A SIMPLE METHOD OF CONSTRUCTING TREE VOLUME TABLES[1]

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INTRODUCTION

A volume table shows for a given species the average contents of trees of different sizes (*5*).[2] In the past, most volume tables have been constructed by separating the field data or samples into diameter and height classes, plotting the class averages, drawing smooth curves and harmonizing them with each other.

Recently Reineke and Bruce[3] have referred tree volume to that of modified cylinders or frustums of ideal solids in the construction of alinement chart volume tables. Three factors, diameter, height, and form of tree, affect tree volume. Within any diameter-height group variations in form produce corresponding variations in volume. These three factors are harmonized through averaging, and the resulting table presents the assumed average trees for the universe from which the sample was taken.

If the sample is a true average for the whole and the mathematical and mechanical work of computation and curve construction have been carefully done, the other parts of the unit from which the sample came can be measured correctly by using the resulting tabular tree volume table or alinement chart volume table.

Bruce (*2*), Bruce and Reineke (*3*), and Reineke (*7*) have shown how alinement charts may be employed in the solution of other problems in forest mensuration. The development of the technic outlined in this paper was suggested by their articles.

Alinement charts have certain advantages over the older methods of volume table construction. Because of the fact that all of the data are used in the construction of a single curve, better curve definition results and fewer data are necessary. When data are deficient in a few diameter or height classes, the projection of the curve through these points, connecting with points having sufficient weight, is possible. It should be remembered, however, that usually the ends of the curves are defined by the smallest number of samples and extensions are open to error. Less time is required by the alinement chart method.

PROCEDURE

A knowledge of the several types of graph paper is assumed. In the preparation of alinement charts involving multiplication or division, some form of logarithmic paper should be used. The scales on such paper simplify the mechanical work of graduating the axes and make logarithmic computations unnecessary.

The present technic of volume table construction is based upon the use of log.-log. paper, since tree volume computation involves a multiplication. The equation of this multiplication is $y = ax^2 \ (h\ f)$,

[1] Received for publication July 27, 1931; issued May, 1932.
[2] Reference is made by numbers (italic) to Literature Cited, p. 539.
[3] REINEKE, L. H., and BRUCE, D. THE PREPARATION OF ALINEMENT CHART VOLUME TABLES. 1928. [Unpublished manuscript, Forest Service.]

in which y equals volume, a is a constant, x is the diameter, 2 is the exponent of x, h is height, and f is form factor (*5*). The substitution of different values of x in this equation, allowing height and form to remain constant, produces a parabolic curve. A parabolic curve on log.-log. paper plots as a straight line.

To simplify the explanation of the technic presented, there are included in this paper the average values from a sample of 209 red oak (*Quercus borealis maxima* M.) trees secured on five logging operations in central Pennsylvania. The data were collected in the usual manner (*1*), and the tree volumes were computed by cubing the logs according to Smalian's parabolic frustum formula. The d. b. h. (diameter at breast height measured at 4.5 feet above ground level) taken outside bark, is an average to the nearest tenth inch of two measurements taken at right angles to each other with tree calipers. Heights, in feet, were measured from the ground level to the tip of the main stem. The volumes shown are total volumes inside bark contained in the stem and limbs, in cubic feet. Volumes do not include stump. Utilization is to a 2-inch top inside bark.

The data, charts, and tables presented are used only for purposes of illustration. Additional samples for this species should be obtained if final charts and tables are to be constructed.

ARRANGING THE DATA

The trees are first classified and listed by d. b. h. height classes. Column totals for each class are then obtained. This method necessitates only one listing of the data and reduces computing time by nearly one-half. The units of classification are entirely arbitrary. In this case the d. b. h. classes used are 3.5 to 4.4 inches, 4.5 to 5.4 inches, etc., and the height classes are 30 to 39.9 feet, 40 to 49.9 feet, etc.

The data are then classified by d. b. h. classes regardless of height (Table 1, columns 3, 4, and 5). The totals found in the first classification are used, and the number of trees involved in each class are noted. The number of trees, d. b. h., height, and volume for each d. b. h. class are totaled and averaged.

In the same way, the trees are classified by height classes regardless of d. b. h. (Table 2, columns 2, 3, and 4) and the average total height and volume in each height class computed. The totals computed in the first classification are again used.

