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Precision Cropping by Remotely Sensed Prototype Plots and Calibration in the Complex Domain

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Description of Cooperation:

The principal investigator, Dr. Anderson designed and implemented an innovative controlled precision cropping program, which incorporated multi-spectral and hyperspectral remote sensing by state of the art airborne imaging equipment. The cooperating investigator, Professor. Peleg was primarily responsible for developing and implementing the innovative data analysis tools. The cooperation was particularly enhanced by Dr. Peleg's three six month sabbaticals at the ARS, Northern Plains Agricultural Research Laboratory, in Sidney, Montana, during the summers of 2000, 2001 and 2002.

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Abstract

This research report describes a methodology whereby multi-spectral and hyperspectral imagery from remote sensing, is used for deriving predicted field maps of selected plant growth attributes which are required for precision cropping. A major task in precision cropping is to establish areas of the field that differ from the rest of the field and share a common characteristic. Yield distribution maps can be prepared by yield monitors, which are available for some harvester types. Other field attributes of interest in precision cropping, e.g. soil properties, leaf Nitrate, biomass etc. are obtained by manual sampling of the field in a grid pattern. Maps of various field attributes are then prepared from these samples by the "Inverse Distance" interpolation method or by Kriging. An improved interpolation method was developed which is based on minimizing the overall curvature of the resulting map. Such maps are the ground truth reference, used for training the algorithm that generates the predicted field maps from remote sensing imagery. Both the reference and the predicted maps are stratified into "Prototype Plots", e.g. 15x15 blocks of 2m pixels whereby the block size is 30x30m. This averaging reduces the datasets to manageable size and significantly improves the typically poor repeatability of remote sensing imaging systems. In the first two years of the project we used the Normalized Difference Vegetation Index (NDVI), for generating predicted yield maps of sugar beets and corn. The NDVI was computed from image cubes of three spectral bands, generated by an optically filtered three camera video imaging system. A two dimensional FFT based regression model $Y=f(X)$, was used wherein Y was the reference map and $X=NDVI$ was the predictor. The FFT regression method applies the "Wavelet Based", "Pixel Block" and "Image Rotation" transforms to the reference and remote images, prior to the Fast Fourier Transform (FFT) Regression method with the "Phase Lock" option. A complex domain based map Y_{fft} is derived by least squares minimization between the amplitude matrices of X and Y , via the 2D FFT. For one time predictions, the phase matrix of Y is combined with the amplitude matrix of Y_{fft} , whereby an improved predicted map Y_{plock} is formed. Usually, the residuals of Y_{plock} versus Y are about half of the values of Y_{fft} versus Y . For long term predictions, the phase matrix of a "field mask" is combined with the amplitude matrices of the reference image Y and the predicted image Y_{fft} . The field mask is a binary image of a pre-selected region of interest in X and Y . The resultant maps Y_{pref} and Y_{pred} are modified versions of Y and Y_{fft} respectively. The residuals of Y_{pred} versus Y_{pref} are even lower than the residuals of Y_{plock} versus Y . The maps Y_{pref} and Y_{pred} represent a close consensus of two independent imaging methods which "view" the same target. In the last two years of the project our remote sensing capability was expanded by addition of a CASI II airborne hyperspectral imaging system and an ASD hyperspectral radiometer. Unfortunately, the cross-noise and poor repeatability problem we had in multi-spectral imaging was exasperated in hyperspectral imaging. We have been able to overcome this problem by over-flying each field twice in rapid succession and developing the Repeatability Index (RI). The RI quantifies the repeatability of each spectral band in the hyperspectral image cube. Thereby, it is possible to select the bands of higher repeatability for inclusion in the prediction model while bands of low repeatability are excluded. Further segregation of high and low repeatability bands takes place in the prediction model algorithm, which is based on a combination of a "Genetic Algorithm" and "Partial Least Squares", (PLS-GA). In summary, *modus operandi* was developed, for deriving important plant growth attribute maps (yield, leaf nitrate, biomass and sugar percent in beets), from remote sensing imagery, with sufficient accuracy for precision cropping applications. This achievement is remarkable, given the inherently high cross-noise between the reference and remote imagery as well as the highly non-repeatable nature of remote sensing systems. The above methodologies may be readily adopted by commercial companies, which specialize in proving remotely sensed data to farmers.

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Achievements

The results presented in this report, demonstrate a substantial improvement in our ability to develop meaningful functional relationships between remotely sensed imagery and vital plant growth attributes which comprise the essential information for enabling precision cropping.

In the first two years of this project our remote sensing capability was limited to multi-band sensing, whereby the Normalized Difference Vegetation Index (NDVI) was our main source of remote sensing information. Although the NDVI is a very useful tool for quantifying photosynthetic vigor in plants, it is a rather crude indicator of subtle growth attributes which are of major interest in precision cropping. Nevertheless, we developed algorithms whereby the NDVI may be used for predictive yield distribution mapping of sugar beets and corn fields with good accuracy.

The remote sensing capability of the project was considerably enriched by additions of new state of the art equipment, which was acquired in the winter of 1999/2000. These include a CASI II hyperspectral imaging system (\$ 280 K) and an ASD hyperspectral radiometer (\$ 60 K).

Hyperspectral imaging constitutes a quantum leap, in comparison to multi-band remote sensing technology. In hyperspectral imaging each pixel has a unique spectrum of about 200 bands, instead of 3 to 8 bands. Thus, with hyperspectral imaging it may be possible to detect and map many plant growth attributes which could be very useful in precision agriculture and can not be detected by multi-band imaging systems. However, hyperspectral remote sensing is a relatively new and complex technology. So we had to develop new *modus operandi* for implementing, running and flying this rather complicated remote sensing system, as well as for storing, processing and analyzing the huge hyperspectral image datasets.

A hyperspectral remote sensing imaging system is effectively a measuring instrument whereby the spectrum of each pixel in the target image may be used to quantify or identify its properties.

A key requirement of any measurement system is that repeated measurements of the same property should provide the same results, if the environmental conditions have not changed. Similarly, hyperspectral image cubes acquired in consecutive flights over the same target, should ideally be identical. In practice, two consecutive flights over the same target usually yield significant differences between the image cubes, i.e. hyperspectral imagery is usually highly non repeatable. The reasons for non-repeatability are: Variations in solar illumination, changes in atmospheric conditions and various error sources in the imaging system proper.

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The most important achievement of this project was the development of *modus operandi* and algorithms, for deriving important plant growth attributes maps from remote sensing multi-spectral and hyperspectral imagery, with sufficient accuracy for precision cropping applications.

This achievement is remarkable, given the inherently high cross-noise between the reference and remote imagery as well as the highly non-repeatable nature of remote sensing systems. We have been able to overcome these problems by over-flying each field twice in rapid succession and developing the Repeatability Index (RI). The RI quantifies the repeatability of each spectral band in the hyperspectral image cube. Thereby, it is possible to select the bands of higher repeatability for inclusion in the prediction model while bands of low repeatability are excluded. Further segregation of high and low repeatability bands takes place in the prediction model algorithm, which is based on a combination of a "Genetic Algorithm" and "Partial Least Squares", (PLS-GA).

A common technique in measuring devices is to take multiple measurements and use their mean value for improving the repeatability or the signal to noise ratio. Similarly, we effectively used two measurements (e.g. image cubes) for improving the overall signal to noise ratio of our system, which generates predicted plant growth attributes maps. Although the prediction accuracy by 2 over-flights appears to be adequate for precision cropping, we recommend 3 over-flights. The additional cost of a third flight should be well worth the improvement in accuracy and prediction robustness of plant growth attributes maps.

The other major achievement of this project was the development and implementation of the "Wavelet Based", "Pixel Block" and "Image Rotation" transforms, as well as the Fast Fourier Transform (FFT) Regression method with the "Phase Lock" option. These are described in our recently published paper: Peleg and Anderson 2002, FFT regression and cross-noise reduction for comparing images in remote sensing, International Journal of Remote sensing, 23 (10), 2097-2124. This methodology can produce "Close Consensus Maps" of yield, leaf nitrate etc., which are akin to a compromise between the reference and predicted maps. In some cases the Close Consensus Map is probably closer to the true distribution of the modeled growth property than the reference map. The above methodologies may be readily adopted by commercial companies, that specialize in proving remotely sensed data to farmers.

The cooperation between the Technion and the USDA-ARS teams was very efficient. Heading the project, Dr. Anderson designed and implemented the innovative controlled precision cropping program, which incorporates remote sensing by state of the art airborne hyperspectral imaging equipment.

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Dr. Peleg was primarily responsible for developing and implementing the innovative data analysis tools. The cooperation was particularly enhanced by Dr. Peleg's three summer sabbaticals at the ARS, Northern Plains Agricultural Research Laboratory, in Sidney, Montana.

Switching from multi-band to hyperspectral remote sensing technology during the last 2 years of the project was advantageous by expanding the scope of detected plant growth attributes e.g. Yield, Leaf Nitrate, Biomass and Sugar Content of sugar beets. However, it disrupted the continuity of the project which was originally planned on a 2 year crop rotation cycle of sugar beets and multiple crops (corn and wheat), as commonly planted in eastern Montana.

Consequently, at the end of the second year we submitted a continuation BARD proposal which was turned down for funding. This severely hampered our ability to validate our findings as originally planned in a 4-year crop rotation cycle. Thankfully, BARD consented to our request for a one year extension of the project without additional funding. This enabled us to develop most of the methodology for implementing and running the hyperspectral remote sensing system and develop the new analytical tools for solving the non-repeatability problem and analyzing the huge hyperspectral image cube datasets.

However, without validation of these tools over a full 4-year crop rotation cycle this project shall remain essentially unfinished. Should the findings of this report prompt the BARD management to encourage us to resubmit our continuation research proposal, we shall be happy to do so.

