PERFECTING A STAND-DENSITY INDEX FOR EVEN-AGED FORESTS[1]

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INTRODUCTION

An adequate expression for density of stocking in even-aged forests has long been sought by foresters. Comparison of total basal area of the stand with yield-table values of basal area for the same age and site quality has been the usual method of evaluating stand density. Other methods have been proposed, but none has given results good enough to warrant general adoption or displacement of the basal-area method. It is the purpose of this paper to present a stand-density index which does not require a yield table and which is not affected by possible errors in shape of the *total basal area-age* curve. This stand-density index, based on the relationship between number of trees per acre and their average diameter, is premised on the characteristic distribution of tree sizes in even-aged stands.

It is a well-established fact that in any given stand a curve showing the relative (percentile) frequency of occurrence of the various tree sizes (diameters) has a characteristic form, often approximating that of the "normal frequency curve" or "normal curve of error" (*1*, *2*, *3*, *4*, *6*, *7*).[2]

STATISTICAL BASIS

This frequency-curve form may differ with species; the departures from normal may embody positive or negative skewness (*6*), or a logarithmic form may be assumed (*4*). Within a given species, however, stands of all ages on all sites have essentially the same characteristic frequency-curve form (*4*, *7*, *9*).

The form of the frequency curve may be described by several statistical measures, those used most commonly being the average (or mean) diameter and the standard deviation, with the coefficients of asymmetry and of excess less generally used (*6*).

For any one general form of frequency curve, as applied to diameter distribution in even-aged stands, the percentile frequency curve for a specific stand is described primarily by the average diameter. The standard deviation and coefficients of asymmetry and of excess provide a further description of the curve, but since they are correlated with the average diameter they are of secondary importance. It is thus possible to a certain extent to describe the relative diameter distribution of a stand by average diameter alone.

COMPARISON OF STAND DENSITIES

The concept of stand description is useful in comparing the density of stocking of various stands. Two stands of the same description

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[2] Reference is made by number (italic) to Literature Cited, p. 637.

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(same average diameter and, by implied correlation, the same standard deviation, etc.) have the same relative distribution of diameters but may differ as to total number of trees per unit area. Obviously, the stand with the greater number of trees is the better stocked or denser stand of the pair and their relative stand densities are directly proportional to the number of trees on them. The number of trees per acre of one stand may be expressed as a percentage of the number of trees of the other; this percentage will indicate the relative stand densities. If data for a sufficient number of stands of the same description (average diameter) are obtained, the stand with most trees per acre can be considered as having 100 per cent stand density. If all other stands are referred to this one, the density of each can be expressed as a percentage of full density.

THE REFERENCE CURVE

The number of trees per acre for full density varies, however, with the average diameter of the stand. Stands of small average diameter have a large number of trees, while stands of large average diameter have relatively few. To determine the density of stands of all descriptions, it is necessary to have a curve showing the number of trees per acre at full density for all average diameters.

This curve of maximum number of trees per acre over average diameter when plotted on ordinary cross-section paper is concave upwards, falling rapidly in the small diameters and flattening as the larger diameters are reached. When plotted on logarithmic cross-section paper, this curve assumes a straight-line form. For many species the slope of this logarithmic straight-line graph is constant, but its elevation differs with species. This curve is represented by the equation,

$$\log N = -1.605 \log D + k$$

in which N is the number of trees per acre, D is their average diameter (by basal area), and k is a constant varying with species. When k is 4.605 the curve passes through the point representing 10 inches average diameter, 1,000 trees per acre, as shown in Figure 1 (solid line). This curve will be referred to hereafter as the "reference curve."

CONFORMITY TO THE REFERENCE CURVES

In Figure 2 is shown a series of California red fir (*Abies magnifica* A Murray) yield-plot values as collected by Schumacher (*9*). The reference curve is a very good fit for the maximum values. Similarly, in Figure 3, a curve parallel to the reference curve but passing through 10 inches, 830 trees, expresses the maxima for yield-plot data for white fir (*A. concolor* Lindl. and Gord.).