TABLE 1.—*Classification of trees by d. b. h. regardless of height*

D. b. h. class (inches)	Trees	Total d. b. h.	Total height	Actual tree volume	Form factor	Tabular volume
	Number	*Inches*	*Feet*	*Cu. ft.*		*Cu. ft.*
4	6	26.2	220.9	9.83		9.89
	4	16.9	180.5	7.42		7.41
	1	4.3	55.0	2.43		2.32
Total	11	47.4	456.4	19.68		19.62
Average		4.31	41.49	1.79	0.427	1.78
5	3	14.0	107.0	5.34		5.45
	3	15.9	157.5	8.98		9.59
	14	72.1	626.1	36.31		36.60
Total	20	102.0	890.6	50.63		51.64
Average		5.10	44.53	2.53	.400	2.58
6	2	11.8	75.6	4.93		5.90
	9	53.0	430.5	32.69		32.78
	4	24.9	213.5	18.95		18.23
Total	15	89.7	719.6	56.57		56.91
Average		5.98	47.97	3.77	.404	3.79

TABLE 1.—*Classification of trees by d. b. h. regardless of height*—Continued

D. b. h. class (inches)	Trees	Total d. b. h.	Total height	Actual tree volume	Form factor	Tabular volume
	Number	*Inches*	*Feet*	*Cu. ft.*		*Cu. ft.*
7	1	6.7	39.5	3.37	----------	4.04
	5	34.5	226.0	20.42	----------	23.52
	16	113.0	883.4	96.23	----------	99.17
	5	36.5	325.2	37.25	----------	40.12
Total	27	190.7	1,474.1	157.27	----------	166.85
Average	----------	7.06	54.60	5.82	0.393	6.18
8	1	7.6	37.5	4.42	----------	4.95
	3	25.2	133.6	23.56	----------	20.98
	17	135.1	940.6	133.74	----------	134.22
	3	24.9	188.6	28.39	----------	29.85
Total	24	192.8	1,300.3	190.11	----------	190.00
Average	----------	8.03	54.18	7.92	.415	7.92
9	3	27.6	146.4	30.25	----------	27.55
	5	45.5	282.5	53.31	----------	52.40
	6	55.3	374.8	71.03	----------	71.60
Total	14	128.4	803.7	154.59	----------	151.55
Average	----------	9.17	57.41	11.04	.419	10.83
10	1	10.4	45.5	12.66	----------	10.70
	13	131.5	742.0	170.64	----------	166.20
	3	30.4	225.4	50.21	----------	52.30
	12	120.3	746.6	162.31	----------	164.90
Total	29	292.6	1,759.5	395.82	----------	394.10
Average	----------	10.09	60.67	13.65	.405	13.59
11	1	11.5	45.5	11.54	----------	12.85
	3	32.8	169.7	43.97	----------	43.15
	11	122.5	711.5	192.57	----------	191.30
	2	21.5	143.8	32.18	----------	36.95
Total	17	188.3	1,070.5	280.26	----------	284.25
Average	----------	11.08	62.97	16.48	.391	16.72
12	1	11.7	57.5	16.49	----------	16.60
	7	83.5	455.0	137.09	----------	138.50
	1	11.9	71.0	22.87	----------	22.00
Total	9	107.1	583.5	176.45	----------	177.10
Average	----------	11.90	64.83	19.60	.392	19.68
13	1	12.6	52.0	16.19	----------	17.50
	3	39.0	198.5	81.10	----------	73.70
	4	51.9	302.1	116.14	----------	113.40
	3	38.6	247.9	104.15	----------	90.10
Total	11	142.1	800.5	317.58	----------	294.70
Average	----------	12.92	72.77	28.87	.436	26.79
14	3	41.8	200.9	76.16	----------	90.30
	4	56.3	293.5	139.45	----------	137.20
	5	70.1	412.7	182.99	----------	186.10
Total	12	168.2	907.1	398.60	----------	413.60
Average	----------	14.02	75.60	33.22	.411	34.46
15	1	15.0	58.5	33.10	----------	30.40
	1	14.6	68.8	39.45	----------	34.30
	7	104.7	532.8	285.61	----------	287.10
	1	15.0	85.0	48.16	----------	45.00
Total	10	149.3	745.1	406.32	----------	396.80
Average	----------	14.93	74.51	40.63	.449	39.68
16	1	16.3	76.0	52.91	----------	50.80
	2	32.2	162.5	115.19	----------	103.50
Total	3	48.5	238.5	168.10	----------	154.30
Average	----------	16.17	79.5	56.03	.516	51.43
17	1	17.2	63.5	45.67	----------	47.00
	4	67.8	291.8	194.02	----------	211.40
Total	5	85.0	355.3	239.69	----------	258.40
Average	----------	17.0	71.06	47.94	.428	51.68
18	1	18.0	68.5	50.39	----------	56.60
	1	17.9	89.0	77.67	----------	69.00
Total	2	35.9	157.5	128.06	----------	125.60
Average	----------	17.95	78.75	64.03	.463	62.80
Grand total	209	1,968.0	12,262.2	3,139.73	----------	3,135.42