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Notes

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1. Introduction

Traditional cropping practices use the field as the minimum area of management. However, soil type, fertility, soil moisture, and other characteristics can vary substantially within fields and affect crop production (Pfeiffer et al., 1993). The development of geographic information system (GIS), global positioning system (GPS), and remote observation spatial analysis tools can enable management of fields and orchards on a site-specific basis. Site-specific cropping aims to improve production efficiency by adjusting crop treatments to conditions existing at specific areas within fields (Vansichen and Baerdemaeker, 1991). Optimization of agricultural inputs can improve economic returns and reduce the introduction of undesirable residues into the environment (Schueller, 1991). The main tool for developing an efficient site-specific cropping plan is reliable and precise information concerning the spatial distribution of soil physical properties and nutrients, plant production, and crop yields within a given field. For example, precision irrigation of tree crops improves irrigation efficiency and water use by supplying different amounts of water to different subsections in a given orchard. Improvement in water use efficiency is especially important in areas such as Israel, where water is the primary factor limiting agriculture production. In order to improve water use efficiency, it is essential to obtain reliable information on site-specific soil conditions, plant canopy size, photosynthetically active radiation (PAR), evapotranspiration, crop cover, and crop yield.

Ground samples are important contributors to our understanding of the dynamics of a field; however, soil and other samples are time consuming to acquire and expensive to analyze. In addition, the accuracy of maps developed from point data depends on mathematical models to emulate the spatial variation of the variables of interest. Spatial modeling requires intensive sampling efforts that increase in complexity as within field variation increases. Yield maps developed from harvesters with built-in grain meters have also been developed and are important because of the economic importance of actual yield. However, remote spectral observations can rapidly provide information about the condition of plants during the growing season. Plants integrate growing conditions and express their responses through the canopies achieved. Thus, vegetation indices calculated from remote spectral observations, which measure the amount of photosynthetic tissue in the canopies, can be good predictors of yield (Wiegand and Richardson, 1990, and Wiegand et al., 1991, 1992, and 1994).

Spatial information systems such as remote sensing, GIS, and GPS are fast and economical tools for recording and analyzing data concerning the spatial variability of plant growth. Variables such as leaf area, greenness, biomass, and plant cover can readily be measured from a remote platform and are all

indicators of a site's yield potential. Image data also provides a comprehensive view of an area that maintains the spatial connectivity between sites, thus reducing the need for complex spatial modeling. Therefore, spatial information technology can improve the efficiency of data collection and contribute to the development of integrated crop management systems that optimize economic and environmental benefits.

The success or failure of precision cropping depends upon the development of meaningful and accurate within field partitions or strata. Practically, the field should be partitioned into a finite number of optimal prototype plots, which are significantly and sufficiently different from each other, wherein each partition will receive a predetermined treatment. The level of detail needed to adequately partition a field must be a function of expected economic return and environmental benefits. The choice of an optimal field partitioning design must be adaptive and automated (suitable algorithms in the precision cropping software). Field partitioning is similar to optimal partitioning of the feature space, for adaptive classification by prototype populations, previously investigated by (Peleg and Ben-Hanan 1993 and Peleg 1994). The same type of procedure is needed for automatically updating field partitions for different crop types and under various growing conditions throughout the year.

2. Precision Cropping Methodology

The study area of the project was expanded to include six commercial fields which belong to cooperating farmers. This includes approximately 114 ha with 274 ground sample sites. Each field was divided into two treatments (with replication when appropriate) of constant rate application of fertilizers and variable rate application. Nitrogen, phosphorus and potassium were measured at a national soils testing laboratory, at 3 depths within the field, and the farmer used the resulting soils data to determine the level of fertilization needed to obtain their desired yield goals for the crops grown. In 1999 four fields totaling 85 ha were planted to sugarbeets, 203 points were sampled and 6 strips received variable rate treatments and 6 strips were fertilized at a uniform rate. And, two fields, totaling 35 ha, were planted to corn, 71 points were sampled and three strips received variable rate treatments and three strips were fertilized at a uniform rate. The 2000 effort, currently underway, consists of four corn and two sugarbeet fields. The methods described in the next subsections, were used to sample the six project fields in 1999 and are currently being used during the 2000 growing season.

2.1 Soil Sampling

Sample points were assigned within the six fields based on a 2 sample per ha offset grid. Points within 7.62m of the field edge were either moved or discarded to minimize edge effects. This left 274 sample locations. Soil samples were taken at each location to a depth of four feet. Two 120 cm soil cores were taken at each sample location and the soil from the 0 to 30 cm sections of both the samples were combined. The same procedure was used for the 30 to 60 cm section. The two 60 to 120 cm sections were combined into one sample. The first and second sections were analyzed for P, K, Mg, Ca, Na, OM, pH, Salts, NO₃-N, Zn, Mn, Cu, Fe, S, B, CEC, and percent K, Mg, Ca and Na saturation. Particle size analysis was also done for sand, silt and clay content. The samples from the section three were analyzed for P, K, Mg, Ca, Na, NO₃-N, CEC, percent saturation for K, Mg, Ca, Na and particle size analysis for sand, silt and clay.

Soil samples were taken immediately following the 1998-growing season. The soils in the five new fields were sampled and analyzed as described; however, only N, P and K were determined for the field previously sampled in 1997. Samples for the 2000-growing season were only analyzed for N,P and K; however, a 10% random sampling of points within the six fields were intensively analyzed using the protocol described above. Sample points in all fields will remain constant throughout the duration of the BARD project.

Fertilization equations for sugarbeets were developed to describe the desired amount of N, P, and K to be applied based on the soil test levels using research data appropriate for the area and input from Imperial Holly Sugar, the area sugarbeet processor. The equations for corn used previously developed guidelines. Fertilizer application maps were developed based on these equations and the strips that were designated to have variable rate application were fertilized with a triple tank variable rate applicator, with the amounts of N, P, and K independently varied using the soil test levels obtained at each sample location. Fertilizers (N, P, K) were applied at uniform rates on the remainder of the fields. The application rates for the uniform areas were determined using the same equations as the variable rate applications, but the average amount of nutrients available across the field was used as the base from which fertilization levels were determined. Variable rate and constant fertilizer applications were made to the four sugarbeet fields in the fall of 1998 and fertilizer applications were made to the two remaining corn fields in early spring, 1999.

During the 2000 and 2001 seasons, two 120 cm soil cores were taken from each location in the fall after small grain harvest. The 2 cores were cut at 30 cm and 60 cm. The top 30 cm from both cores were mixed together to form one sample. The same procedure was used on the 30 to 60 cm and the bottom 60 to 120 cm of the cores. Not all cores were the full 120 cm depth due to a gravel layer in some areas of the fields. The soil samples were sent to a commercial soils laboratory for analysis. Analysis of the first section included: P, K, Mg, Ca, Na, organic matter, pH, salts, NO₃-N, Zn, Mn, Cu, Fe, S, B, CEC, and percent K, Mg, Ca and Na saturation. The samples from the second and third core sections were analyzed for P, K, Mg, Ca, Na, NO₃-N, CEC, percent saturation for K, Mg, Ca, Na and particle size analysis for sand, silt and clay. Particle size analysis was also obtained for sand, silt and clay percentages for all depths. The extensive soil analysis was conducted to identify any possible nutrients or physical properties that could contribute to production differences between the uniform and variable rate treatments.

2.2 Fertilizer application map development

Soil fertility maps were developed for N, P and K using the sample point data, which were interpolated to a surface using an inverse distance interpolation algorithm, constrained to the field boundaries. Equations 1, 2, and 3 were used to convert the soil fertility maps into variable rate fertilizer application maps for that treatment. The same equations were used with the field average N, P and K levels to calculate the uniform application rates for uniform treatment.

Equation for N application:

$$N_{app} = (\text{Yield} \times 4.2) - (\text{OM} \times 16.8) - (4.5(N_{30} + N_{60})) - 9(0.8 * N_{120}) \quad (1)$$

Where N_{app} is the total kg ha⁻¹ of NO₃-N that will be applied, yield is the desired yield in Mg ha⁻¹, 4.2 kg is the amount of NO₃-N needed to produce each Mg of sugarbeets (kg Mg⁻¹), each percent of organic matter (OM) is expected to release 16.8 kg ha⁻¹ NO₃-N during the production year (Holly Sugar recommended conversion rate for the study period), 4.5 kg ha⁻¹ ppm⁻¹ is the factor that converts parts per million (ppm) NO₃-N (cadmium reduction) to kg ha⁻¹ in the top 30 cm of soil (Carson, 1980), N_{30} is the ppm NO₃-N in the 0 – 30 cm layer of soil, N_{60} is the ppm NO₃-N in the 30 – 60 cm layer of soil, 9 kg ha⁻¹ ppm⁻¹ is the factor that converts ppm NO₃-N to kg ha⁻¹ in the bottom 60 - 120 cm of soil, 0.8 is the fraction of the NO₃-N available to the plant in the 60 – 120 cm soil depth and N_{120} is the ppm NO₃-N in the bottom 60-120 cm of soil.

Equation for P application:

$$P_{app} = P[0,20](\text{Yield} \times 1.27) + 10.1(20-P) * 10^{-1} \quad (2)$$

Where P_{app} is the total kg ha^{-1} of phosphorus (P) that will be applied, P is the soil test value ranging from 0 to 20 ppm of P (Olsen-Bicarbonate phosphorus test) for the top 30 cm of soil (Olsen et al., 1954), yield is the desired yield in Mg ha^{-1} and 1.27 is the amount ($\text{kg ppm}^{-1} \text{Mg}^{-1}$) of P needed to produce each Mg of sugarbeets (kg Mg^{-1}).

A build factor is added to increase P levels in the field to 20 ppm. When test levels are above 20 ppm, no additional P was added ($P_{app}=[20 \text{ ppm, max}] +:0$). The conversion factor $10.1 \text{ kg ha}^{-1} \text{ ppm}^{-1}$ is applied to the needed increase in P in ppm (20-P) and divided by 10 to produce a 10 year build factor.

Equation for K application:

$$K_{app} = K[0,200](\text{yield} \cdot 1.65) + 5(200-K) \cdot 10^{-1} \quad (3)$$

Where K_{app} is the total kg ha^{-1} of K that will be applied, K is the soil test value ranging from 0 to 200 ppm of K (ammonium acetate extraction) for the top 30 cm of soil, yield is the desired yield in Mg ha^{-1} and 1.65 is the amount ($\text{kg ppm}^{-1} \text{Mg}^{-1}$) of K needed to produce each Mg of sugarbeets (kg Mg^{-1}).

