A MODIFICATION OF BRUCE'S METHOD OF PREPARING TIMBER-YIELD TABLES [1] [2]

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In Bruce's method of preparing timber-yield tables [2] anamorphosis is used for constructing the curves for site classification, total basal area, average basal area, number of trees, and total cubic volume. Since, by construction, each set of curves is represented by a series of straight lines radiating from a common point, it is evident that the reading of any two lines must have the same ratio at each age. Any curve of such a series can therefore be expressed as a percentage of any other curve of that series, such as the average curve. This attribute of anamorphically constructed series of curves [3] permits their expression in the form of alignment charts, the use of which eliminates the laborious anamorphic plotting of the data and the balancing of a curve for each site-index [4] class, results in increased accuracy, and makes interpolation and checking easier.

The first step in constructing these alignment charts is the preparation, for site classification, of an average curve [5] over age for—

(1) Height of average dominant tree;

and, for the entire stand, similar curves for—

(2) Total basal area, hereafter called stand basal area.
(3) Average basal area, hereafter called tree basal area.
(4) Number of trees.
(5) Volume in cubic feet.

These curves are similar to the graduating curves used by Bruce, but may be drawn in the conventional way rather than as graduating curves. The average points for the tree basal area curve should be computed from the averages of stand basal area and the numbers of trees used for constructing the average curves for these two factors, instead of using averages of tree basal areas of individual plots; otherwise the final curves for tree basal area will not balance.

The curves for tree basal area, numbers of trees, and stand basal area are checked against each other; at any age the product of tree basal area and number of trees should equal the stand basal area.

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[1] Received for publication June 25, 1927; issued December, 1927.

[2] Bruce, D. A method of preparing timber-yield tables. Jour. Agr. Research 32:543–557, illus. 1926. Errata: Figures 7 and 8 should be interchanged and the applied stand graph should be referred to Figure 7 instead of Figure 8.

[3] This applies, of course, only to systems of curves having a common origin.

[4] "Site index" is the height which the average dominant tree will have, or had, at some standard age, usually 50 (or 100) years.

The site-index system should not be considered as a banding system comparable with but more finely subdivided than the ordinary "I, II, III" or "Good, medium, poor" system. Rather, the farmers' classification of farm land as 45-bushel wheat land (in 1 year implied), or 24-bushel corn land (1 year) is entirely comparable with 93-foot (50-year) longleaf pine land. The only difference is that farm-land quality is rated by actual yield, but forest-land quality, because of the variation in yield with density of stocking, must be rated by measurement of some other factor (height) indicative of the productive capacity, but for the most part not appreciably affected by variations in density.

The grouping of plots into 10-foot site-index classes is done only for convenience in preparing the curves, and in the final tables values are given for only the even 10 feet of site index to avoid bulkiness. Values for every foot of site index, or even smaller intervals, could be determined, but intervals less than 1 foot are not practicable.

[5] These curves should be carefully balanced so that the sums of the positive and negative deviations are equal and distributed as evenly as possible throughout the length of the curve.

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In addition to the curves listed above a so-called average *forest form factor-age* curve is drawn for checking purposes. This "forest form factor" is obtained by dividing the cubic volume by the product of stand basal area and the dominant heights [6] used for the site classification (*height-age*) curve.

Age-class averages of cubic volume, height, and stand basal area should be used to derive the average forest form factor for each class instead of computing the form factor for each plot and then averaging the form factors by age classes. The curve thus derived is used to check the stand basal area, height, and cubic volume curves against each other. At any age the volume should equal the product of stand basal area, height of the average dominant tree, and forest form factor.

A site classification alignment chart is now prepared from the *height-age* curve and a percentage alignment chart. Percentage charts may have either arithmetic or logarithmic scales. The logarithmic type gives better intersections and readings of the same relative accuracy throughout and is also less cumbersome to use. For these reasons logarithmic charts should be employed.

To construct the percentage chart lay out three parallel axes, *A*, *B*, and *C* (fig. 1), *B* midway of the others. On *A* and *C* lay out equal logarithmic scales. On *B* lay out a logarithmic scale with cycles half as large as those used on *A* and *B*, so placed that the 100 on this scale lies exactly on the line through 100 on *A* and *C*. (Only the *B* axis need be actually graduated, as will be shown later.) A line passing through any two numbers on *A* and *B* will intersect *C* at the point representing the ratio or percentage $\frac{B}{A} \times 100$.

To convert this chart into the site classification chart all that is necessary is to let the graduations on the *A* and *B* axes represent the height of the average dominant tree in feet, and to superpose on *A* and *C* the scales for age and site index, as follows:

On the *A* axis the ages are marked opposite the graduations corresponding to heights read from the average *height-age* curve, as listed below, resulting in scale *A*.

Age in years	Height of average dominant tree in feet	Age in years	Height of average dominant tree in feet
10	14	60	78
15	26	70	83
20	37	80	87
30	53	90	90
40	64	100	92
50	71		

Each site index expressed as a percentage of the average site index [7] is then marked opposite that percentage on the *C* axis, resulting in scale *C′*. This may also be done graphically by drawing, from the classification age on *A*, a series of lines to *C* passing through the 10-foot intervals on *B*. Their intersections on *C* fix the positions of the 10-foot site-index graduations. The intermediate site-index graduations may be located in the same way. The unnecessary

[6] The dominant height is substituted for average height to permit checking the site-index curves with the volume and stand basal area curves. Curves for average height are not constructed by anamorphosis and are therefore not susceptible to such simple checking.