Other species also conform to the reference curve. Curves parallel to the reference curve are fitted to yield-plot values for the mixed conifer types in California (measured by Dunning, Show, and others) in Figure 4, A, to second-growth Douglas fir (*Pseudotsuga taxifolia* (LaM.) Britt.) in Oregon and Washington (measured by McArdle) in Figure 4, B, and to second-growth Douglas fir in California (measured by Schumacher) in Figure 4, C.

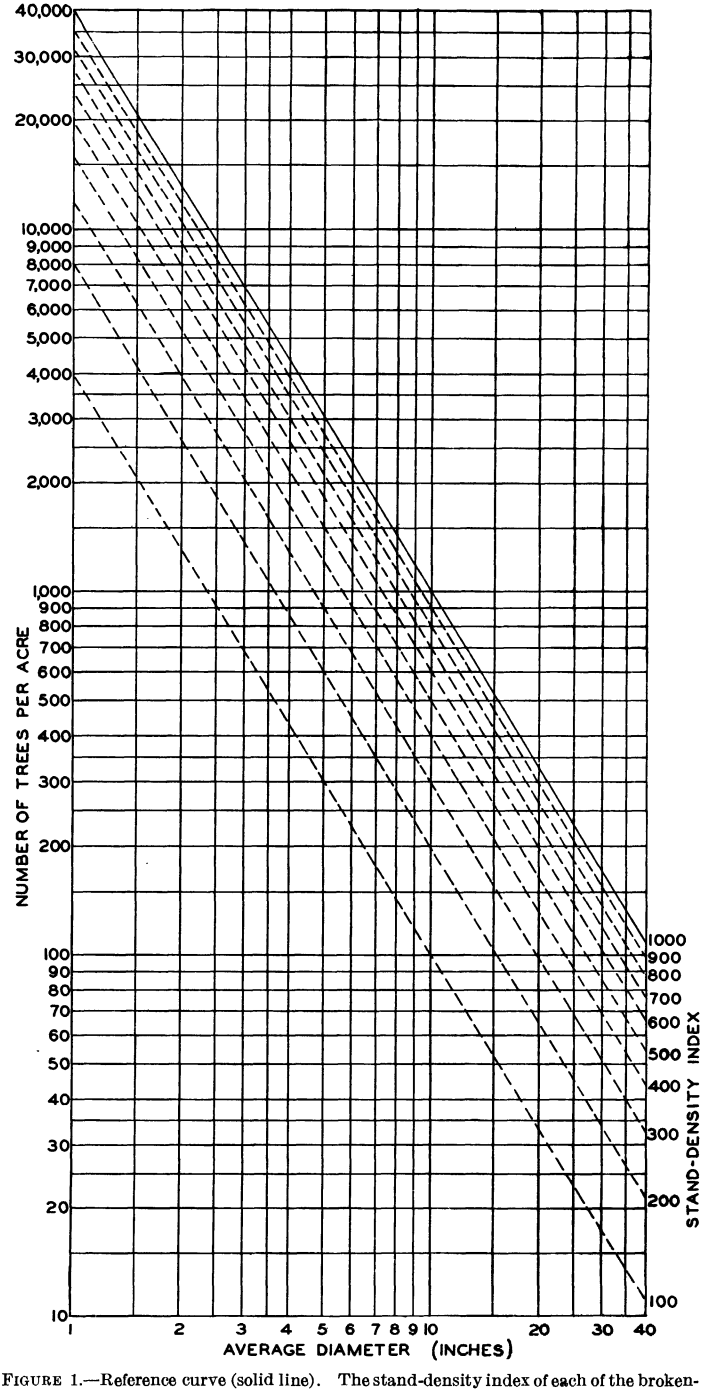

FIGURE 1.—Reference curve (solid line). The stand-density index of each of the broken-line parallel curves is the number of trees indicated by each at 10 inches average diameter

In Figure 5 data are shown for planations of *Eucalyptus globulus* Labill. (*5*), for second-growth ponderosa pine (*Pinus ponderosa* Laws.) as measured by Gallaher, and for second-growth redwood (*Sequoia sempervirens* (Lamb.) Endl.) as measured by D. Bruce. Most of the eucalyptus plantations (fig. 5, A) were widely spaced and very young. They were, therefore, much understocked when measured. The maximum points to which the curve (parallel to the reference curve) has been fitted, represent the few closely spaced or older plantations.

