BARD Final Scientific Report
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Project Title: Scale-dependency and spatial variability of soil hydraulic properties

Investigators

Principal Investigator (PI):

Renduo Zhang

Co-Principal Investigator (Co-PI):

David Russo

Collaborating Investigators:

Institutions

Department of Renewable Resources

**University of Wyoming, Laramie, WY 82071-3354
USA**

**Institute of Soils and Water, Agricultural Research
Organization, The Volcani Center, Israel.**

Keywords *not* appearing in the title and in order of importance. Avoid abbreviations.

Block averaging, disk infiltrometer, fractal and geostatistical characterization, upscaling

Abbreviations commonly used in the report, in alphabetical order:

PSD: particle size distributions, SWRC: soil water retention curve

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Signature
Principal Investigator

Signature
Authorizing Official, Principal Institution

BARD Final Scientific Report

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Publication Summary (numbers)

	Joint IS/US authorship	US Authors only	Israeli Authors only	Total
Refereed (published, in press, accepted)		3	3	6
Submitted, in review, in preparation				
Invited review papers				
Book chapters				
Books				
Master theses				
Ph.D. theses		1		1
Abstracts		3		3
Not refereed (proceedings, reports, etc.)				

Postdoctoral Training: List the names and social security/identity numbers of all postdocs who received more than 50% of their funding by the grant.

Cooperation Summary (numbers)

	From US to Israel	From Israel to US	Together, elsewhere	Total
Short Visits & Meetings				
Longer Visits (Sabbaticals)				

Description of Cooperation:

Our research was designed to create new experimental and theoretical tools for addressing the problems created by the fact that soil hydraulic properties are usually measured at a small scale and used for quantifying flow transport in large scales. Wherever possible, US and Israeli investigators used complementary approaches to the same problem to maximize the effectiveness of the investigation.

Patent Summary (numbers)

	Israeli inventor (s) only	US inventor (s) only	Joint IS/US inventors	Total
Submitted				
Issued (allowed)				
Licensed				

BARD PROPOSAL No. US-3287-01R

FINAL SCIENTIFIC PROGRESS REPORT

Abstract

Water resources assessment and protection requires quantitative descriptions of field-scale water flow and contaminant transport through the subsurface, which, in turn, require reliable information about soil hydraulic properties. However, much is still unknown concerning hydraulic properties and flow behavior in heterogeneous soils. Especially, relationships of hydraulic properties changing with measured scales are poorly understood. Soil hydraulic properties are usually measured at a small scale and used for quantifying flow and transport in large scales, which causes misleading results. Therefore, determination of scale-dependent and spatial variability of soil hydraulic properties provides the essential information for quantifying water flow and chemical transport through the subsurface, which are the key processes for detection of potential agricultural/industrial contaminants, reduction of agricultural chemical movement, improvement of soil and water quality, and increase of agricultural productivity.

The original research objectives of this project were: 1. to measure soil hydraulic properties at different locations and different scales at large fields; 2. to develop scale-dependent relationships of soil hydraulic properties; and 3. to determine spatial variability and heterogeneity of soil hydraulic properties as a function of measurement scales.

The US investigators conducted field and lab experiments to measure soil hydraulic properties at different locations and different scales. Based on the field and lab experiments, a well-structured database of soil physical and hydraulic properties was developed. The database was used to study scale-dependency, spatial variability, and heterogeneity of soil hydraulic properties. An improved method was developed for calculating hydraulic properties based on infiltration data from the disc infiltrometer. Compared with the other methods, the proposed method provided more accurate and stable estimations of the hydraulic conductivity and macroscopic capillary length, using infiltration data collected at short experiment periods. We also developed scale-dependent relationships of soil hydraulic properties using the fractal and geostatistical characterization.

The research effort of the Israeli research team concentrates on tasks along the second objective. The main accomplishment of this effort is that we succeed to derive first-order, upscaled (block effective) conductivity tensor, K'_{ij} , and time-dependent dispersion tensor, D'_{ij} , $i,j=1,2,3$, for steady-state flow in three-dimensional, partially saturated, heterogeneous formations, for length-scales comparable with those of the formation heterogeneity. Numerical simulations designed to test the applicability of the upscaling methodology to more general situations involving complex, transient flow regimes originating from periodic rain/irrigation events and water uptake by plant roots suggested that even in this complicated case, the upscaling methodology essentially compensated for the loss of sub-grid-scale variations of the velocity field caused by coarse discretization of the flow domain. These results have significant implications with respect to the development of field-scale solute transport models capable of simulating complex real-world scenarios in the subsurface, and, in turn, are essential for the assessment of the threat posed by contamination from agricultural and/or industrial sources.

Achievements

Significance of Results

We were able to meet all of our goals in this BARD project, as articulated in the project proposal objectives. In this section we discuss the significance of our results relative to the original expectations.

The US investigators conducted additional field experiment in two fields using different disc sizes (scales) of tension infiltrometers at 100 locations. At each location, measurements were taken on the six scales and two tension values. Soil core samples were collected with the four core sizes at the locations, which coincided with the infiltrometer-measured locations. Based on the field and lab experiments, a well-structured database of soil physical and hydraulic properties was developed. The database was used to study scale-dependency, spatial variability, and heterogeneity of soil hydraulic properties.

