	618	0	618	206.00
91	0	449	0	449	149.67
Total	11,499	10,533	8,313	30,345	10,115.00
Mean	123.6452	113.2581	89.3871	326.29032	108.76344

[1] Those of the 93 plots on which no trees died during the 15-year period are not listed

The hypothesis can be tested for the early period after cutting by the method of analysis of variance. Table 4 summarizes the analysis of the data given in table 3, the sum of squares within periods being equal to $\underset{1}{\overset{kn}{S}}(x-\bar{x}_p)^2$.

TABLE 4.—*Summary of analysis of mortality data in table 3*

Description	Degrees of freedom	Sum of squares	Mean square	Standard deviation	Natural log of standard deviation
Within periods	276	26, 214, 449. 16	94, 979. 89	308. 188	5. 7307
Between periods	2	57, 391. 28	28, 695. 64	169. 398	5. 1323
Total	278	26, 271, 840. 44	94, 503. 02	307. 413	[1] 0. 5984

[1] The value of z, determined as insignificant by the aid of Fisher's table 6. (See footnote 5.)

Where

k=number of plots used in a period
x=mortality
$\bar{x}_p$=mean mortality for period
n=number of periods

the sum of the squares between periods equals

$$k^n{}_1S(\bar{x}_p-\bar{x})^2$$

and

$\bar{x}$=the general mean for all plots.

The lack of significant intraclass correlation indicates that not enough evidence has been accumulated to justify the conclusion that mortality is related to time since cutting. Although the mean mortality by periods shows a progressive decrease, the differences between means can readily be explained as being caused by errors of random sampling and not necessarily by some influence of time.

The foregoing analysis also indicates that the best estimate of the standard deviation of mortality is 307.413. With the standard deviation of mortality known, it is possible to determine the standard error of the mean from the relationship

$$s_m=\frac{s}{\sqrt{n}}$$

where

s=estimated standard deviation
s_m=estimated standard error or mean
n=number of observations.

Substituting the observed figures—

$$\frac{307.413}{\sqrt{279}}=18.40.$$

This establishes the standard error of the mean mortality as 18.40 board feet.

The average growth for a period of 5 years is approximately 300 board feet. Should it be desired to determine the average periodic

mortality within an accuracy of ±5 percent of the periodic growth and with odds 19 to 20 that the true mortality will be within this range, the area which must be sampled to assure this reliability can be determined from the formula—

$$n=\left(\frac{s^2}{s_m}\right)$$

where n=number of acres
s=the best estimated standard deviation of population in question
s_m=desired standard error.

Substituting and solving—

$$n=\left(\frac{307.413}{7.5}\right)^2=1{,}680 \text{ acres.}$$

For units smaller than 1,680 acres it is necessary to cover the area 100 percent to insure the above-stated accuracy of the mean. For larger units the percentage of area to be covered naturally becomes less.

It is sometimes possible to reduce the magnitude of the standard error of a mean by satisfactorily establishing correlation within the sample. In this case it appeared probable that mortality at the end of 15 years might conceivably be related to the amount of reserved volume and the volume of the average tree. A multiple correlation was performed, using the above-mentioned factors as independent variables, and a correlation coefficient of 0.2922 was obtained. The corresponding determination coefficient was 0.0854, which implies that only 8.5 percent of the variance of mortality is associated with the amount of reserved volume and the volume of the average tree.

The coefficient of multiple correlation in this case was so low that it appeared advisable to test it for significance. A summary of this test is given in table 5.

TABLE 5.—*Summary of test for significance of coefficient of multiple correlation between mortality at 15 years, amount of reserved volume, and volume of average tree*

Description	Degree of freedom	Sum of squares	Mean squares	One-half natural log
Regression formula	3	2,787,094.99	929,031.66	6.87095
Deviation from regression	90	29,855,077.88	331,723.09	6.35603
Value of z				[1] 0.51492

[1] Determined as significant by the same test used for z in table 4.

Since the value of z was found to be significant it was concluded that reserved volume and the volume of the average tree left after cutting have some influence on mortality.

The reliability of each regression coefficient was also analyzed, and a summary of the results is given in table 6. It is evident that, while the reserved volume and the volume of the average tree are significantly related to mortality, they do not measure this relation accurately, since in both cases the standard errors are equal to approximately one-half of the corresponding regression coefficients.