A build factor is added to increase K levels in the field to 200 ppm. The conversion factor $5 \text{ kg ha}^{-1} \text{ ppm}^{-1}$ is applied to the needed increase in K in ppm (200-P) and divided by 10 to produce a 10 year build factor. For test levels greater than 200 ppm, no K was applied ($K_{max}=[200 \text{ ppm, max}] +:0$).

All phosphorus and potassium maps were created using samples taken from the top 30 cm of soil. Nitrogen maps were created using soil core data from all three depths when available. When rocks prevented sampling soils at the 30 – 60 cm or the 60 – 120 cm depths, that segment of the equation was assumed to be zero. The application equations described above are unique to the Lower Yellowstone River valley and should not be used in other sugarbeet production areas.

The fertilizer application maps were provided to a custom operator with a multi-bin variable rate applicator. The computer controller used a real-time differential GPS receiver and fertilizer application maps to determine where the applicator was and how much of each product should be dispensed at each location.

2.3 Plant Sampling

Corn and Sugarbeet nutrient statuses were sampled in all six fields during the peak of the growing season. Sugarbeet petiole nitrate levels $\text{NO}_3\text{-N}$ were sampled between 21 July 1999 and 2 August 1999. Corn ear leafy nitrates were sampled on 5 August 1999. Total aboveground biomass and beet

tubers were sampled just prior to harvest, 29 September 1999 to 11 October 1999. Variables measured included total N and C as a percentage of the total aboveground biomass and sugarbeet root weight, percent sugar and percent brei nitrate (excess nitrogen). Corn biomass was not collected prior to harvest.

During the growing seasons of 2000 and 2001 plant samples were also taken. These samples were taken at the same locations as the soil cores. In late July, sugarbeet petiole samples were taken at each sample point by removing a leaf stem from 40 different beets at and around the sample point locations. The Holly Sugar chemistry lab in Sidney, MT, analyzed these samples. Just prior to harvest in October the roots were dug from a 1.858 m² (20 ft²) area at the sample points. The above ground biomass was removed, dried and weighed. The dried beet tops were then ground and analyzed with a LECO carbon/nitrogen analyzer (LECO Corp, 3000 Lakeview Avenue, St. Joseph, MI). The root samples were delivered to the Holly tare lab each day and were cleaned, crowned and analyzed for the percent sugar content and brei nitrate. The Holly Sugar chemistry lab is the same lab used to analyze the commercial grower's beet samples taken from each truckload throughout the harvest campaign.

Sugarbeet yields (Mg ha⁻¹) were calculated for each treatment area from the hand samples previously described. Truck weights from each treatment area and sugarbeet yield monitor data were collected inconsistently throughout the study because of the fast pace of the harvest. When available, accurate data will be presented for qualitative comparison with the hand sample data. The yield monitors harvester real-time differential GPS location information and load cell weights of sugarbeets as they passed over weigh cells on the tank and truck conveyer tracks to calculate production estimates for every point in the field.

2.4 Yield Mapping

Many sugarbeet harvesters and grain combines are now equipped with yield sensing devices. One combine yield monitor was used to measure the amount of corn harvested for each field in 1999. Two sugarbeet pickers were each equipped with a "HarvestMaster" yield mapping system. The sugarbeet systems use load cells to measure the total tons of product produced per acre. Yield maps were produced using the Vision software package.

2.5 Interpolated maps from plant samples

A major task in precision cropping is to establish areas of the field that differ from the rest of the field and share a common characteristic. This task is often accomplished by taking many field samples in a grid pattern and grouping clusters of similar sample values in separate zones.

The field samples are usually located on a grid with no sample points within about 25 meters of the edge of the field to eliminate any effect due to overlap in tillage, spraying or fertilizing operations. Typically, about 3 to 5 sample locations per hectare are used. At each one of the sample locations a small area of the field is sampled and the average value is ascribed to the central point of the sampled field section. Some examples of sampled quantities are: various soil properties, biomass, leaf nitrate, yield etc. A field map is then prepared from these sample points by interpolation. Such maps are basic tools in precision cropping, whereby the approximate distribution of the various properties of interest over the entire field is quantified. In most practical cases the samples grid is irregular which complicates the interpolation task.

The most commonly employed interpolation methods are "Inverse Distance" and "Kriging" (Watson 1992), which can deal with irregularly spaced grids. Both of them are methods of "global" interpolation in the sense that all available sample points participate in the calculation of the values for each interpolated point.

Inverse distance is a fairly simple weighted average interpolation method. The weights W are inversely proportional to the distance L between the interpolated and sampled points $W=1/L$. Thereby, far-away points will have smaller weights while the nearest points will be given larger weights. Two examples of irregular sampling grids in a sugarbeet field are depicted figure 2.1A and 2.2A, e.g. biomass and leaf nitrate respectively. The corresponding maps by the inverse distance method are shown in figure 2.1B and 2.2B. Typically, maps created by the inverse distance method tend to be composed of "bulls eyes" mounds centered at the sampling points and low values spaces in between the "bulls eyes". This is a distinct disadvantage since distributions of attributes in agricultural fields are characteristically gradual. Additionally, this interpolation method is computationally expensive, especially if the grid contains a large number of sampling points.

The Kriging interpolation method (Marcotte1991, Kitanidis 1999) depends on the choice of a correlation function or semi-variogram of the interpolated map. Therefore some *a priori* statistics about the distribution form of the interpolated property must be known in advance or assumed.

For example, assuming a Gaussian semi-variogram function one must specify the desired relative error E , i.e. the normalized value of semi-variogram at $L=0$, as well as the length scales LX and LY in the X and Y directions respectively. The scales LX and LY are assumed maximal distances from the sampling points, at which the interpolated property still does not change. The chosen values for LX and LY have a profound effect on the resulting map, as depicted in figures 2.3 and 2.4.

Figure 2.1 A. Irregular sampling grid of biomass in a sugarbeet field. B. Interpolation by inverse distance. C. Interpolation by Kriging, (scale=20). D. Interpolation by minimal curvature method

Figure 2.2 A. Irregular sampling grid of leaf nitrate in a sugarbeet field. B. Interpolation by inverse distance. C. Interpolation by Kriging, (scale=20). D. Interpolation by minimal curvature method.

The image of this field is composed of 445x306 2m pixels, i.e. its actual size is 890x 612m. Therefore, using $LX=LY=10$, as in Figures 2.3B and 2.4B, assumes that at distances of 20m from sampling points the interpolated property does not change, which is reasonable *a priori*. However, the resulting map displays the familiar “bulls eyes” mounds, similar to inverse distance interpolation. This indicates that the maximal distance at which the interpolated property still does not change is probably larger than 20m.

Setting $LX=LY=20$, as in Figures 2.3C and 2.4C, assumes that at distances of 40m from the sampling points the interpolated property does not change. The “bulls eyes” mounds disappeared and the general form of the distribution of the interpolated properties seems to be more like one would expect in an agricultural field. Nevertheless, it is impossible to tell if the chosen values of $LX=LY=20$ are correct or perhaps they should be increased further?

Thus, setting $LX=LY=50$, as in Figures 2.3D and 2.4D, assumes that at distances of 100m from the sampling points the interpolated property does not change. Comparing Figures 2.3C and 2.4C to Figures 2.3D and 2.4D respectively, one observes that the general forms of the distributions of the interpolated properties are quite similar. However, the maps in Figures 2.3D and 2.4D are more stratified, i.e. the areas of constant values are increased and the differences between them are also increased.

The “Minimal Curvature” is another global interpolation method that can handle irregular grids, e.g. (Briggs 1984). This method was adopted in this project, and is recommended for general usage in precision agriculture because it does not suffer from the above shortcomings and is computationally cheap for large datasets. In spite of this recommendation, we used the inverse distance interpolation method in section 2.2 for developing fertilizer application maps. This was dictated by our use of commercial variable fertilizer spreading equipment, which still uses maps generated by inverse distance interpolation.

In the “Minimal Curvature” method, the weights W are calculated by inversion of a “Green's function” matrix depending on the distances L , between interpolated and sampled points: $W=L^2[\log(L)-1]$. Figures 2.1D and 2.2D depict examples of biomass and leaf nitrate field maps, which were interpolated by the minimal curvature method. Pictorially, imagine that at the sampled grid locations, there are vertical sticks or nails in a board, whose lengths are proportional to the sampled values. Then the surface created by minimal curvature interpolation is like a rubber sheet stretched over the tips of the nails.

Figure 2.3 A. Irregular sampling grid of potassium percentage in a soil sample. B, C and D Interpolation by Kriging, using a Gaussian semi-variogram function with $E=0$ and scales 10, 20 and 50 respectively.

Figure 2.4. A. Irregular sampling grid of biomass in a sugarbeet field. B, C and D Interpolation by Kriging, using a Gaussian semi-variogram function with $E=0$ and scales 10, 20 and 50 respectively.

The rubber sheet contacts all the nails and is stretched in such a way as to minimize the overall curvature of the entire map.

Unlike the “bulls eyes” in inverse distance interpolation, this configuration is much more similar to the natural undulations of typical features in an agricultural field. Note the similarity of these maps to the respective maps in Figures 2.1C and 2.2C, which were derived by Kriging while using the reasonable scale of 20m. Therein lies the main advantage of the minimal curvature interpolation method, which automatically provides maps similar to optimal Kriging, but without the need of any *a-priori* parameter assumptions.

2.6 Remote Sensing

Aerial color photographs were taken of all fields on 12 May 1998. These images were used to identify soil color pattern variations and to develop a georeferenced photographic maps for each field.

A three-camera optically filtered digital video system was used in the first phase of the project. The 3 spectral bands of this system were: Green 555-565 nm (G), Red 625-635 nm (R) and near infrared 845-857 nm. (NIR). The system was airborne on a Cessna Skymaster airplane whereby images were acquired of the fields on 21 July 1999, 2 August 1999, 5 August 1999, 30 August 1999, 9 September 1999 and 21 September 1999. The mid August dates were expected to capture the maximum canopy development for the corn and sugarbeet fields and the September dates was hoped to show differences in residual nitrogen in the sugarbeet fields.

During the winter of 1999/2000, the three-camera system was replaced by a state of the art CASI II hyperspectral imaging system made by ITRES corporation in Canada. Some technical problems were encountered during the initiation of the CASI II system in the 2000 summer season, so effectively it became fully operational only during the 2001 summer season. During the 2001 summer season, 48 bands image cubes of the fields were acquired by the CASI II on 23 May 2001, 6 July 2001, 2 August and 31 August 2001.