The disc infiltrometer has been established as a field device for in-situ measurements of unsaturated infiltration under tensions. Several methods were developed for calculating hydraulic properties based on infiltration data from the disc infiltrometer. The method by Zhang (1998) has the advantage in using infiltrometer data measured in a relatively short time period to estimate soil hydraulic properties. In this study, we improved this method by replacing the piecewise linear approximation of hydraulic conductivity function ($K(h)$), which results in overestimated macroscopic capillary length, with the exponential form of $K(h)$. An efficient and accurate iteration procedure was introduced to solve the highly nonlinear equations of the improved method. Based on the infiltration data in the field experiments, the improved method and other three methods in the literature were utilized to estimate soil hydraulic conductivities at different tensions and macroscopic capillary length. Compared with other methods, the proposed method provided more accurate and stable estimations of the hydraulic conductivity and macroscopic capillary length, using infiltration data collected at short experiment periods.

We developed scale-dependent relationships of soil hydraulic properties using the fractal and geostatistical characterization. Empirical models have been developed to describe the soil water retention curve (SWRC). Applications of the fractal theory may provide a useful tool to fill the gap between the use of empirical models and physical interpretation of their parameters. Especially a more generalized model for SWRC has been developed based on the pore-solid fractal (PSF) distribution. The PSF model covers several exist models as its special cases and theoretically provides a direct way to estimate the SWRC fractal dimension from particle size distributions (PSD). We evaluated the SWRC with the PSF distribution using more than 400

data sets of PSD and SWRC. Then we established a relationship between the fractal dimension and soil texture. Analyses of the data sets showed that the fractal dimension values estimated using the PSD data were consistently smaller than those estimated using the SWRC data. Therefore, using the fractal dimension from PSD to predict the SWRC will result in underestimation. To resolve the problem, the soil data sets of PSD and SWRC were used to establish relationships between the fractal dimensions in the PSF model of SWRC and soil clay content as well as soil texture class. Independent SWRC data sets were used to test the methods. Predicted results using the PSF model of SWRC with the fractal dimension estimated using clay content or soil texture class were compared with the measured soil water retention data. Linear regressions of the predicted and measured SWRC showed good agreement for relatively fine texture soils.

The Israeli research team succeeded to derive first-order, upscaled (block effective) conductivity tensor, K'_{ij} , and time-dependent dispersion tensor, D'_{ij} , ($i,j=1,2,3$) for steady-state flow in three-dimensional, partially saturated, heterogeneous formations, for length-scales comparable with those of the scale of the formation heterogeneity. Furthermore, they were able to determine the averaging domain required in order to obtain asymptotic, effective conductivities and dispersion coefficients devoid of the domain size, and their dependence on soil stratification, soil texture and mean soil water saturation.

The results of this effort suggest that for the physically plausible situation in which the length scale of the formation heterogeneity in the horizontal direction is larger than its counterpart in the vertical direction, the size of the averaging domain required in order to reduce the uncertainty about the upscaled log-conductivity to an acceptable, predetermined level, should increase with increasing stratification and decreasing capillary forces (coarser soil texture); the required averaging domain should increase further in formations in which the correlation length-scale of the soil parameter α is large compared to that of saturated conductivity, K_s .

The results of this effort are of significance with respect to important issues pertinent to flow and transport through unsaturated heterogeneous porous formations, as they allow assessment of (i) the minimal domain size for which the concept of effective properties is appropriate; and (ii) the maximal size of conductivity cells which preserve the heterogeneous structure of the underlying formation with respect to simulation of flow and transport in the vadose zone. New findings of this research suggest that under unsaturated flow conditions, both the time required to reach the asymptotic limits of the principal components of the dispersion tensor, D'_{ij} , and the length-scale needed to reach the ergodic limits of D'_{ij} depend on soil stratification, soil texture and mean soil water saturation, and differ for the longitudinal and the transverse components of D'_{ij} .

Numerical simulations designed to test the applicability of the upscaling methodology to more general situations involving complex, transient flow regimes originating from periodic rain/irrigation events and water uptake by plant roots suggested that even in this complicated case, the developed upscaling methodology essentially compensated for the loss of sub-grid-scale variations of the velocity field caused by coarse discretization of the flow domain. These results have significant implications with respect to the development of field-scale solute transport models capable of simulating complex real-world scenarios in the subsurface, which, in turn, are essential for the assessment of the threat posed by contamination from agricultural and/or industrial sources.

Cooperation

Although the main emphasis of the US group was physical experimentation and that of the Israeli group was numerical and stochastic modeling, the two groups conducted their research from a common design scheme that produced different perspectives on common problems. Water flow and solute transport on the field scale are of sufficient complexity that current methods are often found lacking in representing these processes under natural conditions. Our research was designed to create new experimental and theoretical tools for addressing the problems created by the fact that soil hydraulic properties are usually measured at a small scale and used for quantifying flow transport in large scales. Wherever possible, US and Israeli investigators used complementary approaches to the same problem to maximize the effectiveness of the investigation.

List of Publications

Russo, D., 2003, Upscaled conductivity in gravity dominated flow through variably saturated heterogeneous formations. *Water Resour. Res.* 39(9): 1255, doi: 10.1029/2002WR001857.