[7] This is the height of the average *height-age* curve at the classification age (50, 100, etc., years).

logarithmic (height and percentage) graduations on *A* and *C* can now be erased, leaving only the age graduations on *A'*, the heights on *B'*, and the site indices on *C'*. A line through the age of a plot

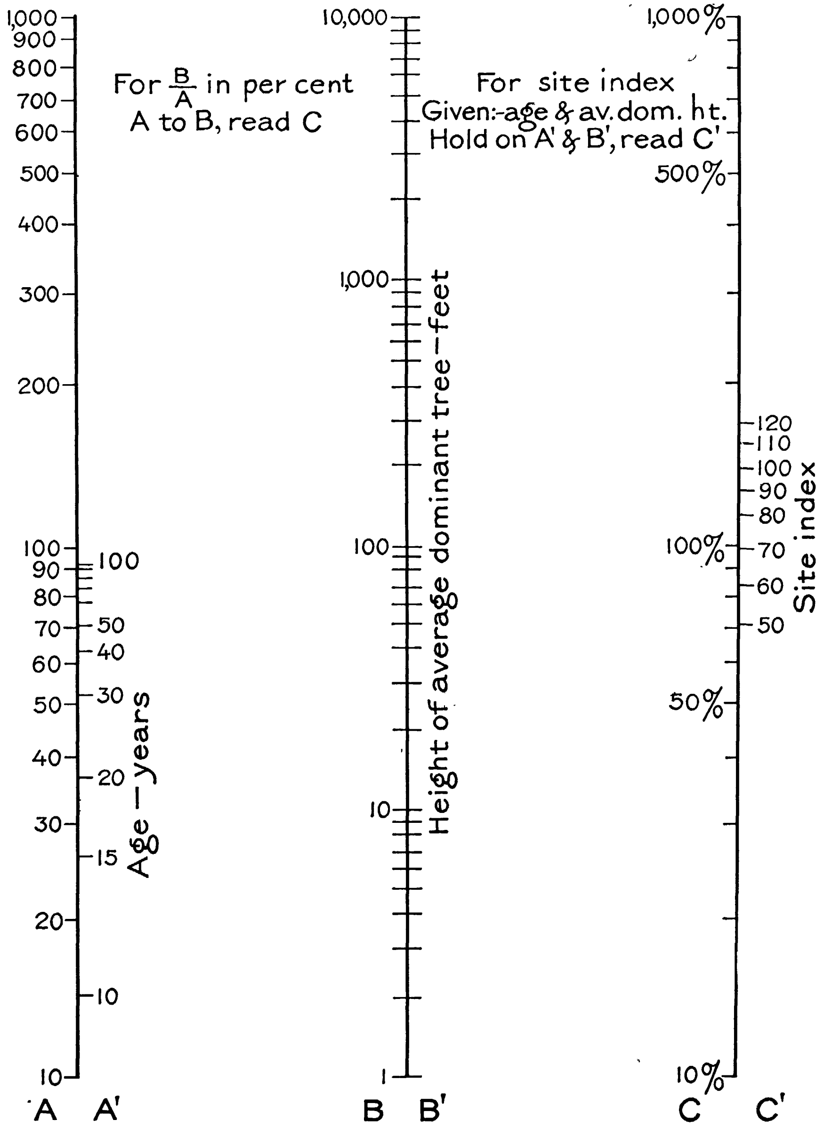

FIG. 1.—Logarithmic chart for computing percentages, converted into chart for classifying site

and the height of its average dominant tree will then intersect *C'* at the site index of that plot.

The next step in the use of this chart is the assignment of a site-index value to each plot, after which the plots are grouped by site-index classes (10-foot classes preferably).

Working first with stand basal area, an estimated stand basal area corresponding to the age of the plot is read from the average *stand basal area-age* curve. (Fig. 2, A.) Both observed and estimated