An ASD full range hyperspectral radiometer was another important equipment acquisition during the winter of 1999/2000. The addition of the new equipment significantly enhanced the quality of the remote sensing data. It eliminated the band-to-band registration errors inherent in the previously used three camera multispectral video system and, most important, the data collected could be corrected to spectral reflectance. The use of spectral reflectance data, as opposed to raw radiance, gave us the ability to develop models that can be applied over space and time

3. Multi-spectral data analysis

In many remote sensing studies it is desired to quantify the functional relationship between images of a given target that were acquired by different sensors, e.g. between LANDSAT and SPOT satellites, (Irish 1998, Sunar et al. 1998, Zhou et al. 1998) or AVIRIS (Bo-Cai Gao 2000) and HYDICE (McKeown et al. 1999) and lower flying Hyperspectral imaging systems, such as CASI (George 1997) and ISA (Kallio et al. 1998). Similar examples of cross-sensor comparisons include Synthetic Aperture Radar (SAR) images versus images of a target in the visual part of the spectrum. Or, in thermal imaging it may be useful to compare a panchromatic image to a thermal image of the same target, (Pelgrum et al. 2000).

Such comparisons are problematic because the "cross-noise" is quite high when pixel values of one image are plotted versus the other. Typically, the correlation coefficient is quite low, even when the compared images look very much alike.

Usually, neither of the compared images can be considered as being an absolutely correct map of the ground target, so neither can be considered as a "true reference". Nevertheless, we can try to quantify the functional relationship between the two images by a suitable regression model function $Y=f(X)$, by arbitrarily choosing one of them as "the reference" Y . Usually, we have more confidence in one of the imaging systems so the image derived by it may be chosen as the reference. In any case, both images are error prone so to compare them we first need to amplify the degree of functional agreement between them, rather than try to fit one error prone image to another error prone image. The underlying assumption of classical regression is that the reference Y is absolutely correct while the predictor X is less exact. Thus, the objective is to fit X to Y by choosing the parameters of $Y=f(X)$, which minimize the "residuals" $(\hat{Y} - Y)$, i.e. the differences between the predicted ($\hat{Y}$) and true Y e.g. (Eubank and Speckman 1990, Krutchkoff 1966, Berkson 1969, Galant 1975, Dennis 1977). When comparing images in remote sensing this objective is not valid because Y itself is error prone. Therefore, our first objective is to modify both X and Y , in a way that minimizes the disagreement between the two information sources. The second objective is to "fuse" the information in the modified X and Y images, while deriving the predicted image Y_{pred} . This predicted image is probably closer to actual field conditions than either X or Y . This approach is similar to PLS regression which attempts to maximize the covariance between X and Y , e.g. (Wise and Gallagher 1998).

We employ the concept of Fast Fourier Transform (FFT) based regression, which was, introduced in three previous publications, (Peleg 1995, 1997 and 1998). Building on the principles developed therein, the FFT regression method is further enhanced in the present project by introducing the "Field Mask" concept and three cross-noise reducing pair-wise image transform algorithms: A "Wavelet Based Transform" (WAVEL), "Pixel Block Transform" (PBLOCK) and "Image Rotation Transform" (IMROT). Either one or all three can be applied to both X and Y in series, thereby amplifying the degree of functional agreement $Y=f(X)$, if any, by reducing the cross-noise, which increases the correlation coefficient between X and Y .

Following pre-processing of X and Y , by WAVEL, PBLOCK and IMROT, the parameters of the regression model function $Y=f(X)$ can be computed by regular or FFT regression.

The FFT regression method is advantageous because in addition to increasing the coefficient of determination, it affords exchanges of image phase matrices. For example, the phase matrix of Y , can be combined with the amplitude matrix of $\hat{Y}$. Similarly, the amplitude matrices of X and Y can be combined with the phase matrix of the field mask, thereby drastically reducing the residuals, and increasing the correlation coefficient.

In the next subsections we shall present these FFT based algorithms and their use for predicting yield distributions in the experimental fields by the remotely sensed images collected during the 1999 season. Yield maps were derived by combine yield monitors which measure the flow rate of the crop, while it is being harvested, e.g. (Blackborne and Marshall 1996, Stafford et al. 1996, Walter et al. 1996), as described in section 2.3. The combine yield monitor maps, were used as the reference. Using the FFT regression method, the cross-noise between the yield monitor maps and the NDVI images was dramatically reduced, whereby predicted yield maps were generated. Since the yield monitor maps are only estimates of the true yield distribution in the field, the predicted yield maps are likely closer to the true yield distribution in the field.

3.1 Conventions and assumptions

The implementation of the image transforms, as well as all other mathematical operations in this report were performed in the MATLAB environment (Anon. 2000). For the sake of brevity we shall not describe the details of the underlying algorithms which the standard MATLAB functions use. We shall use the standard MATLAB "m file" function names and format, so the interested reader can reproduce the results and inspect the algorithm by looking up the appropriate m file in MATLAB.

Upper case bold letters will denote matrices while lower case bold letters will denote vectors. Scalars will be denoted by non-bold lower or upper case letters. Ground based images, (e.g. yield maps) will be designated as "Reference Image" **Y**, while their remotely sensed counterparts **X** will be designated as "Remote Image". Although sometimes both **X** and **Y** are remotely sensed, image **X** is always less reliable than **Y** in some sense. For example, in temporal image comparisons one may think of the later images being remote in time versus the first reference image, e.g. images (b) and (c) in figure 3.1. Similarly, when comparing satellite imagery to airborne imagery the former may be thought as less reliable when compared to the latter. This convention is in line with the main objective of this paper, i.e. to quantify the functional relationships between lesser confidence images and images that are probably closer to actual field conditions.

Throughout this report we assume that the compared image matrices **X** and **Y** contain regions of interest surrounded by a frame of zero pixels, dubbed "zero-frame", as depicted in figure 3.1.

If this is not the case, a zero-frame can be created around the region of interest, by adding rows and columns of zeros to the raw image matrices. To shorten computing times of FFT's in large images, it is advantageous to form Radix 2 matrices, i.e. the number of rows and columns in **X** and **Y** should be powers of 2. In any case, the number of rows and columns in **X** and **Y** must always be the same.

3.2 The Field Mask ("**Fmask**")

The **Fmask**, is a binary image wherein all pixels of the Region of Interest (ROI) are ones and all the pixels of the surrounding zero-frame are zeros. For example, image (a) in figure 3.1 is the **Fmask** of the 2 NDVI images (b) and (c). MATLAB provides many simple ways for creating an **Fmask**. These are elementary procedures in image processing and will not be discussed here. However, the usage of the **Fmask** is central to the main theme of this report, so it deserves special attention. The **Fmask** defines the ROI, which may include only portions of **X** and **Y**. For example, we know *a-priori* that the peripheral edge pixels of a harvester yield monitor map are much more error prone than the pixels inside the field. Thus, prior to the computation of the parameters of the regression function, the **Fmask** is one way to crop off several pixels from the periphery of the raw **Y** image, (i.e. set them to zero). Similarly, we may know *a-priori* that some regions of the remote image include artifacts due to various technical problems in the image acquisition stage. Rather than discard such an image, we can remove the problem areas of **X**, and form the ROI only from the valid part of **X**.

Figure 3.1. Two NDVI images (b) and (c), of a sugarbeet field acquired by an airborne multi-camera imaging system incorporating optical filters in the RED (625-635 nm) and NIR (845-857 nm) spectral bands. These images were acquired 3 days apart (b)- 2.8.99, (c)-5.8.99. Image (a) is the "Field Mask" of (b) and (c). In the scatter-plot (d), the pixel values of (b) are plotted versus (c), excluding the zero pixels of the zero-frame. The scatter-plots in (e) and (f) depict the prediction results by regular linear regression.

The rule of thumb is to include in the ROI, the largest possible parts of X and Y , without substantially increasing the residuals. Obviously, the pixels of Y , which were excluded from the ROI can not be predicted by the model function $\hat{Y}=f(X)$.

The WAVEL, PBLOCK and IMROT transforms will be described in later sections. For reasons to be described therein, some zero-frame pixels, (usually close to the ROI), may not be zero after the image transform operation.

Similarly, some of the zero-frame pixels predicted by $\hat{Y}=f(X)$, may not be zero. Therefore, after each operation on X and Y they are multiplied by the filed mask: $X = X.* Fmask$, $Y = Y.* Fmask$, wherein the MATLAB pixel-wise multiplication operator $.*$ assures that all zero-frame pixels remain zero. In what follows, we shall have to compute various properties of the pixels in the ROI of X and Y , e.g. averages, variances, residuals, the coefficients of determination and correlation etc.

To this end we form respective vectors $\mathbf{x}$ and $\mathbf{y}$ from which the zero pixels of the zero-frame are excluded. The MATLAB function `find.m` is used to extract the ROI pixels from $\mathbf{X}$ and $\mathbf{Y}$ into $\mathbf{x}$ and $\mathbf{y}$:

$$[\mathbf{i}, \mathbf{j}, \mathbf{fones}] = \text{find}(\mathbf{Fmask}); \quad (4)$$

$$[\mathbf{i}, \mathbf{j}, \mathbf{x}] = \text{find}(\mathbf{X} + \mathbf{Fmask}); \quad [\mathbf{i}, \mathbf{j}, \mathbf{y}] = \text{find}(\mathbf{Y} + \mathbf{Fmask}); \quad (5)$$

$$\mathbf{x} = \mathbf{x} - \mathbf{fones}; \quad \mathbf{y} = \mathbf{y} - \mathbf{fones}; \quad (6)$$

Where the vectors $\mathbf{i}$, $\mathbf{j}$ contain the indices of the pixels in the ROI and $\mathbf{fones}$ is a vector of all the ones in the ROI of $\mathbf{Fmask}$. The $\mathbf{Fmask}$ matrix is added to $\mathbf{X}$ and $\mathbf{Y}$ in (5) whereby the zeros in the ROI, if any, are included in $\mathbf{x}$ and $\mathbf{y}$. The original values of the ROI pixels are then restored in $\mathbf{x}$ and $\mathbf{y}$, by subtracting the $\mathbf{fones}$ pixels in (6).