Russo, D., 2003, Block-effective macrodispersion in variably saturated heterogeneous formations. *Water Resour. Res.* 39(12): 1374, doi: 10.1029/2003WR002724.

Russo, D., 2003, Simulation of transport in three-dimensional heterogeneous unsaturated soils with upscaled dispersion coefficients. *Water Resour. Res.* 39(12): 1358, doi: 10.1029/2003WR002245.

Xue,X., **R. Zhang**, and S. Gui, 2004. An improved disc infiltrometer method for calculating soil hydraulic properties. *Can. J. Soil Sci.*, 84: 265-273.

Huang, G., and **R. Zhang**, 2004, Evaluation of soil water retention curve with the pore-solid fractal model. *Geoderma* (in press).

Wang, K. **R. Zhang**, and F. Wang., 2004, Testing the pore-solid fractal model for the soil water

retention function. *Soil Sci. Soc. Am. J.* (in press)

Appendix

Published Papers

Appendix A: Upscaled conductivity in gravity dominated flow through variably saturated heterogeneous formations.

Appendix B: Block-effective macrodispersion in variably saturated heterogeneous formations.

Appendix C: Simulation of transport in three-dimensional heterogeneous unsaturated soils with upscaled dispersion coefficients.

Appendix D: Evaluation of soil water retention curve with the pore-solid fractal model.

Appendix E: An improved disc infiltrometer method for calculating soil hydraulic properties.

Appendix F: Testing the pore-solid fractal model for the soil water retention function.

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WANG ET AL.: THE PORE-SOLID FRACTAL MODEL AND SOIL WATER RETENTION

Testing the Pore-Solid Fractal Model for the Soil Water Retention Function

Kang Wang, Renduo Zhang,* and Fuqin Wang

ABSTRACT

The soil water retention curve is an important hydraulic function for the study of flow transport processes in unsaturated soils. To accurately describe and interpret the hydraulic property, a general soil water retention function was developed based on the pore-solid fractal (PSF) model. The objective of this study was to evaluate the general soil water retention function using data of 65 soils and to compare the PSF function with its special cases, that is, three other soil water retention functions. Defined from the parameters of the PSF, an index of b/q_s was used to quantify the relationship between the PSF and the other soil water retention functions. The PSF function fit all the data sets well, whereas the other retention functions only matched the retention data for some soils, ranging from 11 to 72% of the tested soils. Directly fitting these functions with the data sets showed that for 30 to 40% of the tested soils, these functions gave poorer results than the PSF.

K. Wang, R. Zhang, and F. Wang, State Key Lab. of Water Resources and Hydropower Engineering Science, Wuhan Univ., Wuhan 430072, China; R. Zhang, Dep. of Renewable Resources, Univ. of Wyoming, Laramie, WY 82071-3354, USA. Received 18 July 2004. Soil Physics. *Corresponding author (renduo@uwyo.edu).

Abbreviations: PSF, pore-solid fractal.

The soil water retention function defines the relationship between water content and soil matric potential, and is an important hydraulic property necessary to study water flow and chemical movement in unsaturated soils. Various models have been developed to characterize the soil water retention function (e.g., Brooks and Corey, 1964; Tyler and Wheatcraft, 1990; Rieu and Sposito, 1991). Applications of fractal geometry (Mandelbrot et al., 1984) in soil science have shown that the complicated porous medium can be characterized by the fractal representation. In recent years, fractal approaches have been developed to model soil aggregate, soil mass, soil pore space, and pore surface and particle-size distributions (e.g., Turcotte, 1986; Perfect and Kay, 1991; Bittelli et al., 1999; Perrier and Bird, 2002). Fractal models of soil hydraulic properties, such as the

unsaturated soil hydraulic conductivity and water retention function, have also been proposed (Tyler and Wheatcraft, 1990; Rieu and Sposito, 1991; Gimenez et al., 1997). For example, Tyler and Wheatcraft (1990) derived an expression for soil water retention using the Sierpinski carpet model. Perfect et al. (1998) introduced a retention function related to the Euclidean dimension of space and a fractal dimension.

The PSF model developed by Perrier et al. (1999) and Perrier and Bird (2002) is an extension and generalization of the fractal approach to modeling soil structure. In its most general form, the model illustrates the symmetry between the solid and pore phases. In this respect, it is quite different from a conventional fractal model (Perrier and Bird, 2002). Based on the PSF theory, Bird et al. (2000) derived a soil water retention function. The PSF model yields a general expression for the water retention function. The retention functions of Brooks and Corey (1964), Tyler and Wheatcraft (1990), and Rieu and Sposito (1991) become special cases of the PSF water retention function. Bird et al. (2000) tested the PSF model with only two soils. The objective of this paper was to test the general PSF function with soil water retention data of 65 soils and to compare the PSF function with the models of Brooks and Corey (1964), Tyler and Wheatcraft (1990), and Rieu and Sposito (1991).