TABLE 2.—*Classification of trees by height regardless of d. b. h.*

Height class (feet)	Trees	Total height	Actual tree volume	Tabular volume
	Number	*Feet*	*Cu. ft.*	*Cu. ft.*
30	6	220.9	9.83	9.89
	3	107.0	5.34	5.45
	2	75.6	4.93	5.90
	1	39.5	3.37	4.04
	1	37.5	4.42	4.95
Total	13	480.5	27.89	30.23
Average		36.96	2.14	2.32
40	4	180.5	7.42	7.41
	14	626.1	36.31	36.60
	9	430.5	32.69	32.78
	5	226.0	20.42	23.52
	3	133.6	23.56	20.98
	3	146.4	30.25	27.55
	1	45.5	12.66	10.70
	1	45.5	11.54	12.85
Total	40	1,834.1	174.85	172.39
Average		45.85	4.37	4.31
50	1	55.0	2.43	2.32
	3	157.5	8.98	9.69
	4	213.5	18.95	18.23
	16	883.4	96.23	99.17
	17	940.6	133.74	134.22
	5	282.5	53.31	52.40
	13	742.0	170.64	166.20
	3	169.7	43.97	43.15
	1	57.5	16.49	16.60
	1	52.0	16.19	17.50
	1	58.5	33.10	30.40
Total	65	3,612.2	594.03	589.88
Average		55.57	9.14	9.08
60	5	325.2	37.25	40.12
	3	188.6	28.39	29.85
	6	374.8	71.03	71.60
	12	746.6	162.31	164.90
	11	711.5	192.57	191.30
	7	455.0	137.09	138.50
	3	198.5	81.10	73.70
	3	200.9	76.16	90.30
	1	68.8	39.45	34.30
	1	63.5	45.67	47.00
	1	68.5	50.39	56.60
Total	53	3,401.9	921.41	938.17
Average		64.19	17.38	17.70
70	3	225.4	50.21	52.30
	2	143.8	32.18	36.95
	1	71.0	22.87	22.00
	4	302.1	116.14	113.40
	4	293.5	139.45	137.20
	7	532.8	285.61	287.10
	1	76.0	52.91	50.80
	4	291.8	194.02	211.40
Total	26	1,936.4	893.39	911.15
Average		74.47	34.36	35.04
80	3	247.9	104.15	90.10
	5	412.7	182.99	186.10
	1	85.0	48.16	45.00
	2	162.5	115.19	103.50
	1	89.0	77.67	69.00
Total	12	997.1	528.16	493.70
Average		83.09	44.01	41.14
Grand total	209	12,262.2	3,139.73	3,135.52

GRADUATING THE DIAMETER AXIS

On log.–log. cross-section paper, with the abscissa as d. b. h. and the ordinate as volume, the average values obtained in the second classification (Table 1) are plotted. A curve is fitted to the plotted values after proper weights have been assigned, as shown in the left-hand curve of Figure 1. It may be shown that the variation in the points from a smooth curve is due to differences in the height and form factor of the average tree. Column 6 in Table 1 shows the cylinder form factor of the average tree in each class. Figure 2 shows the