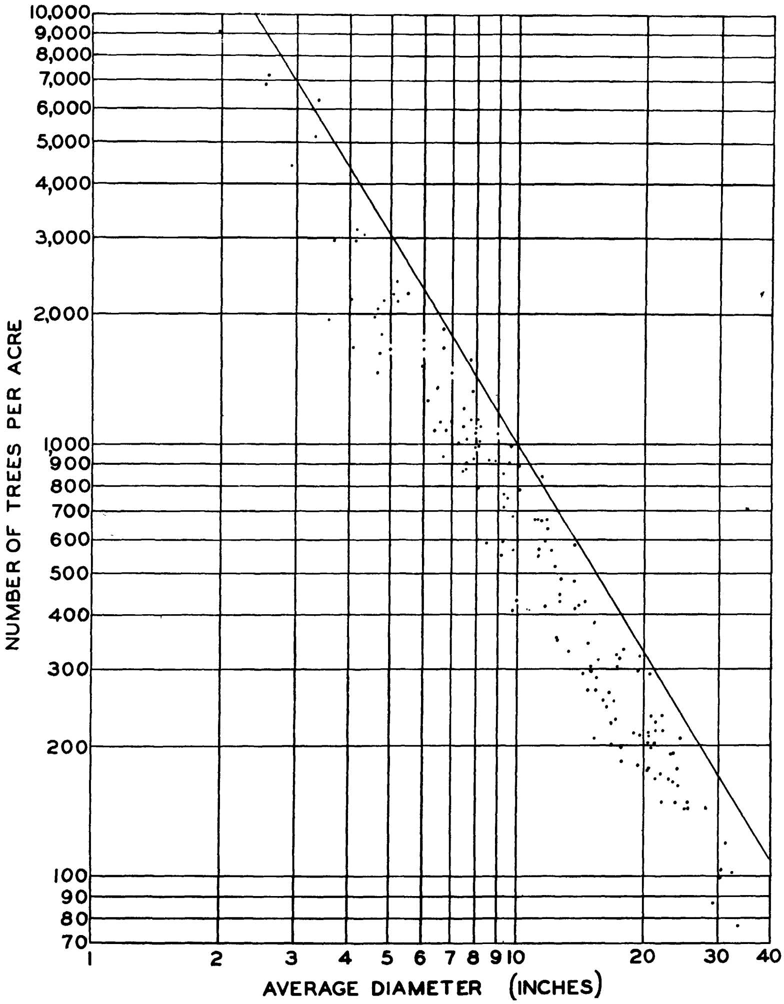

FIGURE 2.—Number of trees—average diameter relation for red fir, with reference curve defining the maxima

The data for ponderosa pine (fig. 5, C) were taken to represent "overstocked" stands. Although these data cover only a small range in average diameter, they fit the curve of standard slope quite well.

The maximum curve for redwood (fig. 5, B) is less well defined. The distribution of plots by average diameter is poor, however, and additional data below 10 inches and above 20 inches are necessary to establish conformity or nonconformity with the reference curve.