MATERIALS AND METHODS

Based on the PSF theory, Bird et al. (2000) derived a soil water retention function. The PSF function is expressed as follows:

INSERT EQUATION [1]

where θ is the volumetric water content ($\text{cm}^3 \text{cm}^{-3}$), h is the suction head (cm), Φ is the total soil porosity ($\text{cm}^3 \text{cm}^{-3}$), p and s denote proportions of regions occupied by pores and solids ($\text{cm}^3 \text{cm}^{-3}$), respectively, $h_{\min}$ and $h_{\max}$ (cm) are the suctions at which the largest and smallest pores explicitly defined within the PSF desaturate, d is the Euclidean dimension, and D is the fractal dimension of the PSF. Setting $d = 3$ and

INSERT EQUATION [2]

we have

INSERT EQUATION [3]

In the model of Bird et al. (2000), the fractal dimension is calculated using cumulative distributions of solid mass. Specifically the following equation is used for the calculation:

INSERT EQUATION [4]

Here $M_{s(i)}$ is the total mass of solid particle size $\leq r_{s(i)}$, which can be obtained directly by summing the solids volume of size $\leq r_{s(i)}$ multiplied by their density d_s , $r_{s(i)}$ is the upper limit of the particle size, i is fractal iteration number, L is the linear size of the studying region, ξ is a constant such that ξL^d equals the bulk volume of the PSF.

It is assumed that soil pores can be full saturated. Then Eq. [3] is rewritten as

INSERT EQUATION [5]

where θ_s is the saturated water content ($\theta_s = \Phi$). If $\beta = \theta_s$, Eq. [5] is equivalent to the equation of Tyler and Wheatcraft (1990):

INSERT EQUATION [6]

The Brooks and Corey (1964) formulation is as follows:

INSERT EQUATION [7]

in which θ_r is the residual water content ($\text{cm}^3 \text{ cm}^{-3}$), h_a is the air-entry suction (cm) and equivalent to $h_{\min}$ in Eq. [5], and λ is the pore-size distribution index. Comparing Eq. [5] with Eq. [7], we can see that they are essentially the same if $\beta = \theta_s - \theta_r$ and $\lambda = 3 - D$. In Eq. [1], when $s = 0$, $\beta = 1$, the PSF reduces to a simple mass fractal structure and the water retention function becomes (Bird et al., 2000)

INSERT EQUATION [8]

which is the model by Rieu and Sposito (1991).

To evaluate the applicability of the PSF function (Eq. [5]) and its special cases, the models of Brooks and Corey (1964), Tyler and Wheatcraft (1990), and Rieu and Sposito (1991), we utilized 65 data sets of soil water retention collected from the literature and our own measurements. Among the data sets, 41 were obtained from the Unsaturated Soil Hydraulic Database (UNSODA) (Leij et al., 1996). Ten data sets were collected from Huang and Zhan (2002), six were from Basile and Guidicelli (1997), and eight were our measured data sets. Huang and Zhan (2000?) measured the data of soil water retention using undisturbed soil samples with the pressure plate method (Hillel, 1998). Basile and Guidicelli (1997) measured the soil water retention data using the classical wind method (Wendroth et al., 1993). We used undisturbed soil samples and the pressure plate method to measure the soil water retention curves. The data sets of soil water retention from the UNSODA (Leij et al., 1996) were measured using several methods, such as the pressure plate method, hanging water column and pressure plate method, pressure outflow method, and tensiometry and TDR method. For all the data sets, soils were classified to clay ($<2 \mu\text{m}$), silt (2–50

μm), and sand ($>50 \mu\text{m}$) based on the USDA classification scheme. Then soil texture was determined according to the USDA triangle. Clay contents of the soils varied between 0 and 72%. To warrant meaningful statistical analyses, each of the data sets included more than 10 data points. The soils were grouped based on texture in Table 1.

To test the soil water retention functions, the models were fit to the soil water retention data by means of a least-squares approach based on Marquardt's maximum neighborhood method (Marquardt, 1963). By minimizing the sum squared error between the measured water content and the estimated water content with the model, the procedure provided the best fitting parameters, the 95% confidence intervals of the parameters, and the value of goodness of fit (r^2).

The detailed calculations and statistics were described as follows. First we fit the PSF (Eq. [5]) to the data sets of soil water retention to obtain values of the parameters θ_s , β , $h_{\min}$, and D . Then the values of β/θ_s and β were used to quantify the relationship between the PSF (Eq. [5]) and the other soil water retention functions (Eq. [6], [7], and [8]). With the confidence intervals of the fitted θ_s and β , we calculated intervals of β/θ_s of the soils. For a soil, if the interval of β/θ_s included 1, we considered that β/θ_s of the soil was not significantly different from 1 or $\beta \approx \theta_s$. If all β/θ_s values within the interval were greater or smaller than 1, we considered β/θ_s of the soil was significantly different from 1, that is, $\beta > \theta_s$ or $\beta < \theta_s$. It should be pointed out that the measured saturated water content is more accurately the "satiated water content" because of trapped air.

The fractal dimension D plays an important role in the soil water retention curve functions. Therefore, the models of Tyler and Wheatcraft (1990) and Rieu and Sposito (1991) were also used to fit the data of soil water retention so that we can compare the fractal dimensions by independently fitting with the models (Eq. [5], [6], and [8]). A t test was applied to assess the significant difference of the fractal dimensions (Karlen and Colvin, 1992). The differences between measured soil water retention data and fitted values with the models were evaluated using a statistical scheme (a 95% confidence interval band) similar to that described by Logsdon et al. (2002). For model comparison, we also utilized the root mean square errors between measured data and predicted retention functions with the different models.