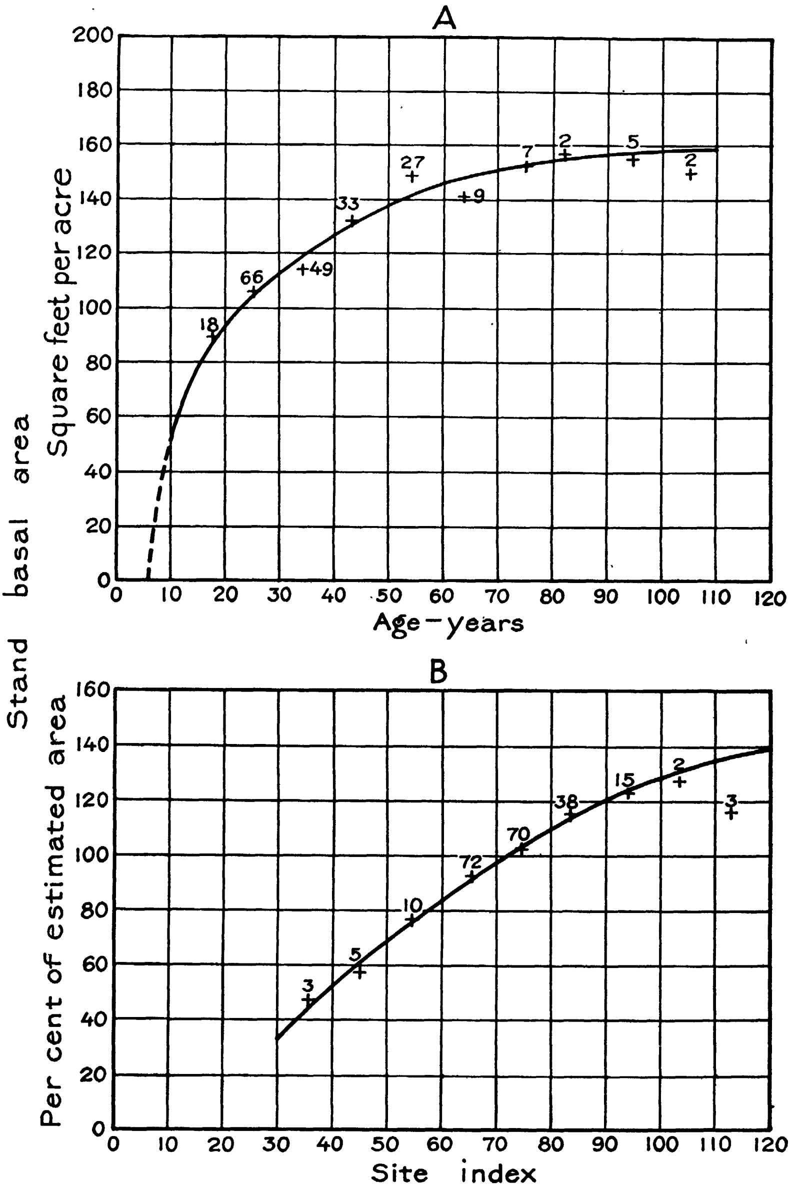

FIG. 2.—Curves used for preparing alignment chart stand basal area: A, average curve over age, used to graduate age axis in Figure 3; B, average curve over site index, used to graduate site-index axis in Figure 3

areas are then totalled by site-index classes and the sums of the observed values expressed as percentages of the sums of the estimated. Table 1 illustrates this. These percentages are than plotted over the average site index of the class and curved, as in Figure 2, B. Similar curves are prepared for tree basal area, number of trees, and cubic volume, and are checked against each other. For each site index the percentage of stand basal area should equal the product of the percentage of number of trees and percentage of tree basal area.[8] The cubic volume curve could be checked through forest form factor, but ordinarily such a check is not needed, since the *volume per cent-site index* curve is usually a well-defined and straight or nearly straight line. Readings from the curve for stand basal area are then superposed on a percentage alinement chart, as was done for the site classification chart; the ages as obtained from the average *stand basal area-age* curve appearing on the *A* axis, and the site indices, from the *stand basal area per cent-site index* curve, appearing on the *C* axis. Similar charts are derived for tree basal area and diameter, number of trees, and total volume of peeled stem wood in cubic feet.

TABLE 1.—*Computations of stand basal areas, by site index classes, as percentages of stand basal area of average site index*

Plot No.	Number of plots	Site index	Age	Stand basal area: Observed	Stand basal area: Estimated from average curve over age	Stand basal area: Observed in per cent of estimated
			Years	*Sq. ft.*	*Sq. ft.*	*Per cent*
8		31	16	36	83	
46		36	24	56	102	
31		39	21	42	97	
Total	3	106		134	282	
Average		35.3				47.5
112		40	32	77	116	
80		41	27	46	108	
168		49	44	110	133	
119		47	34	48	119	
25		48	20	48	94	
Total	5	225		329	570	
Average		45.0				57.7
55		50	25	51	104	
153		57	40	86	128	
Total [a]	10	545		935	1,223	
Average		54.5				76.5

SUMMARY

	3	35.3		134	282	47.5
	5	45.0		329	570	57.7
	10	54.5		935	1,223	76.5
	72	65.2		8,183	8,835	92.6
	70	74.1		8,894	8,715	102.1
	38	83.4		5,354	4,620	115.9
	15	94.0		1,937	1,573	123.1
	2	103.0		232	182	127.5
	3	113.0		311	268	116.0
Total	218			26,309	26,268	

[a] Totals are for 10 plots in this group, of which only 2 are shown.

[8] If the average curves over age have not been perfectly balanced these curves will not pass through 100 per cent at the average site index. Thus a simple check is combined with an automatic correction for any inaccuracies in balancing the curves drawn in the first steps.