3.3 Cross-Noise reduction

The main problem in comparing images in remote sensing, on a pixel by pixel basis, stems from the relatively large "Cross-Noise", that is present even when the compared images seem to be visually identical. Subsequently, the term "Cross-Noise" and a Cross-Noise Index, shall be quantified rigorously, in the context of hyperspectral data analysis. Presently, its intent is to convey a measure of the abundance of pixels in the compared images, that can not be fitted to a regression model function line or curve $\mathbf{Y} = f(\mathbf{X})$. Deferring the usage of the Cross-Noise Index for hyperspectral imagery, herein we shall only use simple image similarity indices, based on conventional similarity measures of two vectors:

Minimum pixel value of $\mathbf{x}$ or $\mathbf{y}$ - min.

Maximal pixel value of $\mathbf{x}$ or $\mathbf{y}$ - max.

Mean value of $\mathbf{x}$ or $\mathbf{y}$ - av.

Standard deviation of $\mathbf{x}$ or $\mathbf{y}$ - std

Residuals vector $\hat{\mathbf{y}} - \mathbf{y}$ - **res** ($\hat{\mathbf{y}}$ is the predicted $\mathbf{y}$)

Standard deviation of **res** - stdRES

Correlation Coefficient - r

Coefficient of determination - $R = (\text{Variation due to } \hat{\mathbf{Y}} = f(\mathbf{X})) / (\text{Total Variation in } \mathbf{Y})$

Obviously, if two images $\mathbf{X}$ and $\mathbf{Y}$ are identical then their min, max, av and std values must be the same, while **res** and stdRES should be zero. The index r is a measure of predictability of $\mathbf{Y}$ from $\mathbf{X}$ by linear regression. The index R is more general than r since it is not confined to linear estimation,

e.g. (Draper and Smith 1981). When the prediction is perfect $R=1$. Values of $R<1$ indicate underestimation of the total variation in Y by $\hat{Y}=f(X)$, while $R>1$ indicates overestimation of Y . Note for example that by visual inspection, it is practically impossible to distinguish between the two NDVI images (b) and (c) in figure 3.1. The listed min, max, av and std values of their x and y vectors are seen to be very similar. So, one would expect that it should be relatively easy to predict y from x using simple linear regression. Yet, the plot of x versus y in figure 3.1 (d) shows considerable disagreement between them, i.e. the "cross-noise" is quite large. Although the coefficients $r=0.82$ and $R=1.04$ are reasonably good, the plot of the residuals in figure 3.1(f) and their relatively high standard deviation ($\text{stdRES}=0.0387$); indicate a relatively large prediction error. Note particularly, that the cross-noise is largest when x or y are negative or close to zero.

The $\text{NDVI}=(\text{NIR}-\text{RED})/(\text{NIR}+\text{RED})$ is widely used in remote sensing for quantifying growth vigor of vegetation e.g. (Anderson et al. 1993, 1996, Yang and Anderson 1996, 1999, Todd et al. 1998 and Thenkabail et al. 2000). It is an excellent index of photosynthetic activity in the plant canopy due to the chlorophyll absorption band around 650-660 nm. (Gitelson et al. 1997). Negative or close to zero NDVI pixel values, usually indicate that there is little or no photosynthetic activity at this point. Usually, this means that such a pixel images bare ground, whence the NDVI ceases to function as a vegetation index. When it is negative or close to zero, we can only say that there are most probably very few or no green plants at this point. Hence, the large cross-noise between X and Y in figure 3.1(d), in the $\text{NDVI}<0.1$ region. In the next sections we show that it is possible to reduce the cross-noise in the low NDVI range, whereby the correlation coefficient is dramatically increased.

Normally, when images which were acquired by different platforms are compared, or when a combine monitor map is compared to an NDVI image the cross noise is very much larger than in the example of figure 3.1, e.g. values of $r = 0.3$ are not uncommon. Thus, in most cases, comparison between images is practically impossible without some cross-noise reduction. In the next sections we describe pair-wise cross-noise reducing image transforms of X and Y , whereby such comparisons are made possible.

3.4 The Wavelet based transform (WAVEL)

The use of wavelets in image processing is well documented in the literature, e.g. (Mohany 1997, Zhou et al. 1998). The MATLAB Wavelet Toolbox provides a variety of ready "m functions" for

processing images by the wavelet transform. The WAVEL transform is composed from the following m functions:

$$[C,S] = \text{wavedec2}(X,1, \text{wavelet}); \quad (7)$$

$$Xa = \text{wrcoef2}('a',C,S,\text{wavelet},1); \quad (8)$$

$$Hd = \text{wrcoef2}('h',C,S,\text{wavelet},1); \quad (9)$$

$$Vd = \text{wrcoef2}('v',C,S,\text{wavelet},1); \quad (10)$$

$$Dd = \text{wrcoef2}('d',C,S,\text{wavelet},1); \quad (11)$$

The function `wavedec2` computes the coefficient matrices **C** and **S** given the input matrix **X**, using the selected **wavelet**. The function `wrcoef2` is then used to construct the approximations image **Xa**, the horizontal details image **Hd**, the vertical details image **Vd** and the diagonal details image **Dd**. Figure 3.3c is the composite image of figures 3.2b-d, which is the difference matrix **Xdif** and depicts what has been removed from the original image **X**, while constructing the transformed image **Xa**: (Fig 3.3b):

Figure 3.2. Decomposition of image **X** in figure 3.1(c), into approximations (**Xa**), horizontal details (**Hd**)*5, vertical details (**Vd**)*5 and diagonal details (**Dd**)*5.

$$\mathbf{X}_{dif} = \mathbf{X} - \mathbf{X}_a = \mathbf{H}d + \mathbf{V}d + \mathbf{D}d; \quad (12)$$

To apply the WAVEL transform to the reference image, $\mathbf{X}$ is replaced by $\mathbf{Y}$, in equations (7)-(12).

The details images $\mathbf{H}d$, $\mathbf{V}d$ and $\mathbf{D}d$, which were removed from $\mathbf{X}$ are on a much finer dimension scale, relatively to the scale of $\mathbf{X}$. As a corollary, imagine viewing an object with the naked eye and a magnifying glass.

In the former case we only see the "approximations" of the object while in the latter we see only the "details". Yet, the reflected radiation that the imaging system captures includes both the details and the approximations. In comparing images we are usually interested in the approximations while the details usually represent clutter.

As an example of the WAVEL transform application, consider the four images in figure 3.2, which depict the decomposition of the remote NDVI image $\mathbf{X}$ of figure 3.1(c), into $\mathbf{X}_a$, $\mathbf{H}d$, $\mathbf{V}d$ and $\mathbf{D}d$. The images 2(b), 2(c) and 2(d) accentuate the directional nature of $\mathbf{H}d$, $\mathbf{V}d$ and $\mathbf{D}d$.

Figure 3.3(a) shows $\mathbf{X}$ before the WAVEL transform operation while image $\mathbf{X}_a$ in 3(b) is the $\mathbf{X}$ image after the WAVEL transformation.

Figure 3.3 (a) Image $\mathbf{X}$ before WAVEL transformation, (b) Image $\mathbf{X}$ after WAVEL transformation. (c) Difference between (a) and (b) multiplied by 5.

The image in figure 3.3(c) is the difference X_{dif} between 3(a) and 3(b). To better see the low values of the detail image (mostly close to zero) values in images Hd , Vd , Dd and X_{dif} were multiplied by 5.

The physical dimensions of the field in figure 3.3 are 434x280m. The pixel size is 2x2m so matrices X and Y have 217 rows and 140 columns. By this resolution, the pixel outlines of X in figure 3.3(a) should be visible, which is not the case. Observe however, that after the WAVELET operation the pixels of X_a are clearly visible. The NDVI images X and Y were created from separate camera images RED and NIR, i.e. not through the same lens. Then they were geo-corrected, before the NDVI computation. As a result, there is considerable pixel blurring which is on a much smaller scale than the 2x2 meters pixel resolution. This "pixel smearing" increases the cross-noise between X and Y .

Thus, in the example of figures 3.3(a) and 3.3(b), the input image had: $\min=-0.154$, $\max=0.48$, $av=0.393$, $std=0.0645$, while the transformed image had: $\min=-0.0962$, $\max=0.454$, $av=0.390$, $std=0.0663$. Note that the minimum of the transformed image increased to almost zero, while before the transformation it was -0.154. With reference to figure 3.1(d), observe that this increase to $\min = -0.0962$, effectively eliminates most of the negative NDVI pixels, which are responsible for a large portion of the cross-noise.

Replacing X and Y by X_a and Y_a and repeating the comparison by regular regression (as in figure 3.1(d) and (e)), results in a significant increase in r (0.82 to 0.91), a decrease in $stdRES$ (0.0387 to 0.0278), while R is somewhat increased (1.04 to 1.07). The increase in R is due the inefficiency of the regular regression algorithm, which will be explained in a later section.

Similar cross-noise reductions, i.e. increases in r and decreases in $stdRES$ were observed in numerous image comparisons. Particularly, in LANDSAT versus multi-camera NDVI images, as well as sugarbeet and corn combine monitor yield maps versus multi-camera NDVI images.

The operation of the WAVELET transform strongly depends on the chosen wavelet. In this study the order 1 Daubechies (**db1**) wavelet was used. For other types of image comparisons one may want to experiment with other wavelet types. The advantage of the **db1** wavelet is that the mean and the variance values of the transformed image remain essentially unchanged

3.5 Pixel Block Transform (PBLOCK) Unlike the rather easy image comparisons example of figures 3.1, 3.2 and 3.3, most image comparisons in remote sensing are much more difficult, i.e. the correlation coefficient is usually lower than 0.5.

Figure 3.4. Images (a) and (d) are NDVI's of the same sugarbeet field as in figure 3.1. Image (a) was acquired by an airborne multi-camera imaging system incorporating optical filters in the RED (625-635 nm) and NIR (845-857 nm) spectral bands, on July 21, 1999, (Reference). The NDVI image (d) was computed from a LANDSAT 7 dataset: RED (630-690 nm), NIR(760-900 nm), which was acquired on July 8 1999, (Remote). Scatterplot (c) is x vs. y before PBLOCK T., and (f) is x vs. y after PBLOCK T.