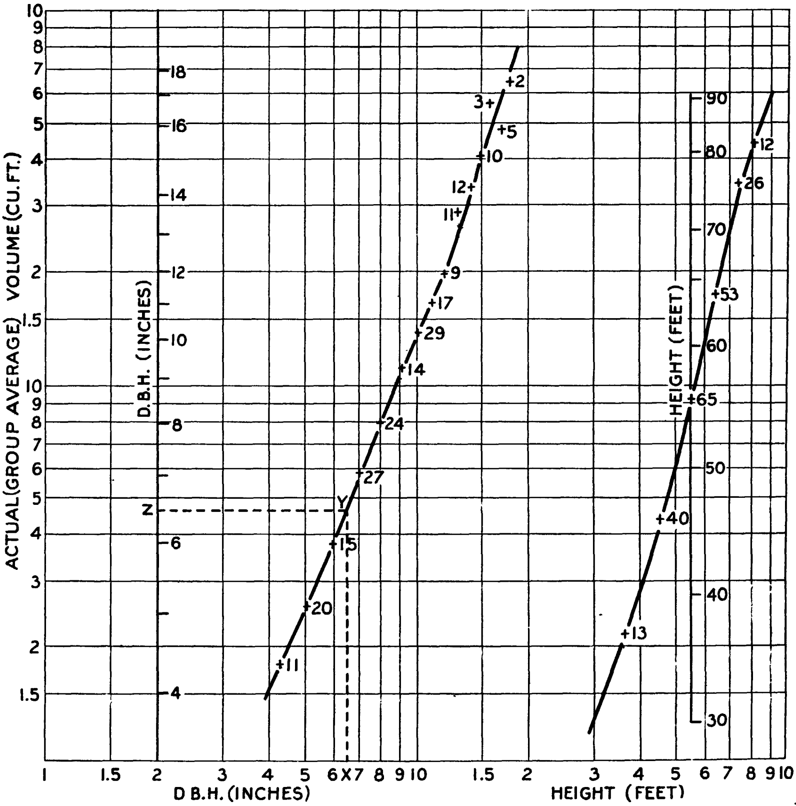

FIGURE 1.—Curves showing method of establishing diameter and height axes

extent of these variations and their relation to diameter. It is readily seen that slight changes in form and average height cause the plotted points to deviate from a smooth curve. The diameter classes above 12 inches show more radical changes in form factor, hence greater deviation from the trend of the other plotted values. In cases where more data are available the curve tends to become smoother owing to better sampling.

By using any convenient point near the left of the paper (in this case at abscissa value 2) the d. b. h. axis is established. If, now, the d. b. h. volume curve is used as a graduating curve, the graduations for d. b. h. may be placed on this axis. Even inches only are graduated in Figure 1, to avoid confusion. The graduations are obtained by tracing vertically from the desired d. b. h. value on the abscissa to the graduating curve intersection, then horizontally to the axis. The dotted line (fig. 1) X–Y–Z shows the method of locating the 6.5-inch graduation.

GRADUATING THE HEIGHT AXIS

Utilizing the values obtained in the third classification (Table 2), a curve of volume on height is plotted. Volume is the dependent or ordinate value and height is the independent or abscissa value. Selecting any convenient point near the right side of the graph paper, the height axis is set up. Using the height-volume curve as a graduating curve, the height values are placed on the height axis in the same manner as for the d. b. h. axis. Again, divergence of the plotted values from a smooth curve is due to fluctuations in the form factor of the average tree in the height classes.