In each of the preceding groups of data, the curve has been fitted to represent the maximum values rather than the average. Ordinarily the average values of any group of data will define a curve better than the maxima, especially if the conditions of random sampling are met. In yield studies, however, biased sampling, as to stand density, is the rule. Plots are selected for maximum stocking and poorly stocked plots are not desired, but in some circumstances normality standards must be lowered to secure adequate representation of stands of certain ages or site quality. This is usually neces-

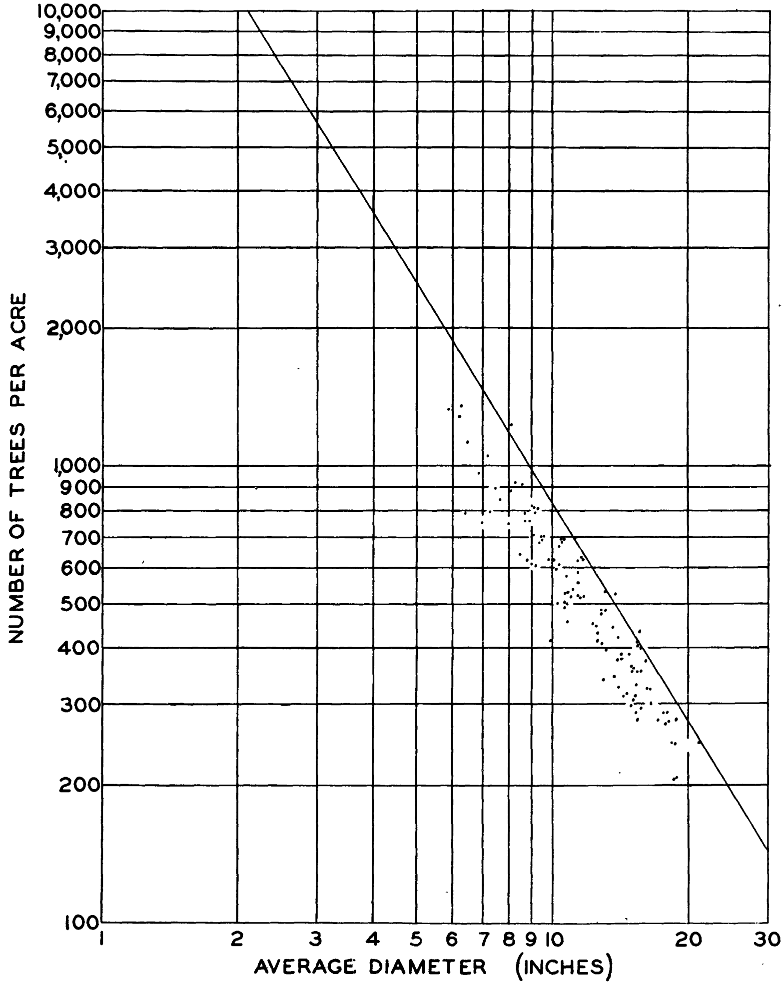

FIGURE 3.—The maximum stand-density index for white fir is 830, as shown by the curve defining the maxima and paralleling the reference curve

sary with the larger, older stands; since they are more mature, many of the better stands are cut, leaving only the poorly stocked ones which are less desirable commercially. The result of such a selection would be a lowering of average stand density for large stands, which would lower the large-diameter end of the number of trees—average diameter curve. This distortion is avoided by fitting to the maxima.

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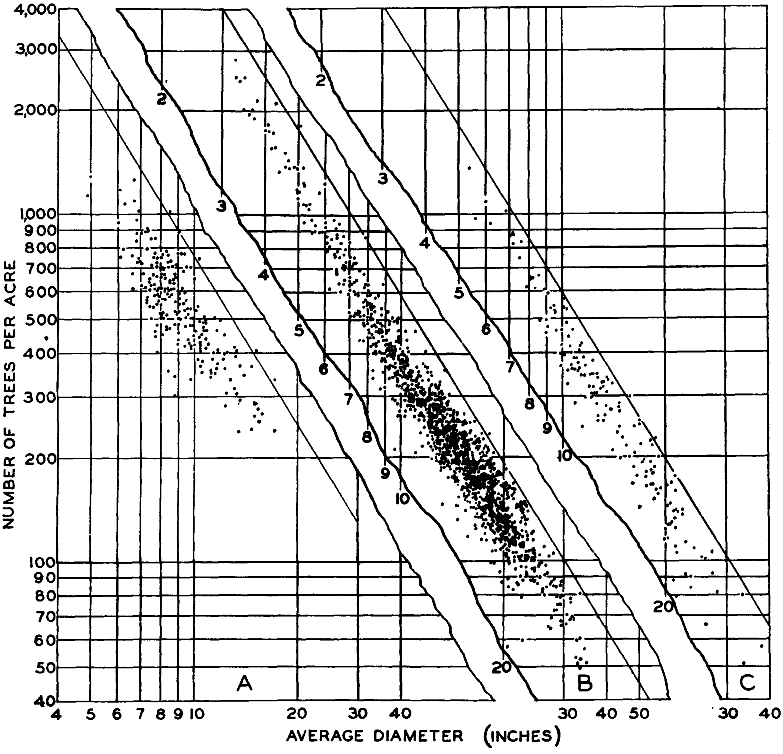