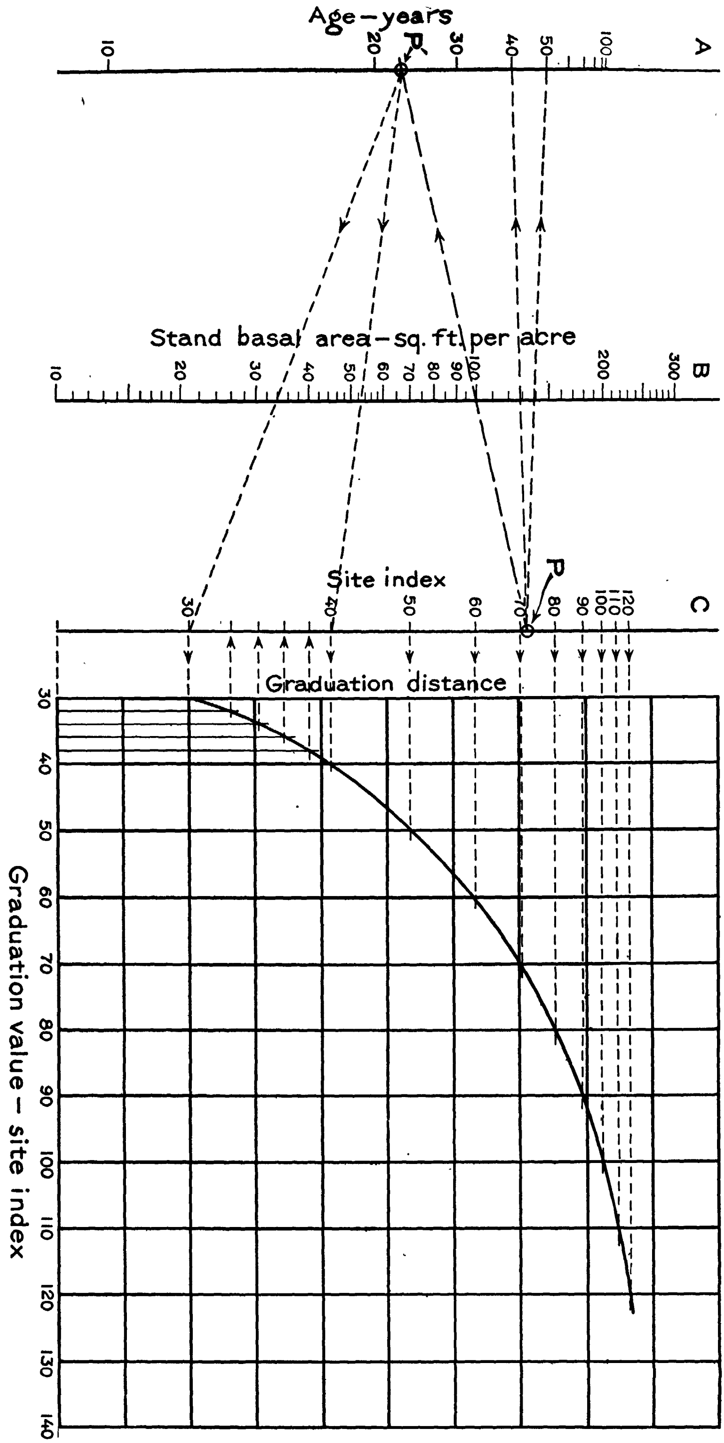

FIG. 3.—Alignment chart for stand basal area, indicating the location of the major graduations by intersection, and the preparation and use of a graduating curve for locating minor graduations for site index

To avoid the necessity for first constructing percentage charts, a somewhat different procedure may be followed. The three parallel axes are first drawn, not necessarily at uniform intervals. On the central axis a logarithmic scale of stand basal area is entered, as in Figure 3. Any point P is then temporarily marked on C as the position of the average site-index graduation. Lines drawn from P and intersecting B at values read from the average *stand basal area-age* curve will intersect A at the location of the corresponding age graduations.

For locating the site index graduation on C the graduations of B are temporarily considered as percentages, and a line from P, through 100 per cent, locates a point P' on A. Lines from P', intersecting B at percentages, read from the *stand basal area per cent site-index* curve, will intersect C at the position of the corresponding site-index graduations.

Charts for each of the other factors are similarly constructed. The chart for tree basal area can be read directly in terms of average breast-high diameters by marking the diameters along the B scale where the corresponding basal areas appear, as in Figure 4. Percentages for tree basal area should be computed from totals of stand basal area and number of trees by site-index classes.

This chart for diameters is also used for obtaining partial-stand values in terms of percentages of the corresponding total-stand values. The curves described by Bruce [9] are used for this.

A point (P, fig. 4) is permanently marked to the right of the A' axis. A series of axes parallel to X are drawn (B, C, D, etc.), their number depending on the number of ratios to be represented. Percentages are then marked on these axes at their intersection with lines, from P, through the corresponding average breast-high diameters of the total stand on X'. The same procedure is followed for the average diameters of the partial stands, except that the curves used by Bruce (difference between partial and total stand diameters over total stand diameter) are replaced by curves of partial-stand diameters over the entire-stand diameter.

Only a few graduations on the axis of any of the alignment charts need be located by intersection. The distances of these from any fixed point on the axis are then curved over the value of the graduation (age, site, index, diameter, etc.) and the distances of the intermediate graduations transferred from this curve. Figure 3 illustrates the location of the intermediate site-index graduations from the curve passing through the distances of the 10-foot graduations.

When the aggregate checks of the various factors are made it will be necessary in the case of tree basal area to weight the values for each plot by the observed number of trees; this is equivalent to comparing the observed stand basal area with that obtained by multiplying estimated tree basal area by the observed number of trees.

By Bruce's method the fit of the radiants indicates whether or not anamorphosis can be legitimately employed in preparing the curves. A consistent failure of the points to fall into straight lines indicates that the method is not suitable.