As mentioned above, the fractal dimension D was calculated by fitting Eq. [5] to the data of soil water retention. The fractal dimension D can also be evaluated by fitting Eq. [4] to solid distribution data. Taking log-transforms for both sides of Eq. [4] and fitting solid distribution data with the log-transformed equation, we can obtain the fractal dimension from the slope ($3 - D$) of the fitted line. Following the procedure by Logsdon and Karlen (2004), we conducted a paired

comparison t test to check statistical significance for the fractal dimensions evaluated from Eq. [4] and [5].

RESULTS AND DISCUSSION

Table 2 summarizes the parameters (θ_s , β , $h_{\min}$, and D) from fitting Eq. [5] to the data sets of soil water retention for each group of the soils, including the mean, standard deviation, maximum, and minimum of the parameters. The PSF function fit the data well for all the soils. The values of goodness of fit (r^2) between the measured and fitted soil water content values ranged from 0.928 to 0.999 (Table 2). Calculated values of β/θ_s of the soils are also presented in Table 2, ranging from 0.67 to 1.64.

For soils with $\beta \approx \theta_s$, the model of Tyler and Wheatcraft (1990) (Eq. [6]) characterizes the retention functions as well as the PSF (Eq. [5]). There were seven such soils, including one sand soil, one silty loam soil, one silty clay loam soil, one loam soil, one clay loam soil, and two sandy loam soils, which accounted for 11% of the total tested soils.

For soils with $\beta/\theta_s > 1$, using Eq. [6] (i.e., setting θ_s equal to β in Eq. [5]) overestimated the soil water retention function as shown in Fig. 1a for a sand soil ($\beta = 0.35$ and $\theta_s = 0.30$). This category included 24 soils, accounting for 37% of the total tested soils. On the contrary, for the soils with $\beta/\theta_s < 1$, using Eq. [6] underestimated retention functions. As an example, Fig. 1b compares the measured soil water retention data with fitted curves of Eq. [5] and calculated curves of Eq. [6] for a clay soil with $\beta = 0.41$ and $\theta_s = 0.56$. There were 34 soils with $\beta/\theta_s < 1$, accounting for 52% of the total tested soils.

For the 58 soils with $\beta/\theta_s > 1$ or < 1 , we fit Eq. [6] independently to the soil retention data. The values of the fractal dimension from the fitting process are listed in Table 3. For 32 soils, Eq. [6] fit the measured data equally well as Eq. [5]. However, the fitted $h_{\min}$ values with the two models were quite different.

Among the soils with $\beta/\theta_s > 1$, Eq. [6] fit the retention data poorly for seven soils (one silt soil, two silty loam soils, two clay loam soils, and two clay soils). As an example shown in Fig. 2 for a silty loam soil, the fitting parameters of the PSF (Eq. [5]) were $\theta_s = 0.44$, $\beta = 0.72$, $h_{\min} = 29.48$ cm, and $D = 2.89$, and the fitting parameters of Eq. [6] were $\theta_s = 0.44$, $h_{\min} = 5.88$ cm, and $D = 2.90$. For 11 data sets, Eq. [6] fit the measured data well within the high suction region, but underestimated soil water content at the lower suction region. Figure 3 compares the measured water retention data of a silty clay loam soil and fitted curves with Eq. [5] ($\theta_s = 0.60$, $\beta = 0.64$, $h_{\min} = 28.26$ cm, and $D = 2.85$) and with Eq. [6] ($\theta_s = 0.60$, $h_{\min} = 17.92$ cm, and $D = 2.86$).

Within the soils with β/θ_s , fitted values of the fractal dimension with Eq. [6] for two sand soils were 1.07 and 1.71, respectively. Theoretically, the fractal dimension should range between 2 and 3 for a three-dimensional soil structure. For eight soils (one sand soil, one clay soil, one sandy clay loam soil, one silty loam soil, two sandy loam soils, and three clay loam soils), Eq. [6] did not fit the measured data well.

If $(\theta_s - \theta_r)/\beta = 1$, the general PSF function reduces to Brooks and Corey (1964) model (Eq. [7]). Again we used β/θ_s as an index to compare the applicability of these two models. For soils with $\beta/\theta_s \leq 1$, the residual water content in Eq. [7] was calculated directly from $\theta_r = \theta_s - \beta$ and Eq. [7] works equally well as Eq. [5]. In Table 1, there were 47 soils satisfying the condition, accounting for 72% of the tested soils. These soils also included the soils with $\theta_r = 0$, for which Tyler and Wheatcraft (1990) model also works well.

For soils with $\beta/\theta_s > 1$, using Eq. [7] should cause errors. When we used Eq. [7] to fit retention data of the soils with $\beta/\theta_s > 1$, we obtained negative values of fitted residual water contents, which were physically meaningless. For instance, for a soil with $\theta_s = 0.44$ and $\beta = 0.72$, the fitted residual water content was equal to $\theta_s - \beta = -0.28$. If the residual water content was set to be zero, the fitting results were poor (Fig. 4). This category included 18 soils (two sand soils, two sandy loam soils, two silty clay loam soils, two silt soils, three clay soils, four clay loam soils, and six silty loam soils), accounting for 28% of the tested soils.