In these cases, the increase in r by WAVEL is usually not sufficient for image comparisons. The PBLOCK transform is much more powerful in decreasing cross noise.

The algorithm of PBLOCK is quite simple:

1. Divide the compared image matrices X and Y , into N by N pixel blocks each.
2. Average the pixel values in each block
3. Remove non-zero pixels from the zero-frame, if any.
4. Preserve av of X and Y after the transformation.

The choice of the block size N depends on the quality of the compared images. A larger N increases the correlation coefficient but also removes more details from the compared images. Thus, one should experiment with increasingly larger N values, until the increase in the correlation coefficient and decrease of the residuals is adequate for comparing the images. In the study reported herein we used 8 by 8 pixel blocks. In MATLAB steps 1 and 2 are accomplished by the `blkproc.m` function, which for $\mathbf{X}$ and $\mathbf{Y}$ is respectively:

$$\mathbf{X} = \text{blkproc}(\mathbf{X}, [N \ N], 'mean2(\mathbf{B}) * \text{ones}(N, N)'); \quad (13)$$

$$\mathbf{Y} = \text{blkproc}(\mathbf{Y}, [N \ N], 'mean2(\mathbf{B}) * \text{ones}(N, N)'); \quad (14)$$

Where $\mathbf{B}$ is the N by N , block matrix. Steps 3 and 4 are performed on $\mathbf{X}$ and $\mathbf{Y}$ respectively by (15) and (16):

$$\mathbf{X} = \mathbf{X} * \mathbf{Fmask}; \quad \mathbf{Y} = \mathbf{Y} * \mathbf{Fmask}; \quad (15)$$

$$\mathbf{X} = (\text{avP}(\mathbf{x})/\text{avN}(\mathbf{x})) * \mathbf{X}; \quad \mathbf{Y} = (\text{avP}(\mathbf{y})/\text{avN}(\mathbf{y})) * \mathbf{Y}; \quad (16)$$

Where $\text{avP}(\mathbf{x})/\text{avN}(\mathbf{x})$ and $\text{avP}(\mathbf{y})/\text{avN}(\mathbf{y})$ are simply the means ratios of the $\mathbf{x}$ and $\mathbf{y}$ vectors before and after the `blkproc.m` operations, in (13) and (14).

A typical example of the PBLOCK transform operation is illustrated in figure 3.4. The images (a) and (d) are NDVI's of the same sugarbeet field as in figure 3.1. Image (a) was acquired by the same airborne multi-camera imaging system incorporating optical filters in the RED (625-635 nm) and NIR (845-857 nm) spectral bands, on July 21, 1999. The NDVI image (d) was computed from a LANDSAT 7 dataset: band 3, RED (630-690 nm) and band 4, NIR(760-900 nm), which was acquired on July 8 1999. In both cases the spectral bandwidths are at half maximum of the indicated band.

The LANDSAT 7 pixel size is 30 by 30 meters while the multi-camera pixel size was 2 by 2 meters. Since the matrices $\mathbf{X}$ and $\mathbf{Y}$ must be of same size it was necessary to resample the LANDSAT 7 images at the 2 by 2 meter resolution. Then, they were geo-referenced thereby establishing a pixel to pixel correspondence with the multi-camera images. To improve visual contrast, 0.2 was subtracted from the matrices in images 4(d) and 4(e). This subtraction is not included in the numerical analysis. It may be seen, that apart from the same field shapes, the reference image 4(a) $\mathbf{Y}$, bears little resemblance to the remote image $\mathbf{X}$ in figure 3.4(d). Clearly, it is practically impossible to compare $\mathbf{X}$ and $\mathbf{Y}$, before the PBLOCK transform, due to the very low correlation coefficient (0.386), as depicted in figure 3.4(c). The dramatic increase of the correlation coefficient (0.833), after the PBLOCK transform, is evident in figure 3.4(f). Note particularly, that the problematic negative

NDVI pixels in figure 3.4(c) were eliminated by the averaging effect of the PBLOCK transform in figure 3.4(f).

Figure 3.5. Demonstration of the Image Rotation Transform (IMROT). (a) Sugarbeet yield monitor map (reference). Image (b) is (a) after rotation, (d) NDVI image (remote) and (e) is (d) after rotation. Image (c) is the unrotated field mask and (f) is (c) after rotation. The rotation increased the correlation coefficient to 0.421.

In most cases it is prudent to combine the WAVEL transform with the PBLOCK transform for further increasing the correlation coefficient. In the present example, following PBLOCK by WAVEL increased the correlation coefficient from 0.833 to 0.859 (not shown).

3.6 Image Rotation Transform (IMROT)

The image rotation transform can further reduce cross-noise in image pairs that have many slanted lines, i.e. lines which are not parallel or perpendicular to the frame of the image. Such images are common in agriculture wherein parallel lines are created in the direction of cultivation, e.g. images (a) and (d) in figure 3.5.

The flight direction for optimal illumination of the target during image acquisition is usually motivated by the sun azimuth and elevation angle. Thus, sometimes the flight path can not be

parallel or perpendicular to the direction of cultivation, as is the case in the example of figure 3.5(a) and 3.5(d).

The reference image **Y** in figure 3.5 (a) is a yield map which was generated by a sugarbeets harvester yield monitor, harvest date 10 October 1999. The harvester monitor weighs the flow of sugarbeet tubers as they are harvested. Knowing the forward speed and the harvested swath, each pixel value of the yield map is computed in terms of sugarbeets tons per hectare, (t/ha). In this case: min=0.177 t/ha, max=89.5 t/ha, av=60.1 t/ha and std=14.1 t/ha.

The remote NDVI image **X** was computed from RED and NIR images which were acquired by the multi camera video imaging system on 30 August 1999 (e.g. figure 3.1). Negative NDVI pixels were set to zero, because they contained no meaningful information concerning green biomass, which resulted in figure 3.5(d): NDVI; min=0, max=0.462, av=0.379 and std=0.0565.

Image rotation functions are usually available in most commercial image processing programs.

The IMROT transform uses the MATLAB `imrotate.m` function, whereby the **Fmask**, **X** and **Y** are rotated counter clockwise by a selected angle (degrees) :

Fmask=`imrotate(Fmask, angle, 'bilinear');` (17)

X=`imrotate(X, angle, 'bilinear');` (18)

Y=`imrotate(Y, angle, 'bilinear');` (19)

The `imrotate.m` function is essentially an interpolation routine wherein the rotated pixel positions can be optimized by the 'bilinear' or by the 'nearest neighbor' strategy. The IMROT algorithm employs 'bilinear'. The angle was 41 degrees, whereby after the rotation the cultivation lines are horizontal. Due to the interpolative nature of the `imrotate.m` function, the edge pixels of the field mask may not remain exactly ones. Therefore it is necessary to threshold the rotated **Fmask** by `im2bw.m`, before using the `.* Fmask` operator for cleaning up the edge pixels of the rotated **X** and **Y** images:

Fmask=`double(im2bw(Fmask));` (20)

X = **X**.***Fmask**; **Y** = **Y**.***Fmask**; (21)

X = (avP(x)/avN(x))***X**; **Y** = (avP(y)/avN(y))***Y**; (22)

Where avP(x)/avN(x) and (avP(y)/avN(y) are simply the means ratios of the **x** and **y** vectors before and after the `imrotate` operations in (18) and (19).

The unrotated and rotated field masks are depicted in figure 3.5(c) and 3.5(f) respectively. The rotated Y and X images are depicted in figure 3.5(b) and 3.5(e) respectively. Note that before the rotation the correlation coefficient was $r=0.364$ and after the rotation $r=0.421$.

Although both these r values are too low for any kind of image comparisons, when the IMROT transform follows PBLOCK and/or WAVEL, the r value may be improved.

In the present example, applying IMROT after WAVEL gives $r=0.521$ instead $r=0.475$ when WAVEL is used alone. Similarly applying IMROT after PBLOCK gives $r=0.801$ instead $r=0.794$ when PBLOCK is used alone. Finally, when PBLOCK, WAVEL and IMROT were applied in series, the correlation coefficient was 0.817.

3.7 Image comparisons by FFT Regression

Two regression model functions $Y=f(X)$ were tried in this research, a linear and non-linear model defined by equations (23) and (24) respectively:

$$Y = A * X + B; \quad (\text{Linear model}) \quad (23)$$

$$Y = A * X^B; \quad (\text{Non-Linear model}) \quad (24)$$

When $B < 0$ the derivative is negative so the slope of Y by (24) is decreasing. When $B > 0$ the derivative is positive so the slope is increasing and when $B=1$ the nonlinear model automatically reverts to $Y = A * X$.

In the training stage, the residuals obtained with (24) are usually lower than by (23). However, in the validation stage, the residuals by (24) were higher than by (23).

This is probably because the linear parameters are less sensitive to small image changes. So for long term prediction (23) is preferable while for one-time image comparisons (24) gives better results.

In regular and in FFT based regression, the parameters A, B, of formulae (23) or (24) can be easily found by the MATLAB non-linear least squares minimizing function `leastsq.m`.

For the linear case, model (23), regular regression:

$$[p] = \text{leastsq}('P1 - (p(1).*P2 + p(2))', [\text{guess1}, \text{guess2}], Y, X); \quad (25)$$

For the non-linear case, model (24) regular regression:

$$[p] = \text{leastsq}('P1 - (p(1).*P2.^p(2))', [\text{gues1}, \text{gues2}], Y, X); \quad (26)$$

For the linear case, model (23), FFT regression:

$$[p] = \text{leastsq}('abs(\text{fft2}(P1)) - abs(\text{fft2}(p(1).*P2 + p(2)))', [\text{guess1}, \text{guess2}], Y, X); \quad (27)$$

For the non-linear case, model (24), FFT regression:

$$[p] = \text{leastsq}('abs(\text{fft2}(P1)) - abs(\text{fft2}(p(1).*P2.^ab(2)))', [\text{guess1}, \text{guess2}], Y, X); \quad (28)$$

In all cases, the user must supply an initial solution, $A \sim \text{guess1}$, $B \sim \text{guess2}$, while the vector $\mathbf{p}$, which is returned by `leastsq.m`, holds the optimal parameters $A=\mathbf{p}(1)$, $B=\mathbf{p}(2)$.