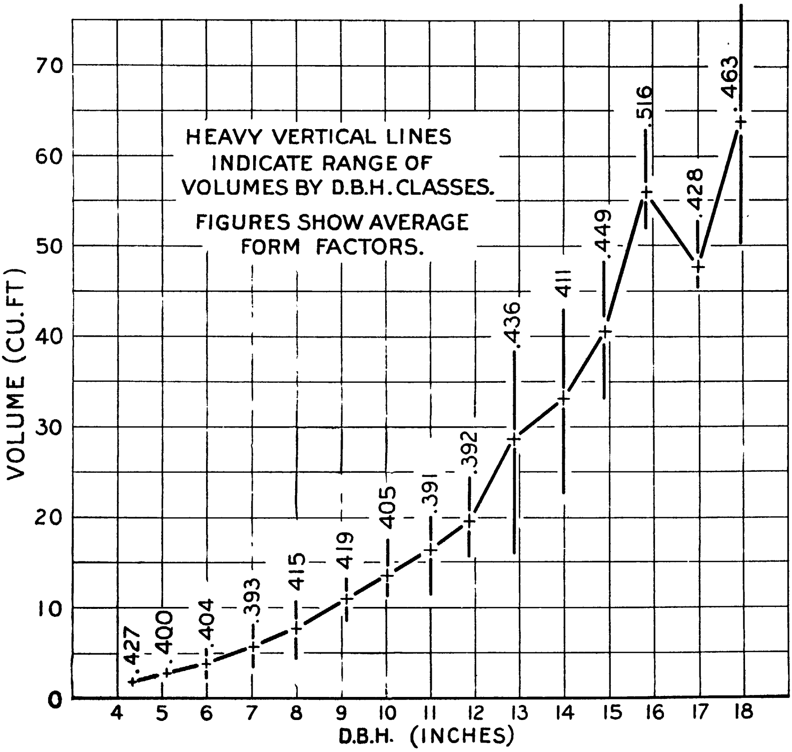

FIGURE 2.—Curve showing extent of tree volume variations and average form-factor values for each diameter class

LOCATING THE VOLUME AXIS

To locate the volume axis in cubic volume tables, any convenient d. b. h. and height may be assumed and the volume of a cylinder of these dimensions computed. Another cylinder 2 or 3 inches larger or smaller than the first is then assumed and the height necessary to produce a cylinder having a volume equal to that of the cylinder first assumed is computed. Construction lines between the d. b. h. and height values of these two cylinders are drawn, and the intersection of the two lines in the location of the volume axis. Repetition of this process for several assumed d. b. h. height values will show the exact position of the axis.

It will be found in this case that the volume axis is parallel to the d. b. h. and height axes and nearer to the d. b. h. than to the height axis. Its exact location depends upon the range of the d. b. h. and

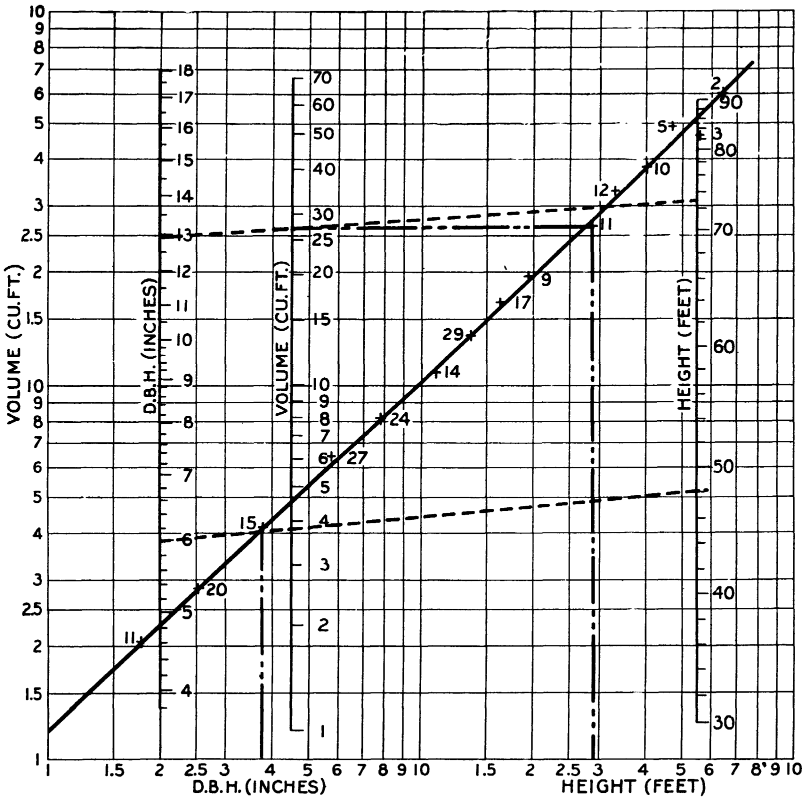

FIGURE 3.—Method of establishing values which locate the volume graduating curve. This drawing shows the form of an alinement chart volume table

height values and the spacing of their respective axes. A check of the location of volume axis is necessary in constructing a volume table.