Theoretically, the PSF reduces to the model of Rieu and Sposito (1991) (Eq. [8]) when $\beta = 1$. As shown in Table 2, 77% of the tested soils have β values less than or equal to 0.5; 17% of the tested soils have β values between 0.5 and 0.6; two soils (3% of the tested soils) have β values between 0.6 and 0.7; one soil has a β value of 0.72; and one soil has a β value of 0.84. For all the soils, values of the 95% confidence intervals of β were smaller than 0.85. Since the β values of the soils were significantly different from 1, the model of Rieu and Sposito (1991) was not comparable with the PSF function for these soils. As pointed out by Perrier et al. (1996), the condition of $\theta_s \rightarrow 1$ as $h \rightarrow \infty$ in the model of Rieu and Sposito (1991) was related to a Menger sponge with an infinite number of recursive steps and to the case that solid proportion was equal to zero in the PSF model. Probably this case is relatively rare in the real soils.

We used Eq. [8] (i.e., fixing $\beta = 1$ in Eq. [5]) to fit all the soil data sets. For 22 soils (one loamy sand soil, one loam soil, two silt loam soils, three sand soils, three silt soils, three sandy loam soils, four clay loam soils, and five clay soils), the fitted soil water retention curves were significantly different from the measured data. Figure 5 shows an example of the fitting curve with Eq. [8] ($\theta_s =$

0.24, $h_{\min} = 8.50$ cm, and $D = 2.96$) and compares with the fitting curve of Eq. [5] ($\theta_s = 0.24$, $\beta = 0.21$, $h_{\min} = 26.60$ cm, and $D = 2.49$).

We also compared the root mean square errors between measured data and predicted retention functions with the models of PSF (Eq. [5]), Tyler and Wheatcraft (1990) (Eq. [6]), and Rieu and Sposito (1991) (Table 3) (Eq. [8]). The mean and maximum values of the error for Eq. [5] were smaller than those for Eq. [6] and [8]. Table 3 also lists the values of fitted fractal dimension using Eq. [6] and [8]. For most of the soils, the fractal dimensions of Eq. [8] were >2.90 . The range of the fractal dimension values of Rieu and Sposito (1991) model is much narrower than those of the PSF and Tyler and Wheatcraft (1990) models. Independent t tests indicated that the fractal dimensions between the PSF and Rieu and Sposito (1991) models were significantly different at the 95% confidence interval, whereas the difference of fractal dimensions between the PSF and Tyler and Wheatcraft (1990) models was not significant. We realized that the reason for Eq. [5] to give the best fitting results (the higher r^2 values and smaller root mean square errors) was because Eq. [5] has a larger number of fitted parameters than the other equations. This also resulted in a slightly larger standard deviation of the fitted fractal dimensions from Eq. [5] than from Eq. [6] and [8] (Tables 2 and 3).

Fractal dimension values by fitting Eq. [4] to available particle distributions are listed in Table 4 for 13 soils. The largest mass fractal dimension was 2.93 for a clay soil, while the smallest value was 2.45 for a loamy sand soil. The fractal dimension was related to soil texture and showed an increasing tendency with the clay content. For these soils, the mean and variance of the fractal dimension values were 2.742 and 0.0147, respectively. For the same soils, the mean and variance of the fractal dimension values by fitting Eq. [5] to soil water retention data were 2.798 and 0.0226, respectively (Table 4). A paired comparison t test showed that the differences of the fractal dimension values fit with Eq. [4] and [5] were significant at a level of significance of 0.05. Since the fractal dimension is a sensitive parameter, a small difference of D can result in large deviations of estimated retention functions from measured data (Fig. 6).

CONCLUSIONS

A total of 65 data sets of soil water retention collected from the literature and our measurements were applied to evaluate the applicability of the soil water retention function proposed by Bird et al. (2000). The general PSF model fit all the data sets well with the values of goodness of fit (r^2) ranging from 0.928 to 0.999. The special cases of the PSF function, that is, the retention functions of Brooks and Corey (1964), Tyler and Wheatcraft (1990), and Rieu and Sposito (1991), were also tested using the data sets.

The index of β/θ_s , ranging from 0.67 to 1.64, was used to determine the applicability of Tyler and Wheatcraft (1990) model (Eq. [6]). Only for soils with $\theta_s/\beta \approx 1$, does Eq. [6] describe the soil retention function well; otherwise, it either underestimates or overestimates retention functions. For the tested data sets, Eq. [6] described the soil retention curve well for 11% of the soils, whereas the equation underestimated the retention function for 52% of the soils and overestimated for 37% of the soils. Equation [6] was also used to fit the data sets independently. The results showed that the model fit the data reasonably well for 60% of the tested soils, including two sand soils with fitted fractal dimensions smaller than 2. For 40% of the soils, the equation provided less accurate results of fitted soil water retention curves than the PSF.