FIGURE 4.—Maxima curves for: A, Mixed conifer stands in California; B, Douglas fir in Washington and Oregon; C, Douglas fir in northern California. Note that the maximum stand-density index is almost identical (approximately 595) for both groups of Douglas fir

Where random sampling is secured, as in strip cruises, fitting to average points is more satisfactory. Figure 6 represents such a case, the data resulting from a strip cruise by G. H. Barnes in second-growth lodgepole pine in British Columbia (*1a*). The numbers in Figure 6 indicate the number of 1-acre plots represented by each average point. The solid line is parallel to the reference curve; the broken (straight) line corresponds to the curve fitted by Barnes on semilogarithmic paper.

In all the above examples, the reference curve, or one parallel thereto, has fitted reasonably well.[3] This is also true for longleaf pine (fig. 7, C) and for loblolly pine (fig. 7, B), based on yield-study plots. In Figure 7, A, showing measurements at 5-year intervals of

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[3] This is also true for southern white cedar (*Chamaecyparis thyoides* (L.) B. & P.) and for northern white pine (*Pinus strobus* L.), not illustrated.

two permanent sample plots in loblolly pine, the same slope is indicated. The slope for slash pine, Figure 7, D, is slightly steeper than for the reference curve (broken line), although it is none too well defined. For shortleaf pine, however, the slope is definitely steeper. (Fig. 7, E.)

This possible nonconformity of slash pine, and the definite nonconformity of shortleaf pine, may be due, perhaps, to the influence of fire. Although unburned plots were sought for the yield study, few of them were entirely free of fire damage, and the chances of fire injury were obviously greater in the older, larger stands.

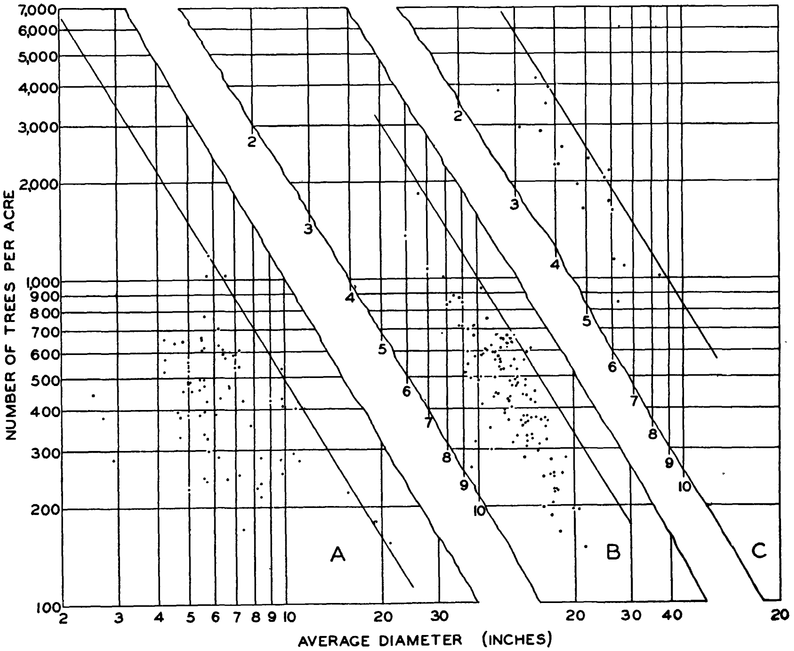

FIGURE 5.—Curves, parallel to the reference curves, for: A, Eucalyptus (in plantations); B, redwood; C, ponderosa pine