In figures 6 and 7 the net regression lines of mortality over reserved volume and mortality over the volume of the average tree are plotted with the corresponding residuals. It is clear from this that an increase

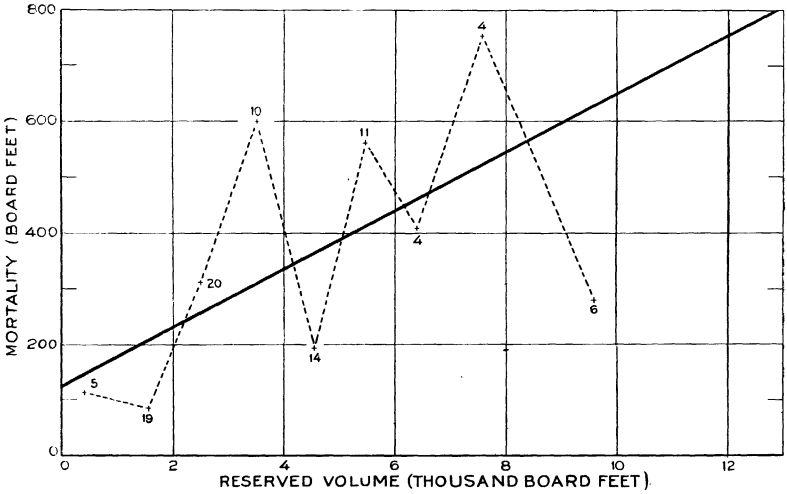

FIGURE 6.—Net regression line of mortality over reserved volume with residuals.

in either reserved volume or the size of the average tree leads to greater mortality. Casual observation of the plotted data alone indicates that this relationship is not very strong, and although a straight

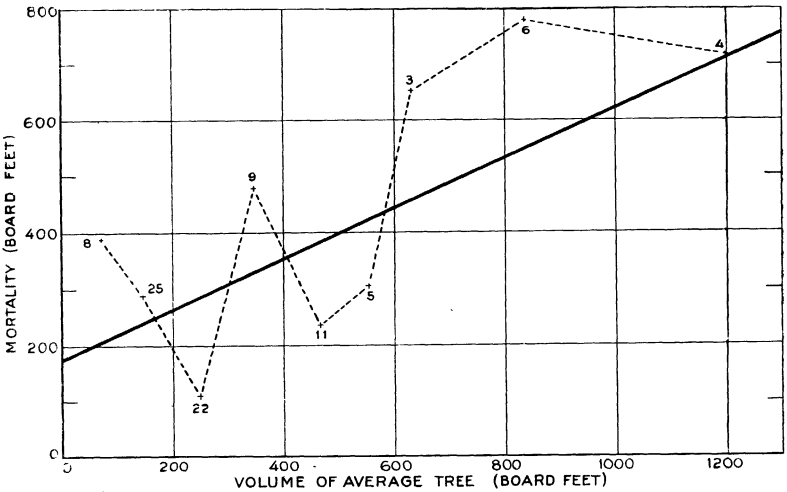

FIGURE 7.—Net regression line of mortality over volume of average tree with residuals.

line has been fitted to the data, it cannot be definitely assumed that the relationship is proportional.

TABLE 6.—*Summary of test for reliability of regression coefficients of correlation between mortality at 15 years, amount of reserved volume, and volume of average tree*

Variables	Net regression coefficients	Standard error	Regression coefficient / Standard error	Standard error / Regression coefficient ×100
X_1 and X_2	0.0530	0.0258	2.0542	48.7
X_1 and X_3	.4500	.2239	2.0099	49.8

SUMMARY

Analysis of factors influencing yield of timber stands indicates that the increment per acre at the end of 15 years after cutting is closely related to the volume of the reserved stand and the volume of the average tree. These two variables alone explain approximately 80 percent of the variance of increment; the remaining 20 percent is attributed to such factors as the degree of release, site differences, and differences in vigor of reserved stands.

By holding constant the influence of the size of trees left after cutting, it was found that the yield increased with an increase in reserved volume and that this increase came at a constantly decreasing rate. With the influence of reserved volume held constant, the increment decreased as the size of the average tree increased, and this decrease came at a constantly decreasing rate. This naturally follows from the fact that with a given reserved volume the amount of growing surface increases with a decrease in the average size of the trees left after cutting. Furthermore, as the size of the average tree decreases the average age also decreases, and from the nature of the curve of growth with time it follows that, in the average, more growth can be expected from a given volume of young trees than for the same volume of older trees.

The average mortality per acre for a period of 5 years was found to be 108.8 board feet with a standard error of 18.4 board feet. No significant relationship was found between mortality and years after cutting. There was, however, evidence of correlation between mortality and the two independent factors, reserved volume and volume of the average tree left after cutting, since an increase in either variable resulted in an increase in mortality. This relationship was not strong enough, however, to indicate definitely whether or not it was proportional.

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