The index of β/θ_s was also applied to compare the PSF retention function with the model of Brooks and Corey (1964) (Eq. [7]). As $\beta/\theta_s \leq 1$, the general PSF function reduces to Eq. [7]. For about 72% of the tested soils, $\beta/\theta_s \leq 1$, Eq. [7] matched the retention data of the soils as accurately as the PSF function. For the rest of the soils, $\beta/\theta_s > 1$, Eq. [7] resulted in negative values of the residual water content, which was physically meaningless.

Theoretically, the PSF reduces to the model of Rieu and Sposito (1991) (Eq. [8]) when $\beta = 1$. However, the fitted β values from the PSF, ranging from 0.21 to 0.84, were not closed to 1 for all the soils. From this point of view, the model of Rieu and Sposito (1991) was not comparable with the PSF function for the tested soils. Directly fitting Eq. [8] with the data showed that for 37% of the soils, the equation gave poorer results than the PSF. For most of the soils, the fractal dimensions from the fitting process were great than 2.90.

The fractal dimension by fitting soil water retention data was compared with the fractal dimension evaluated using solid distribution data. For most of the soils, the fractal dimension values from solid distribution data were smaller than those from soil water retention data and the fractal dimensions from the two fitting processes were significantly different, which showed some inconsistencies in the PSF model.

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Fig. 1. Measured soil water retention data (circles), fitted curve of Eq. [5] (solid line), and calculated curve of Eq. [6] (dash line) for (a) a sand soil with $b/q_s > 1$ and (b) a clay soil with $b/q_s < 1$.

Fig. 2. Comparison of measured water retention data (circles) of a silty loam and fitted curves with Eq. [5] (solid line) and with Eq. [6] (dash line).

Fig. 3. Comparison of measured water retention data (circles) of a silty clay loam soil and fitted curves with Eq. [5] (solid line) and with Eq. [6] (dash line).

Fig. 4. Measured soil water retention data (circles) of a sand soil and fitted curves with Eq. [5] ($q_s = 0.39$, $b = 0.53$) (solid line) and Eq. [7] (setting $q_r = 0$) (dash line).

Fig. 5. Comparison of measured water retention data (circles) of a loam soil and fitted curves with Eq. [5] (solid line) and with Eq. [8] (dash line).

Fig. 6. Measured water retention data (circles), fitted curve with Eq. [5] (solid line), and calculated curve with D from Eq. [4] (dash line) for (a) a sandy loam soil and (b) a sandy clay loam soil.

Table 1. Basic information of the soil data sets.

Texture	Number of data sets	Clay content		Sand content	
		Mean $\pm$ SD [†]	Max./Min. [‡]	Mean $\pm$ SD	Max./Min.
		%			
Sand	6	1.3 $\pm$ 1.6	4.9/0.0	95.5 $\pm$ 3.6	100/93
Loamy sand	2	5.9	7.9/4.0	82.1	83.0/81.1
Sandy loam	7	13.8 $\pm$ 3.1	18.1/10.0	62.6 $\pm$ 5.2	70.5/54.2
Sandy clay loam	3	28.6 $\pm$ 2.4	26.9/32.0	60.1 $\pm$ 8.3	70.4/50.0
Silt	4	3.6 $\pm$ 3.8	9.2/0.0	2.2 $\pm$ 3.4	8.0/0.0
Silt loam	9	13.6 $\pm$ 4.3	24.2/9.0	12.3 $\pm$ 9.0	20.4/1.4
Clay loam	9	35.5 $\pm$ 5.1	37.5/31.8	34.4 $\pm$ 5.1	42.0/29.4
Loam	6	18.0 $\pm$ 3.0	23.8/15.4	38.4 $\pm$ 6.7	45.0/34.1
Silty clay loam	5	33.0 $\pm$ 3.6	28.9/39	6.06 $\pm$ 3.58	10.0/0.7
Sandy clay	1	42.0		51.0	
Silty clay	2	46.1	47.0/45.2	3.1	6.0/0.2
Clay	11	52.8 $\pm$ 10.3	72.0/40.9	15.5 $\pm$ 10.3	33.0/0.2

[†]Standard deviation.

[‡]Maximum and minimum values.

Table 2. Summary statistics of the fitted parameters of the PSF.

Texture	q_s		b		$h_{\min}$		D		q_s/b	r^2
	Mean $\pm$ SD [†]	Max./Min. [‡]	Mean $\pm$ SD	Max./Min.	Mean $\pm$ SD	Max./Min.	Mean $\pm$ SD	Max./Min.	Max./Min.	Max./Min.
	cm ³ cm ⁻³		cm							
Sand	0.33 $\pm$ 0.07	0.41/0.23	0.34 $\pm$ 0.11	0.53/0.21	16.9 $\pm$ 8.70	28.2/4.66	2.28 $\pm$ 0.26	2.64/2.01	1.36/0.88	0.999/0.934
Loamy sand	0.44	0.44/0.43	0.41	0.42/0.40	15.50	22.5/8.4	2.82	2.85/2.78	0.96/0.88	0.991/0.968