EFFECT OF AGE AND SITE QUALITY

Before accepting this relationship of number of trees per acre to average diameter as an index to stocking, it is necessary to examine into the possible effect of age and site quality. Multiple linear correlations were computed for several species with number of trees per acre in a percentage of the maximum curve as the dependent variable, and with height of average dominant tree of the stand and total age of stand as independent variables. Age and dominant height were used in preference to site index, since the latter would be influenced by any improper shaping of the site-index curves. The regression equations, with the statistical measures, are given in Table 1.

TABLE 1.—*Regression equations and statistical measures resulting from multiple linear correlations of trees per acre, dominant height, and total age of stand*

Species	Regression equation	Alienation coefficient	Correlation coefficient	Ratio of correlation coefficient to probable error
Douglas fir (in California)	Percentage number of trees = −0.067 age +0.011 dominant height+51. 075.	0. 990	0. 141±0. 0501	2. 81
White fir	Percentage number trees=0.006 age+ 0.095 dominant height+56.491.	. 978	. 208± . 071	2. 93
Red fir	Percentage number trees=0.229 age− 0.194 dominant height+61.98.	. 968	. 072± . 049	1. 47

For Douglas fir and white fir, the regression coefficients are very small. For red fir they are considerably larger, but the correlation coefficient is the smallest (0.072) and least significant (correlation coefficient only 1.47 times its probable error). For each species the correlation coefficient is small and insignificant, since it does not exceed three times its probable error.

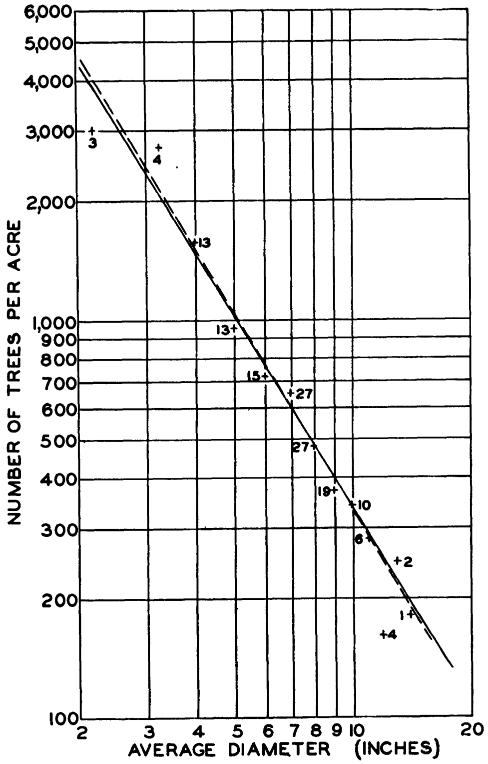

FIGURE 6.—Curve (solid line), parallel to reference curve, fitted satisfactorily to points based on strip cruises in lodgepole pine stands in British Columbia; the broken line corresponds to the curve fitted by Barnes on semilogarithmic paper

Apparently, there is no significant or appreciable correlation between age or site quality and the number of trees per acre for a given average diameter. It is, then, safe to use the *number of trees—average diameter* curve as a standard to which similar values of an individual stand may be compared to determine its density of stocking.

PERCENTAGE STOCKING

The numerical expression of density of stocking may take two forms. The obvious way to express this measure is to establish the position of the *number of trees—average diameter* curve for maximum or 100 per cent stocking for a given species and to express individual stand values as a percentage of this maximum curve. The value thus derived may be termed the "percentage stocking." This is a simple, usable measure, but has the disadvantage of requiring prior determination of the maximum curve. Comparison of one species with another is less simple, because of differences in maxima.