With the modifications here presented the applicability of anamorphosis can be tested as follows: The observed values and the estimated

[9] Bruce, D. Op. cit. p. 549, fig. 5; p. 551, fig. 6.

values corresponding to the age and site index of each plot are grouped, summed, and averaged by 10-year age—10-foot site-index classes. The deviations of the observed averages from the estimated aver-

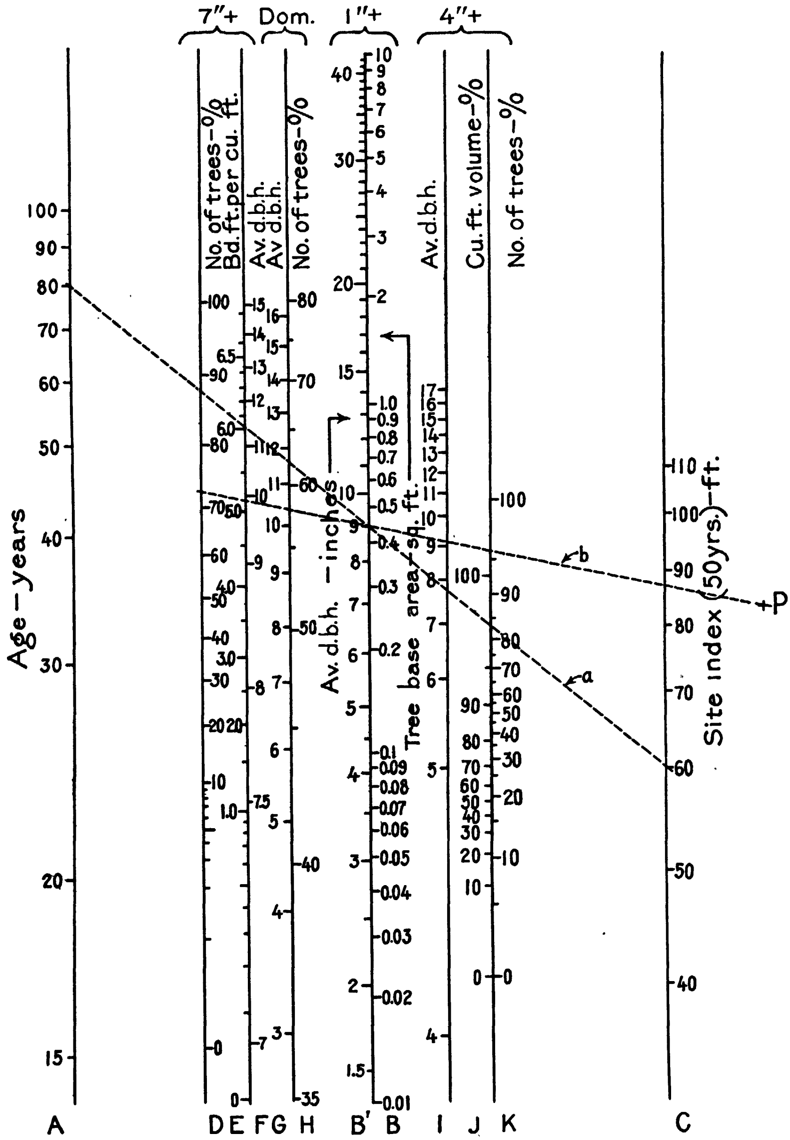

FIG. 4.—Alignment chart for tree basal area with scale reading in terms of average d. b. h. (*B'*). The additional axes, *D* to *K*, are graduated to show the partial stand values in per cent of the total stand values when a straight line is passed through *P* and the average d. b. h. of the total stand

ages are plotted over age by site-index classes. Failure of the method to fit will be indicated by a series of progressively changing curves. This is not the case with stand basal area in the illustrative example as shown in Figure 5. Such a test, so essential in the development