Sandy loam	0.44 ± 0.14	0.69/0.2 6	0.43 ± 0.13	0.59/0. 25	19.8 ± 11.1	40.8/0.3 7	2.66 ± 0.24	2.87/2. 08	1.06/0. 84	0.993/0. 936
Sandy clay loam	0.40 ± 0.09	0.52/0.3 1	0.32 ± 0.03	0.36/0. 29	15.3 ± 16.2	37.8/1.0 2	2.91 ± 0.05	2.95/2. 84	0.94/0. 69	0.989/0. 980
Silt	0.30 ± 0.08	0.44/0.2 4	0.32 ± 0.11	0.50/0. 23	63.1 ± 29.0	86.6/13. 6	2.67 ± 0.10	2.78/2. 55	1.14/0. 92	0.928/0. 971
Silty loam	0.45 ± 0.10	0.67/0.2 7	0.50 ± 0.10	0.72/0. 42	31.2 ± 8.51	48.7/25. 3	2.77 ± 0.08	2.89/2. 66	1.33/0. 78	0.996/0. 961
Clay loam	0.44 ± 0.07	0.56/0.3 3	0.44 ± 0.11	0.60/0. 32	26.1 ± 22.3	77.1/0.5 4	2.86 ± 0.08	2.96/2. 66	1.22/0. 73	0.998/0. 925
Loam	0.46 ± 0.12	0.68/0.2 9	0.40 ± 0.08	0.54/0. 29	21.1 ± 21.0	65.0/0.3 6	2.80 ± 0.05	2.84/2. 70	1.00/0. 76	0.970/0. 992
Silty clay loam	0.50 ± 0.12	0.66/0.3 3	0.50 ± 0.13	0.65/0. 30	33.2 ± 14.3	55.2/18. 0	2.84 ± 0.04	2.89/2. 78	1.07/0. 91	0.987/0. 956
Sandy clay	0.41		0.25		66.3		2.88		0.94	0.944
Silty clay	0.43	0.53/0.3 3	0.39	0.46/0. 32	22.0	22.6/21. 3	2.91	2.99/2. 82	0.92/0. 87	0.989/0. 988
Clay	0.51 ± 0.09	0.69/0.3 8	0.48 ± 0.14	0.84/0. 33	43.9 ± 85.9	313.2/5. 02	2.90 ± 0.08	2.95/2. 68	1.50/0. 70	0.994/0. 952

†Standard deviation.

‡Maximum and minimum values.

Table 3. Comparison of fitted fractal dimension values among the models.

Texture	<u>Mean/Maximum root mean square error</u>			<u>Fractal dimension fit with Eq. [6]</u>		<u>Fractal dimension fit with Eq. [8]</u>	
	Eq. [5]	Eq. [6]	Eq. [8]	Mean ± SD†	Max./Min. n. ‡	Mean ± SD	Max./Min.
Sand	0.016/0.038	0.025/0.0 47	0.042/0.1 26	2.20 ± 0.61	2.75/1.06	2.85 ± 0.13	2.96/2.77
Loamy sand	0.011/0.018	0.018/0.0 31	0.016/0.0 30	2.69	2.77/2.61	2.94	2.96/2.91

Sandy loam	0.022/0.048	0.030/0.062	0.025/0.055	2.72 ± 0.13	2.87/2.43	2.92 ± 0.05	2.82/2.96
Sandy clay loam	0.004/0.005	0.006/0.007	0.005/0.008	2.92 ± 0.05	2.96/2.85	2.98 ± 0.01	2.99/2.97
Silt	0.024/0.034	0.033/0.047	0.034/0.040	2.68 ± 0.07	2.76/2.59	2.95 ± 0.01	2.97/2.94
Silty loam	0.011/0.022	0.026/0.042	0.022/0.045	2.80 ± 0.05	2.89/2.73	2.94 ± 0.02	2.97/2.91
Clay loam	0.011/0.017	0.012/0.020	0.012/0.022	2.91 ± 0.02	2.96/2.87	2.97 ± 0.01	2.99/2.96
Loam	0.007/0.013	0.010/0.023	0.018/0.044	2.86 ± 0.03	2.90/2.84	2.94 ± 0.04	2.97/2.86
Silty clay loam	0.015/0.023	0.020/0.028	0.020/0.032	2.85 ± 0.04	2.91/2.80	2.95 ± 0.01	2.97/2.93
Sandy clay	0.001	0.003	0.003	2.91		2.97	
Silty clay	0.006/0.009	0.007/0.009	0.008/0.010	2.91	2.93/2.88	2.97	2.98/2.95
Clay	0.011/0.037	0.016/0.041	0.018/0.044	2.92 ± 0.03	2.96/2.77	2.97 ± 0.02	2.98/2.93

†Standard deviation.

‡Maximum and minimum values.

Table 4. Comparison of fractal dimensions obtained from Eq. [5] and [4].

Soil texture	Fractal dimension	Fractal dimension
	from Eq. [5]	from Eq. [4]
Loamy sand	2.43	2.45
Sandy loam	2.65	2.62
Sandy loam	2.87	2.73
Sandy loam	2.71	2.68
Sandy loam	2.81	2.67
Sandy clay loam	2.95	2.79
Clay loam	2.96	2.83
Loam	2.76	2.77
Loam	2.83	2.73

Loam	2.70	2.77
Loam	2.84	2.82
Clay	2.94	2.86
Clay	2.93	2.93