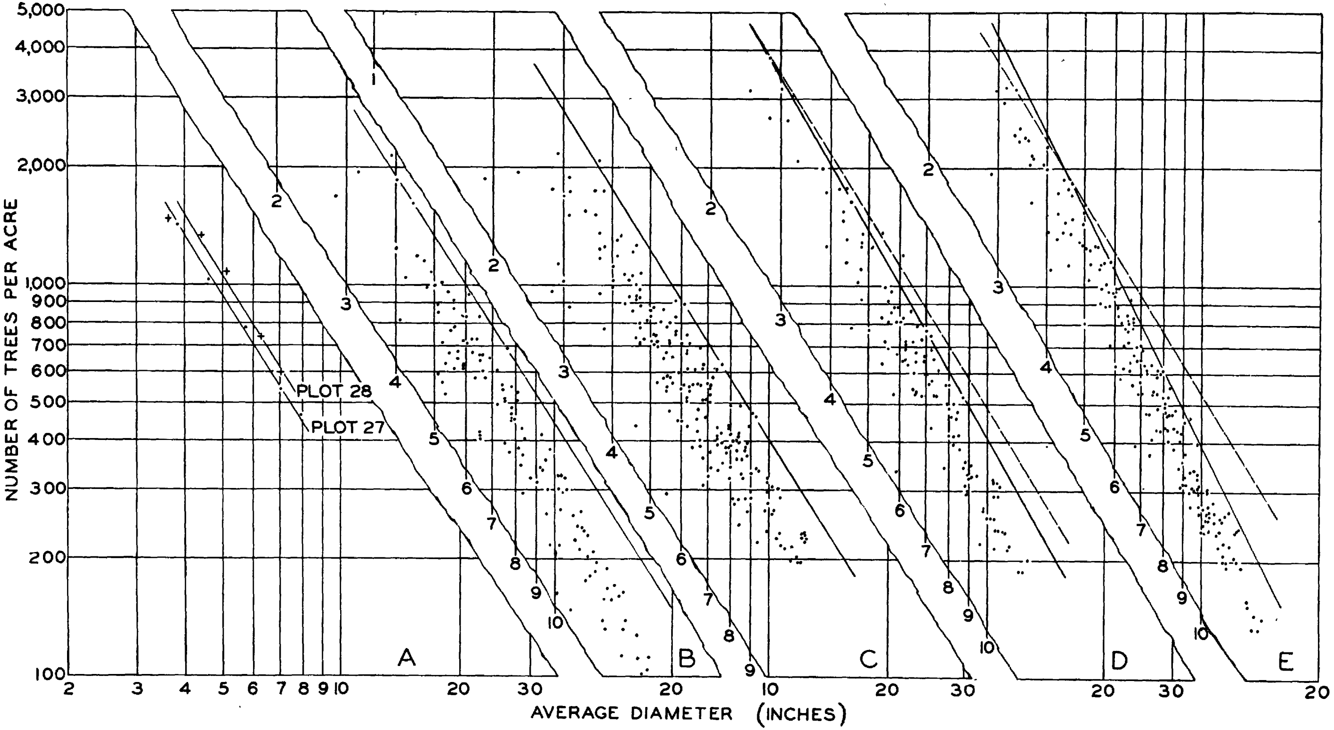

FIGURE 7.—Agreements with and divergences from the reference curve in different species. Parallelism is found in (A) loblolly pine at 5-year intervals, on permanent plots, (B) temporary yield-plot data for loblolly pine, and (C) longleaf pine; divergence is noted to a slight degree in (D, dotted line) slash pine, and more definitely in (E, dotted line) shortleaf pine

STAND-DENSITY INDEX

A second expression for density, which may be termed the "stand-density index," permits direct comparison between species in addition to expressing varying stand densities within a species. Using the reference curve as the basis for all species for which the slope is the same as that of the reference curve,[4] the number of trees per acre in a given stand is plotted on a graph similar to Figure 1 (with the broken-line curves omitted) and a line is drawn through this point, parallel to the reference curve, until it intersects the 10.0 inch ordinate. The number of trees at this point of intersection is taken as the stand-density index.[5] Since the reference curve passes through 1,000 trees per acre at 10.0 inches, this value is ten times the percentage relation between the stand and the reference curve.