GRADUATING THE VOLUME AXIS

Utilizing the average values in Table 1 (Columns 3, 4, and 5), place a straight edge on the d. b. h. and height values and mark the intersection on the volume axis, as shown by the dotted construction lines for the 6-inch and 13-inch d. b. h. classes in Figure 3. Using the abscissa as the actual tree volume, the average volume value is plotted on the abscissa horizontally opposite the intersection obtained by the pairing of the average d. b. h.-height values. The double dot and dash construction lines in Figure 3 show the method of locating the volume values for the 6-inch and 13-inch classes, and these are respectively weighed with the number of trees in those classes.

A smooth curve is fitted to the plotted volume values thus obtained. It will be noted that the divergence from a straight line in these plotted values is less than in the case of the first curve, since d. b. h. is now associated with height, and the latter tends to influence the form factor in the reverse direction.

The volume graduations may now be placed on the volume axis, the volume curve being used as a graduating curve. In Figure 3, the major volume values are shown on the volume axis.

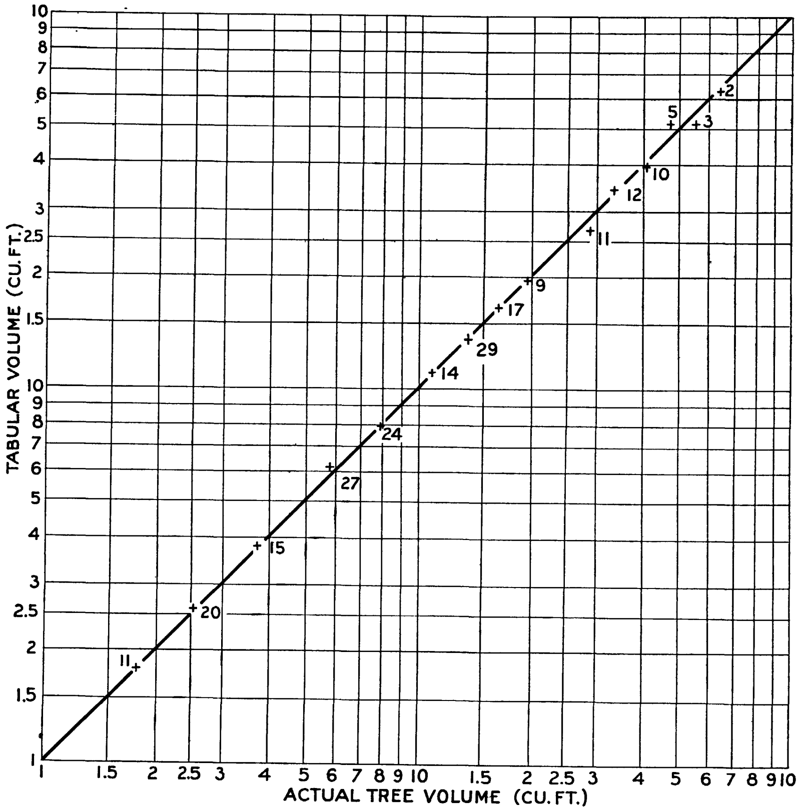

FIGURE 4.—Curve showing relation of tabular volume to actual tree volume

DETERMINATION OF ACCURACY

In order to check the accuracy of the graphic work (4) and to determine the limits of applicability of the table, it is necessary to read from the completed chart the volume of each individual tree used in its construction.

To determine the volume of any tree, a straight edge is laid on the chart intersecting the d. b. h. and height values on their respective axes. The volume of the tree is read at the point where the straight edge intersects the volume axis. These values are shown as tabular values in column 7 of Table 1.

For purposes of comparison with the actual tree volumes, the tabular values are totaled by classes. The grand total of the actual tree volumes is compared with the grand total of the tabular volumes. To compute the "aggregate percentage deviation" of the table, the difference between the actual and tabular volumes is obtained. This difference multiplied by 100 is divided by the total volume of the actual trees. A plus or minus sign is assigned to the result depending on the greater volume, tabular or actual.

The usual limit of accuracy for standard volume tables applicable over a large area is 1 per cent. In this particular case the aggregate percentage deviation is −0.134 per cent, which is well within the required limit.