Images predicted by regular regression will be denoted by $\mathbf{Y}_{reg}$, i.e. when A and B are computed by function (25) or (26). Similarly, images predicted by FFT regression will be denoted by $\mathbf{Y}_{fft}$, i.e. when A and B are computed by function (27) or (28).

The respective vectors of the residuals will be denoted by $\text{res}(reg)$ and $\text{res}(fft)$.

From functions (27) and (28), it may be deduced, that in FFT regression the parameters A and B are computed by transforming $\mathbf{X}$ and $\mathbf{Y}$ into the complex domain using the two dimensional FFT transform, as proposed in Peleg (1998). In each iteration stage, the 2D FFT transform (`fft2.m`), is applied whereby matrices $\mathbf{X}$ and $\mathbf{Y}$ are decomposed into absolute matrices $\mathbf{X}_{abs}$, $\mathbf{Y}_{abs}$ (amplitudes) and phase matrices $\mathbf{X}_{phas}$, $\mathbf{Y}_{phas}$. Only the amplitude matrices $\mathbf{X}_{abs}$, $\mathbf{Y}_{abs}$ are used in the optimization process.

The mathematical validity and justification for doing so was detailed in Peleg (1998).

For the linear case, i.e. model (23) and function (27), it can be simply explained as follows: From the linearity properties of the Fourier Transform we know that adding a constant to all members of a matrix $\mathbf{X}$, or multiplying a matrix $\mathbf{X}$ by a constant, does not change the phase matrix $\mathbf{X}_{phas}$.

Therefore, if a linear relation such as $\mathbf{Y}=\mathbf{A}*\mathbf{X}+\mathbf{B}$ exists between $\mathbf{X}$ and $\mathbf{Y}$, it must be invariant to changes in $\mathbf{X}_{phas}$ and $\mathbf{Y}_{phas}$.

In other words, if $\mathbf{Y}=\mathbf{A}*\mathbf{X}+\mathbf{B}$ holds for all elements of $\mathbf{X}$ and $\mathbf{Y}$, then $\mathbf{X}_{phas}=\mathbf{Y}_{phas}$.

Conversely, large differences between $\mathbf{X}_{phas}$ and $\mathbf{Y}_{phas}$, do not necessarily mean that a functional relationship $\mathbf{Y}=\mathbf{A}*\mathbf{X}+\mathbf{B}$ does not exist between $\mathbf{X}$ and $\mathbf{Y}$. In fact, even a little cross-noise may render $\mathbf{X}_{phas}$ very much different from $\mathbf{Y}_{phas}$. This is the main reason why the correlation coefficient of the pixels in two images is usually very low, even when they look very much alike.

In practical image comparisons, $\mathbf{X}_{phas} \neq \mathbf{Y}_{phas}$ because cross-noise is always present. When $\mathbf{X}$ and $\mathbf{Y}$ are decomposed by the FFT transform, the cross-noise is shared between the amplitude matrices $\mathbf{X}_{abs}$, $\mathbf{Y}_{abs}$ and phase matrices $\mathbf{X}_{phas}$, $\mathbf{Y}_{phas}$. If we perform the least squares optimization process only on $\mathbf{X}_{abs}$ and $\mathbf{Y}_{abs}$, we exclude the cross noise in $\mathbf{X}_{phas}$ and $\mathbf{Y}_{phas}$. Thereby, the resulting values of the parameters A and B are less influenced by cross-noise. No generality is lost by this, since $\mathbf{X}_{phas}$ and $\mathbf{Y}_{phas}$ do not carry any valid information about the true values of A and B .

When using the linear model (23), the A and B values by FFT regression yield coefficients of determination values R, that are closer to 1, while the correlation coefficient r is the same as in linear regression. That is, by FFT regression the variation due to the model $Y_{fft}=A*X+B$, is closer to the total variation in Y. A small price is paid for this, because the residuals by regular regression are usually slightly less than in FFT regression. However, the benefit of increasing R is normally well worth the slight increase in the residuals.

Essentially, the same results are obtained with the non-linear model (24), if the parameter B is relatively close to 1. When B differs much from 1, the results are unpredictable because the underlying linearity assumptions are violated.

Normally, the correlation coefficient between the compared images is quite low so the residuals by both methods are too high anyway. Fortunately, in two special cases of image comparisons, the FFT regression method provides an elegant means for drastically reducing the residuals and significantly increasing the correlation coefficient.

Consider first, a special case when the phase of the reference image Y is constant, i.e. Y_{phas} does not change in time. This means that the relative reflectance of all the pixels in the reference image does not change, although it can go up and down in unison.

In this case, we can correct the predicted image Y_{fft} by combining its amplitude matrix with the phase matrix of the reference image Y_{phas} , whence using MATLAB's FFT functions is easily implemented by:

$$Y_{plock} = \text{abs}(\text{fft2}(Y_{fft})) .* (\exp(Y_{phas} * \text{sqrt}(-1)))); \quad (29)$$

$$Y_{plock} = \text{real}(\text{ifft2}(Y_{plock})); \quad (30)$$

The amplitude matrix of Y_{fft} is computed by the first member of equation (29), i.e. $\text{abs}(\text{fft2}(Y_{fft}))$. Then the phase matrix Y_{phas} is "locked" to the amplitude matrix by the pixel-wise multiplication operator (.*). The resulting "phase locked" matrix, Y_{plock} is complex, so the inverse transform operation by function (30), completes the correction of Y_{fft} by the reference phase Y_{phas} . Since the phase matrix of Y_{plock} and Y is the same, the cross-noise between them is much smaller than the cross-noise between Y_{fft} and Y.

A practical example of this case is comparing a single reference image Y, to time spaced remote images X. For example, comparisons of a yield monitor map of a field Y, to NDVI images X, which were acquired at different times of the crop's growth period.

In the second case, we modify Y_{fft} as well as the reference image Y , by similarly combining their amplitude matrices with the phase matrix of the field mask F_{phas} :

$$F_{phas} = \text{angle}(\text{fft2}(F_{mask})); \quad (31)$$

The phase locked image Y_{pred} is then used as a predictor of the phase locked reference image Y_{pref} , whence both are obtained similarly to Y_{plock} :

$$Y_{pred} = \text{abs}(\text{fft2}(Y_{fft})) \cdot (\exp(F_{phas} \cdot \sqrt{-1})); \quad (32)$$

$$Y_{pred} = \text{real}(\text{ifft2}(Y_{pred})); \quad (33)$$

$$Y_{pref} = \text{abs}(\text{fft2}(Y)) \cdot (\exp(F_{phas} \cdot \sqrt{-1})); \quad (34)$$

$$Y_{pref} = \text{real}(\text{ifft2}(Y_{pref})); \quad (35)$$

Since the phase matrix of Y_{pref} and Y_{pred} is the same, the cross-noise between them is much smaller than the cross-noise between Y_{fft} and Y .

This approach is particularly suitable when it can be assumed that both Y_{fft} and Y are excursions about a mean uniform image that can be represented by the F_{mask} .

Such an assumption is indeed valid in comparing images of yield distributions, e.g. harvester yield monitor and NDVI images.

The mean yield predicted by Y_{fft} is effectively identical to the reference mean of Y . So, if we multiply the F_{mask} by this common mean, it can be viewed as a model of the field, wherein the yield is uniformly distributed. Then, images Y_{fft} and Y portray the yield distribution in the field, by a similar but not identical pattern, about this uniform yield distribution. Actually, there is no need for such a multiplication because F_{phas} is invariant to a multiplication of F_{mask} by a constant.

The variances of the matrices Y_{pref} and Y_{pred} are usually smaller than the variances of matrices Y and Y_{fft} , due to the mitigating effect of their combination with F_{phas} . This creates a clustering effect which amplifies the agreement between Y_{pref} and Y_{pred} , so they are closer to a consensus. In precision agriculture this is beneficial in two respects:

- a. The prediction confidence and robustness of both Y_{pref} and Y_{pred} is increased by the clustering effect, albeit the increased consensus is confined to a smaller portion of the variance.
- b. The original reference Y is usually fragmented into small contiguous patches of widely varying yields, which precision agriculture practices can not follow. The predicted yield map Y_{pred} is much less fragmented than Y .

These advantages make Y_{pref} a better estimator of the yield distribution than the original reference map Y , while Y_{pred} is the next best. Most important, since F_{phas} does not change with time, Y_{pred}

can be derived by remote sensing alone. This affords predictions of the yield distribution in a field well ahead of the harvest time. Additionally, the mean value of Y_{pred} , is a good predictor of the mean crop yield per hectare, as exemplified in the next section.

3.8 Example of FFT regression (Yield monitor map vs. NDVI)

In this section, we present a detailed example of the procedure for deriving a predicted yield map Y_{pred} , of a sugarbeet field. An NDVI image X was acquired on August 2, 1999 and a reference yield map Y , from a harvester yield monitor was obtained on October 10, 1999. In figure 3.6 (a) and 3.6 (d), the yield monitor map and NDVI images are depicted respectively.

The pixel values in X and Y , excluding the pixels of the zero-frame were transferred to the respective vectors x and y . The scatterplot in figure 3.6 (c) shows the Raw Reference vector y plotted versus the Raw Remote vector x . Although the images in figure 3.6(a) and (d) look quite similar the correlation coefficient before preprocessing is very low, $r_0=0.368$. Note particularly the abundance of zero pixels along the x and y axes. That is, for many zero yield points in the harvester monitor map the NDVI images predicts average yield. Conversely, for many zero NDVI points (many of which were negative before being set to zero), the yield monitor map indicates normal yield values. In both cases, such zero and near zero yield points are most probably artifacts which should be excluded from the comparison between X and Y . Indeed, after pre-processing by the PBLOCK, WAVEL and IMROT transforms most of the previously zero points (black), are gray in images (b) and (e) of figure 3.6.

Although the visual similarity between images (b) and (e) still remains pretty much the same as between (a) and (d), the plot of y versus x in figure 3.6 (f) is quite different from plot (c), before pre-processing stage. Now the functional relationship between the harvester yield monitor values on the horizontal axes, (sugarbeet tubers t/ha) and the NDVI values on the vertical axes is quite evident. The correlation coefficient was increased to $r_t=0.785$ from $r_0=0.368$, although a considerable amount of cross-noise still remains. Nevertheless, the plot in figure 3.6(f) clearly demonstrates that a functional relationship does indeed exist between Y and X . Without the pre-processing stage this functional relationship could not be discovered. This functional relationship is remarkable, because it demonstrates that there is a direct link between photosynthetic vigor and the yield of sugarbeet tubers more than 2 months prior to harvest.