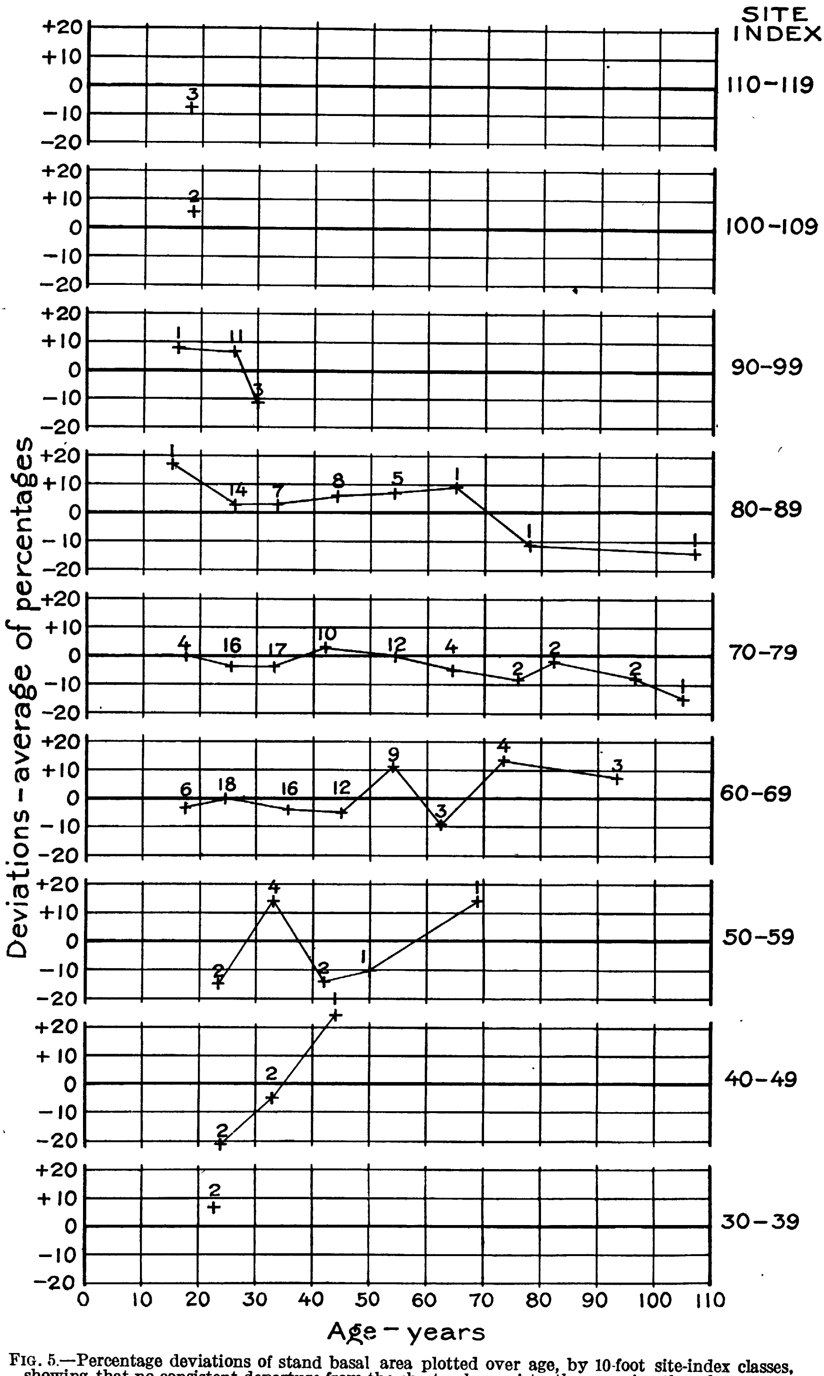

FIG. 5.—Percentage deviations of stand basal area plotted over age, by 10-foot site-index classes, showing that no consistent departure from the chart values exists, thus proving that the method used introduces no distortion

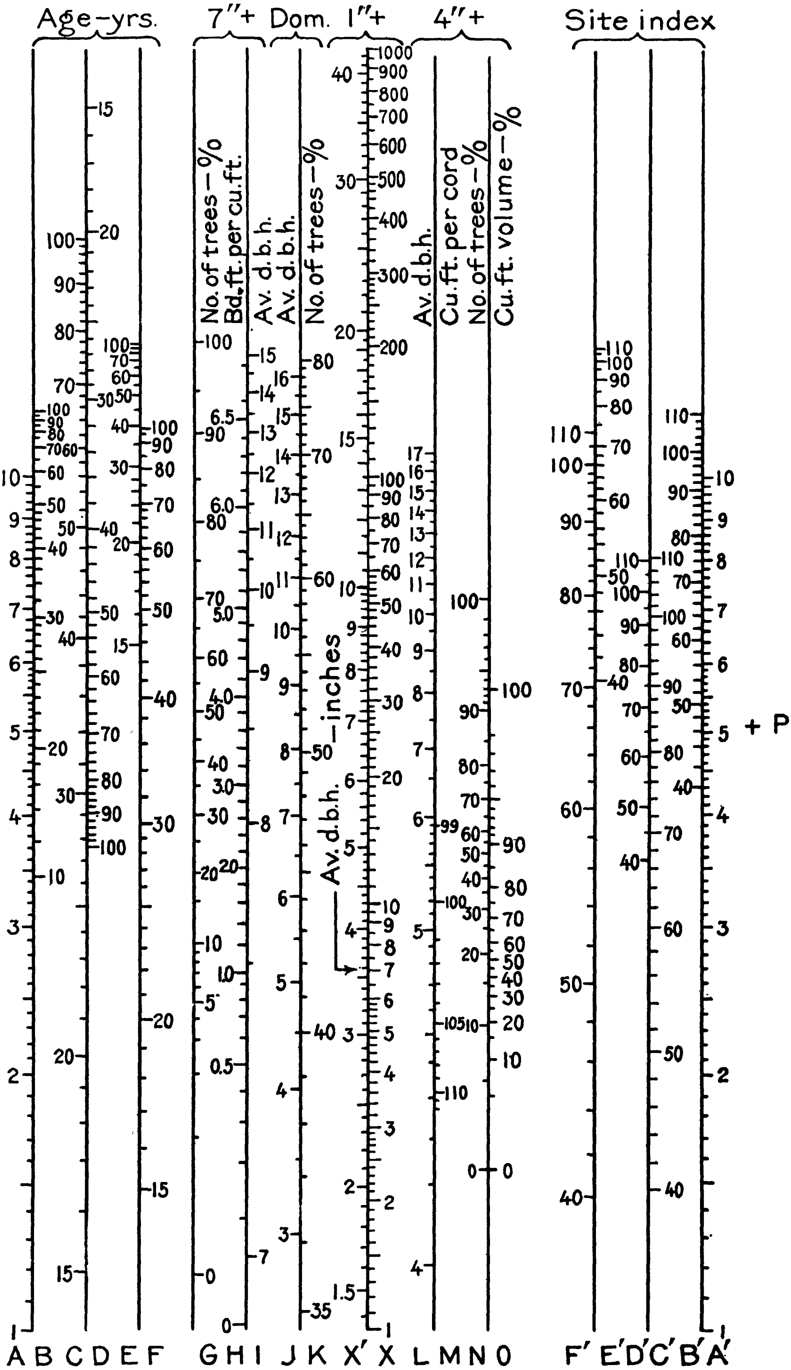

FIG. 6.—Composite chart for total-stand and partial-stand values. All total-stand values are read on the central scale. The two outer scales, *A* and *A′*, are used with *X* for multiplying total-stand values by partial-stand percentages to get partial-stand values. (See appendix for instructions for using this chart

of a new technic, is of little value now. Experience with nine species of diversified characteristics has justified the assumptions basic to the method.