A low aggregate deviation does not necessarily mean that the table as constructed is accurate, since too low volumes in the smaller diameters might offset too high volumes in the larger diameters, or vice versa. A graph of tabular volume (Table 1, column 7) plotted on actual tree volume (column 5) on log.–log. paper will plot as a straight 45-degree line through 1–1 provided the work is correct. Figure 4 shows a graph in which the plotted values are the averages by d. b. h. classes from Table 1.

Failure of these plotted values to produce a 45-degree line through 1–1 means that the volume axis should be regraduated in those portions as indicated by divergence from the 45-degree line.[4] This is accomplished by reading the ordinate volume value first and then the abscissa value from the curve as the corrected volume graduation. Reading of these corrected volume values at intervals depending upon their magnitude, will allow replotting over the abscissa values on the chart and a new volume graduating curve is produced. The volume axis may now be regraduated and the new individual tree volumes reread from the chart. The graduations on both the diameter and height axis should be carefully checked if considerable variation in tabular values is noted. Recomputation of the aggregate percentage deviation should produce a lower value.

The average percentage deviation is found by determining the percentage deviation of each individual tree volume from its chart volume. The total of these individual deviations taken without regard to sign, multiplied by 100, and divided by the number of trees, gives the average percentage deviation. The limit of this average deviation for standard volume tables should not exceed ±10 per cent. The table here produced gives an average percentage deviation of ±7.74 per cent.

PREPARATION OF THE FINAL TABLE

The volume table may be read from the alinement chart and tabulated in the conventional form. (Table 3.) Volumes are read for any desired d. b. h. and height interval in the same manner as explained above.

Using the alinement chart itself for determining tree volume makes interpolation unnecessary. For practical application it is, therefore, simpler to use the chart in its finished form rather than in the conventional table form.

[4] REINEKE, L. H., and BRUCE, D. Op. cit.

DISCUSSION

Previous alinement chart technic for volume table construction has utilized base charts for standard solid figures, cylindrical, parabolic or cone frustums, depending on the type of table desired. The technic has been based upon correlation of the tree volumes with these base charts. The technic here developed departs from previous practice in that base charts are not utilized.

Previous technic in volume table construction has first correlated the dependent variable, volume, followed by a fitting of the independent variables, d. b. h. and height. The present technic departs from previous practice again in that the independent variables are set up first, then the dependent variable, volume, is correlated with the two independents. Meyer (*6*) has simultaneously developed the same general principle of correlation but used a base chart with which to correlate first the independent, then the dependent variables.

Comparison of the present technic with previous technic, using identical data, indicates a saving of time in construction of graphs and in reduced correlations. In the comparisons made, the average percentage deviations obtained were in each case reduced by the present technic, the reduction ranging from 0.4 to 1.1 per cent.

TABLE 3.—*Merchantable volume* [a] *of red oak stand Pennsylvania, 1930*

D. b. h. (inches)	Volume (cubic feet) of trees of total height (feet) indicated							Basis number of trees
	30	40	50	60	70	80	90	
4	1.24	1.52	1.86					11
5	1.81	2.22	2.75					20
6	2.60	3.20	3.93					15
7	3.60	4.50	5.47	6.60				27
8	4.70	5.80	7.07	8.80				24
9		7.20	8.95	10.90	13.00			14
10		8.95	10.90	13.20	15.60			29
11		10.55	12.75	15.40	18.50			17
12			15.00	18.10	21.90	25.00		9
13			17.90	21.70	26.20	30.00		11
14			21.60	26.50	32.00	36.30	39 00	12
15			25.80	31.50	38.00	43.00	46.00	10
16				37.00	44.40	50.00	54.00	3
17				42.70	51.00	58.00	62.00	5
18				49.00	58.00	65.50	60.00	2
								209

[a] Volume includes stem and limbs inside bark above a 1-foot stump. Utilization limit is 2 inches inside bark. Heavy line indicates range of basic data. Aggregate percentage deviation: Table 0.134 per cent low. Average percentage deviation: ±7.74 per cent. Data collected in 1930 by A. C. McIntyre and T. A. Liefeld.

SUMMARY

A technic for the construction of alinement chart volume tables has been developed. Graduating curves for d. b. h. and height are plotted on log.–log. cross-section paper, and these two independent variables are then correlated with the dependent variable, volume, to produce the finished chart.

Except in the case of meager data, no axis regraduation is necessary since the initial graduations conform strictly to the variations in form factor of the trees measured.

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