In practice, determination of the 10-inch intercept value by means of the parallel line as described above is not necessary. To Figure 1 has been added a series of broken-line curves parallel to the reference curve and intersecting the 10-inch ordinate at 100, 200, 300 trees, etc., and so labeled. When the average diameter and number of trees per acre of a stand are plotted on this graph, the stand-density index (to the nearest 50 trees per acre) is given by the parallel line nearest the plotted point. By interpolation between curves, stand-density index may be read easily to the nearest 10 trees per acre.

Stand-density index can also be expressed as a percentage of the reference curve. It is deemed more desirable, however, to use the number of trees as the index. This is a quantitative, not a relative, measure and permits a better visualization of stand conditions. Furthermore, the use of number of trees as the index avoids confusion with the percentage stocking values discussed previously.

It is deemed desirable, therefore, to limit the use of percentages to percentage stocking values, for comparison of stands of the same species. For interspecies comparisons, the stand-density index expressed in number of trees is desirable, not only because it is a quantitative measure, permitting better visualization of stand conditions, but because it is directly proportional to the percentage relationship (ten times the latter), thus incorporating any advantages of the percentile values, yet avoiding confusion with percentage stocking.

CONCLUSIONS

The method of determining density of stocking in even-aged stands, which has been described, has the advantages of simplicity, freedom from correlation with age and site index, and general applicability. The equation derived, expressing the relationship between number of trees per acre and average diameter, satisfies the data for 12 of the 14 species investigated and departs but slightly for the thirteenth. Additional species should be investigaged for conformity with results presented here. For species, or groups of species, not conforming to this equation, separate reference curves may be established, but it is recommended that the number of trees at an average

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[4] For species in which the slope differs from that of the reference curve, a curve paralleling its maximum curve and passing through 1,000 trees at 10.0 inches can be used as a reference curve.

[5] This stand-density index is determined very much as is site index, since it is the number of trees per acre with an average diameter of 10.0 inches which the stand had or will have, assuming that the change in number of trees and average diameter progresses parallel with the reference curve.

diameter of 10 inches be maintained as the basis for stand-density index.

Similar investigations of selection stands will be of value. Schæffer (*9*) points out that the distribution of trees by diameter in selection stands can also be represented by a straight line on logarithmic graph paper.

SUMMARY

The correlation between average diameter of an even-aged stand and the other statistical measures by which the diameter distribution is described permits the use of average diameter alone as a basis for comparing the densities of stands. Of a group of stands of the same average diameter, and, therefore, having the same distribution of diameters, the stand with the greatest number of trees per unit area is obviously the most completely stocked. Age and site quality have no significant effect upon this relationship.

For a given species, the maximum number of trees that it is possible for a stand to have is correlated negatively with the average diameter. The curve representing this relationship assumes a straight-line form when plotted on logarithmic paper and is termed the reference curve.

In 15 groups of data representing 14 species (13 coniferous), the slope of the *number of trees—average diameter* curve was identical for 12 species. The slope for slash pine was slightly greater; that for shortleaf pine was appreciably greater. The heights of these curves, at a given diameter, varied between species.

By means of the reference curve "stand-density index" is derived for a given stand by plotting its number of trees and diameter, passing a line, parallel to the reference curve, through this point, and reading the number of trees per acre at its intersection with the 10-inch ordinate. This expression may be used for comparisons between species and within a species.

Percentage stocking is the percentage expression of the ratio between the number of trees per acre in a given stand and the number, for the same diameter, taken from the maximum curve for the species involved. This expression should be used only for comparisons within a species.

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