Separate charts need not be made for each factor; they may be combined into a single chart by using a common central axis for all factors and separate outside axes for each factor, as shown in Figure 6. In this case the readings on the central logarithmic scale are multiplied or divided by 10 or 100, as necessary for the factor considered.

An additional pair of scales used in conjunction with the central

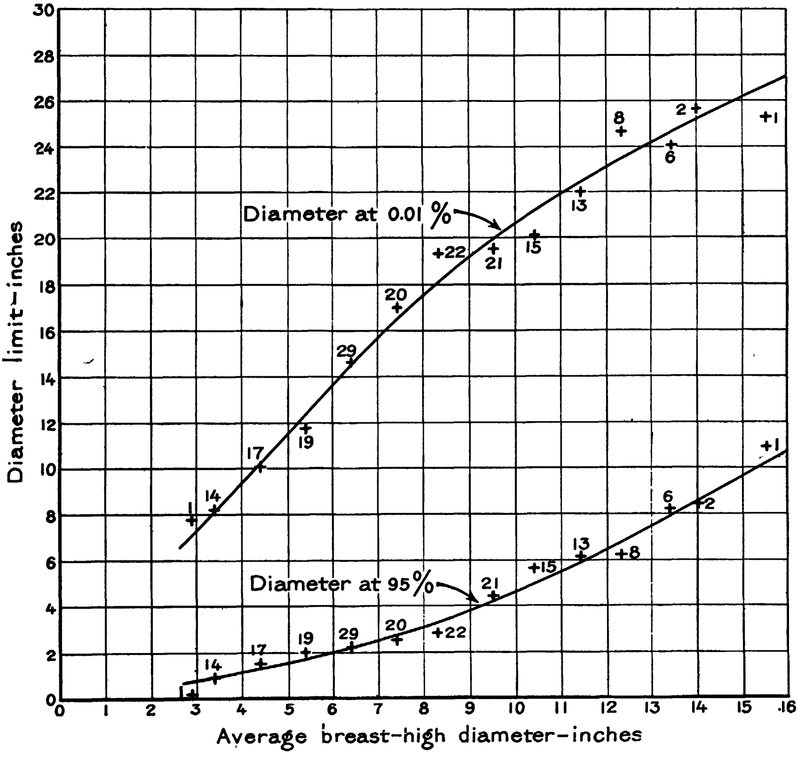

FIG. 7.—Intercept curves for stand-table graph. The diameters at which the basic frequency curves intercept the 95 and 0.01 ordinates are plotted over the average diameters (by basal area) of the respective frequency curves. The curves fitted to these points are used to construct the *A* and *B* scales of Figure 8

one can be profitably added to this chart for multiplying total stand values by percentages to get partial stand values (*A* and *A′*, fig. 6). These will be ordinary logarithmic scales with cycles matched with those of the central scale.

Stand tables may also be included by using a series of percentage scales, but a more usable and simpler way of presenting them is by means of the stand-table graph employing "frequency" paper where such paper fits the data. In a number of cases the arithmetic paper specifically described by Bruce has failed to represent the data by straight lines. With one species the logarithmic type was resorted to in order to straighten the frequency curves. For another species neither type of paper gave satisfactory results at first. Straight lines were finally obtained by plotting the percentages on logarithmic paper over the diameter *plus a constant*. This constant was the same for stands of any average diameter. For still another species the constant added varied with the average diameter of the stand, the

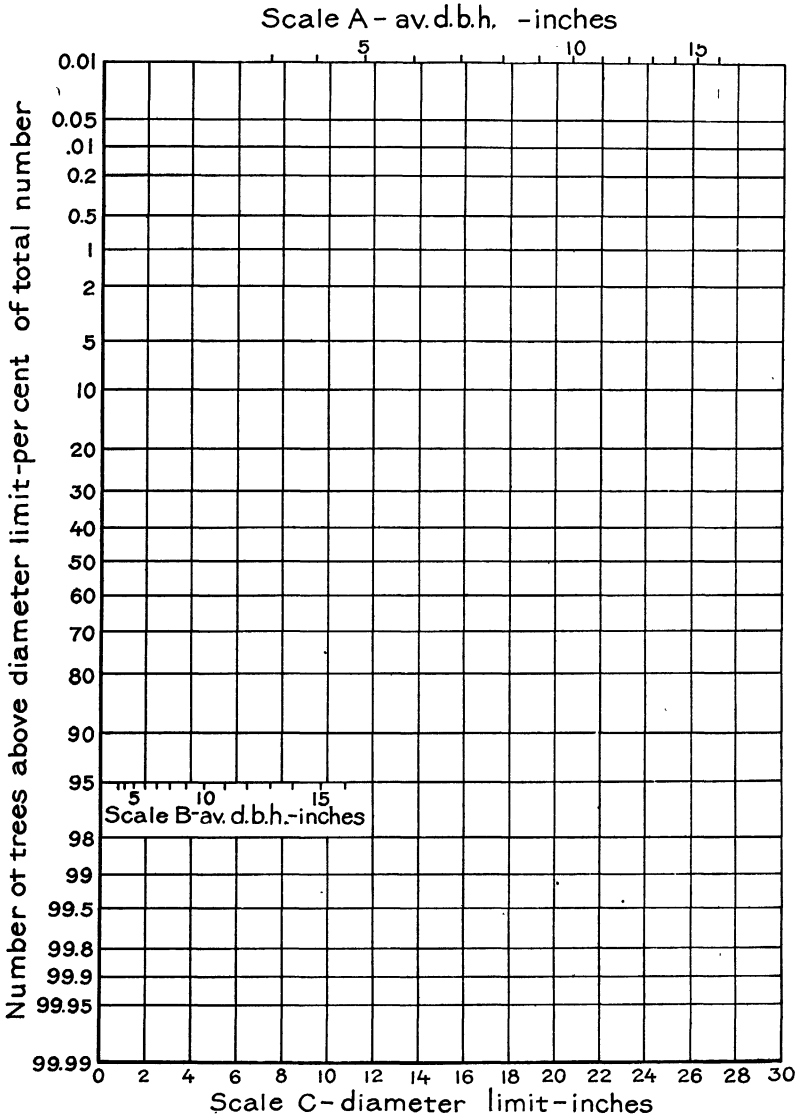

FIG. 8.—Stand-table chart. The frequency distribution in a stand will be shown by a straight line passing through the *A* and *B* scales at the average diameter (by basal area) of the stand

best results being obtained when the constant added for each stand was the same as its average diameter.

The pair of charts used by Bruce for presenting the stand tables graphically can be simplified and combined in one chart, easier to use, by substituting two additional scales to take the place of his two curves of standard deviation and average diameter. To do this, two somewhat similar curves are prepared from the readings of the basic frequency curves [10] at percentages as far apart on the chart as convenient—in the example given here the readings of 95 and 0.01 per cent are used. The diameters at which each frequency curve intercepts the two percentage lines selected are plotted over the average diameter for this frequency curve. A curve is fitted to each of the two series of points, as shown in Figure 7. Reading back from these two curves, two scales of average diameter are constructed along the percentage lines. These are scales *A* and *B* in Figure 8. To locate the line representing the frequency distribution in a stand of given average diameter all that is necessary is to draw a line passing through this average diameter on both of the *A* and *B* scales.

It will be noted that the frequency graph presented here has percentages along the vertical scale and diameters along the horizontal scale, in keeping with the standard practice of using abscissas for the independent variable and ordinates for the dependent variable. The frequency paper as printed does not conform to this standard.

SUMMARY

The modification here reported of Bruce's method of preparing timber-yield tables consists, chiefly, in expressing the conventional system of curves by means of alignment charts. The curves developed by Bruce's method may be converted into alignment-chart form, but several advantages accrue from deriving the alignment charts directly from the basic data. Chief among these advantages are the reduction in the number of curves to be fitted, with a consequent increase in definition of each curve, and the saving in labor made possible by the ease of interpolating the alignment charts for any age and site index. In addition to the saving of labor in preparing the yield tables the alignment-chart form permits the presentation of final results in greatly condensed form; a single sheet of pocket notebook size is sufficient to carry the results with enough accuracy for field use.

To insure greater accuracy an additional cross check between height, stand basal area, cubic volume, and "forest form factor" has been incorporated. Several minor errors in Bruce's method have been corrected. A simpler method of computing and graphically presenting stand tables has been described.

The method, as modified, has no disadvantages not inherent in Bruce's method, except that partial-stand values can not be read directly but must be obtained through converting factors. It has all the advantages of Bruce's method plus the advantages resulting from the use of alignment charts.

[10] The basic curves should be prepared from the data grouped by average-diameter classes, instead of by age-site index classes as used by Bruce. No variation in form of the curves has been detected between stands of the same average diameter but of different age or site quality. The age-site index grouping was desirable, in technic development, for analysis. Since the soundness of the technic has been established, however, it is more desirable to use the grouping by average diameter, with its resultant stronger, fewer curves.

APPENDIX

INSTRUCTIONS FOR USING ALIGNMENT CHART YIELD TABLE

For—	Hold age on—	Hold SI on—	Read—	Multiply by—
A. For site classification hold age on *B*, hold height of average dominant on *X*, and read site index on *B'*.				
B. Height of average dominant	B	B'	X	------
C. Entire stand:				
1. Average d. b. h., inches	C	C'	X'	------
2. Tree basal area, square feet	C	C'	X	0.01
3. Number of trees per acre	D	D'	X	10
4. Basal area, square feet per acre	E	E'	X	------
5. Volume, total, less bark, cubic feet per acre	F	F'	X	100
D. Partial stand: For any of the five factors listed under *C* determine its entire-stand value first. Determine average d. b. h. (*C*–1) also. Pass a line through this average d. b. h. (on *X'*) and the point *P'*.				
1. For the stand 4 inches plus—				
Read average d. b. h. on	L	------	------	------
Read per cent number of trees on	N	------	------	------
Read per cent volume (cubic feet) on	O	------	------	------
Read cubic feet per cord on	M	------	------	------
2. For the stand 7 inches plus—				
Read average d. b. h. on	I	------	------	------
Read per cent number of trees on	G	------	------	------
Read board feet per cubic foot on	H	------	------	------
3. For the dominant stand—				
Read average d. b. h. on	J	------	------	------
Read per cent number of trees on	K	------	------	------

Multiply the entire-stand values by the percentages and ratios read, holding the entire-stand value on *A*, the percentage or ration on *A'*, reading the partial-stand value on *X* pointing off as with a slide rule.

NOTES.—The cubic feet per cord values (*M*) represent the ratio of cubic feet (entire stem, less bark) to cords (to 3-inch top d. i. b.)

The board feet per cubic foot values (*H*) represent the ratio

$$\frac{\text{Board feet stand 7-inch plus}}{\text{Total cubic feet entire stand.}}$$