3.9 Amplitude and phase images

For a better understanding of the FFT regression principle, it is interesting to compare the amplitude and phase matrix images X_{abs} , Y_{abs} , F_{abs} , X_{phas} , Y_{phas} , F_{phas} of X , Y and the F_{mask} , before and after the pre-processing stage, as depicted in figure 3.7 and 3.8 respectively. These images are visual demonstrations of the 2D FFT operator. The first rows (a), (b) and (c) in figure 3.7 and 3.8, are images of the amplitude matrices X_{abs} , Y_{abs} , F_{abs} before and after the pre-processing stage respectively. Similarly, the second rows (d), (e) and (f) in figure 3.7 and 3.8, are images of the phase matrices X_{phas} , Y_{phas} , F_{phas} before and after the pre-processing stage respectively. The bottom rows (g), (h) and (i) in figure 3.7 and 3.8, are images of the phase matrix differences ($Y_{phas} - X_{phas}$), ($Y_{phas} - F_{phas}$) ($X_{phas} - F_{phas}$) before and after the pre-processing stage respectively. For each pixel in the original image, the 2D FFT operator creates a sinusoid which is uniquely defined by its amplitude and phase angle. The pixels in the X_{abs} , Y_{abs} , F_{abs} images are these amplitudes while the pixels in the X_{phas} , Y_{phas} , F_{phas} are the phase angles in radians. Given the amplitude and phase matrices, the original image can be reconstructed exactly by the inverse 2D FFT operator.

The first element of the first row in the amplitude image $A(1,1)$ is the mean value of the pixels in the image. These mean values are listed besides the image titles in the first rows (a), (b) and (c) in figure 3.7 and 3.8 respectively. Normally, the values of $A(1,1)$ are much bigger than the rest of the amplitudes. For visual gray scale display in the (0-1) range, the amplitude matrices were divided by $A(1,1)$ and then multiplied by 2000.

The values of the phase matrices vary between $\pm \pi$ but for visual gray scale display in the (0-1) range, the phase matrices were divided by π and displayed as absolute values of (0-1).

The mean values of ($Y_{phas} - X_{phas}$), ($Y_{phas} - F_{phas}$) ($X_{phas} - F_{phas}$) are listed besides the image titles in the bottom rows (g), (h) and (i) in figure 3.7 and 3.8 respectively.

By visual comparison of the amplitude images in figure 3.7 and 3.8 it is seen that the general shape of X_{abs} , Y_{abs} , F_{abs} is similar within the figures but very different between figures 3.7 and 3.8.

This is mainly due to the change in the aspect ratio and spatial orientation of the ROI from figure 3.7 to 3.8. The size of the images were increased from 445x306 to 537x525, whereby the extra zeros that were added to the zero-frame because of the image rotation by IMROT are accommodated.

However, this does not change the numbers of pixels and their relative positions within the ROI.

In figure 3.7, the image of Y_{abs} is different from X_{abs} and F_{abs} , which are quite similar. The opposite is true in figure 3.8. There, X_{abs} and Y_{abs} are quite similar while F_{abs} is different. This observation is in line with the image transformations in the pre-processing stage, which strives to increase the functional relationship between X and Y by reducing the phase difference ($Y_{phas} - X_{phas}$). Therefore, image (i) is the darkest in figure 3.7 while image (g) is the darkest (i.e. closer to zero) in figure 3.8.

Figure 3.6. (a) Yield monitor image Y (harvest time October 10, 1999). (d) NDVI image X (acquired on August 2, 1999). Images (b) and (d) are Y and X after pre-processing by PBLOCK, WAVEL and IMROT. Scatterplots (c) and (d) depict y versus x before and after pre-processing respectively.

The added benefit is that in figure 3.8(h) ($Y_{phas} - F_{phas}$) = 1.49 and 3.8(i) ($X_{phas} - F_{phas}$) = 1.47. This means that the yield distribution of the transformed X and Y images is similarly different from the uniform yield distribution portrayed by the F_{mask} .

It is interesting to note that the phase images (d), (e) and (f) in figure 3.7 and 3.8 look quite monotonous and seem to be quite similar, which is not the case in reality, as can be observed by the phase difference images (g), (h) and (i). They look monotonous because they encode a rather

monotonous image, i.e. a canopy of sugarbeet plants in a field. When the features within the ROI vary more drastically, the shapes of the phase images are also more "interesting".

Figure 3.7. Images of amplitudes, phase and phase differences matrices derived by the 2D FFT operator. Images (a), (b), (c), are amplitudes of X , Y and F_{mask} respectively. Images (d), (e), (f), are respective phase matrices and (g), (h), (i) are phase differences ($Y_{phas} - X_{phas}$), ($Y_{phas} - F_{phas}$), ($X_{phas} - F_{phas}$).

Figure 3.8. Same as figure 7, after pre-processing by PBLOCK, WAVEL and IMROT.

3.10 Regular regression

Following the preprocessing stage as outlined in the previous two subsections, the increase of the correlation coefficient to $r=0.785$ is sufficiently large for generating a predicted yield map by regular regression. The results are depicted in 3.9 wherein image (a) is the reference yield map Y while image (b) is the yield map predicted by regular regression Y_{reg} . The linear model of equation (23) was used to derive the parameters $A=151.6638$ and $B=0.1509$ by formula (25).

The mean phase difference between Y and Y_{reg} is $dP=0.677$, as shown above in figure 3.8(g). The coefficient of determination is $R=1.07$, i.e. Y_{reg} overestimated the variance in Y by 7%. The standard deviation of the residuals was $stdRES=7.81$ t/ha, whence the predicted yield distribution error for the 95% confidence interval is $\pm 2 * 7.81 = 15.62$ t/ha. Given the mean yield of about 60 t/ha this constitutes an error of $\pm 25\%$, which is fairly high for precision agriculture applications.

The true mean yield in this field was 59.267 t/ha, as determined independently by weighing the harvested sugarbeet tubers. The mean reference yield determined from Y was 60.1 t/ha while the mean predicted yield from Y_{reg} was 59.524 t/ha, which is quite accurate. Note that figures 3.6(e) and 3.9(b) look identical because they have the same phase matrix, which remains unchanged by the $151.6638 * X + 0.1509$ operation. For the same reason the shapes of the plots in figures 3.6(f) and 3.9(c) look essentially alike.

3.11 FFT regression

The results by FFT regression using the reference phase lock option of formulae (29) and (30) are depicted in figure 3.10. The linear model of equation (23) was used wherein the FFT regression parameters $A=152.5360$ and $B=0.0840$ were found by formula (27). The phase matrix of Y_{plock} is identical to Y_{phas} so $dP=0$. The correlation coefficient has risen to $r=0.941$ and the coefficient of determination is $R=0.963$, i.e. Y_{plock} underestimated the variance in Y by 3.7%, versus 7% overestimation by Y_{reg} . The standard deviation of the residuals was $stdRES=3.88$ t/ha. whence the predicted yield distribution error for the 95% confidence interval was only $\pm 2 * 3.88 = 7.76$ t/ha i.e. about half of the error by regular regression. The mean yield predicted from Y_{plock} was 59.939 t/ha which is practically identical the reference by Y , i.e. 60.1 t/ha.

Note particularly that the reference and predicted Y_{plock} are much closer to the 45° identity line than Y_{reg} in figure 3.9(c).

Figure 3.9. Results of regular regression. Image (a) is the reference yield map Y , image (b) is Y predicted by $Y_{reg} = 151.6638 \cdot X + 0.1509$, shown in plot (c). Plot (d) depicts the residuals.

Figure 3.10. Results of FFT regression with Y_{phas} lock option. Image (a) is the reference yield map Y , image (b) is Y predicted by Y_{plock} , shown in plot (c). Plot (d) depicts the residuals.

The main drawback of FFT regression with the reference phase lock option is that it can not be used for subsequent yield predictions because Y_{phas} changes from year to year. However, since the F_{mask} phase matrix remains the same from year to year, the F_{phas} lock option is better suited for yield distribution predictions by remote sensing over time.

The results by FFT regression while using the field phase lock option i.e. formulae (32) to (35), are depicted in figure 3.11. The linear model of equation (23) was used with the same parameters A, B, as in the reference phase lock option. Since the same phase matrix F_{phas} is used by Y_{pref} and Y_{pred} , $dP=0$ as in the previous case. The correlation coefficient $r=0.79$, is less than in the previous case. This drop in r is not consequential as can be seen from the value of the coefficient of determination $R=1.03$, i.e. Y_{pred} overestimated the variance in Y_{pref} by 3%, which is even less than in the previous case. Furthermore, the standard deviation of the residuals dropped to $stdRES=3.52$ t/ha so the predicted yield distribution error for the 95% confidence interval was only $\pm 2 \times 3.52 = 7.04$ t/ha. The reduced value of r is misleading because it was caused by the reduced range of the Y_{pref} and Y_{pred} values in comparison to their Y and Y_{plock} counterparts. Such range reductions are typical in the field phase lock option, as was previously explained in section 8, e.g. compare the plot in figure 3.10(c) and 3.11(c).

Note particularly that the problematic zero and close to zero yield readings that are present in figure 3.9(c) and 3.10(c) have been eliminated from Y_{pref} and Y_{pred} .

This is not the case in the maximal yield readings. They are essentially the same as in figure 3.10(c). The minimum yield predicted by Y_{pref} and Y_{pred} is about 20 t/ha which is probably more realistic than the abundance of zeros in Y . This is so because Y_{pref} and Y_{pred} are influenced by the phase of the field mask, which does not include zero values in the ROI. Furthermore, by comparing figures 3.9(a) or 3.10(a) with 3.11(a) it is seen that the former are fragmented into small contiguous patches of widely varying yields. The yield maps Y_{pref} and Y_{pred} in figure 3.11 are much less fragmented than Y , which is advantageous to precision agriculture practices.

Most important, the confidence in Y_{pref} and Y_{pred} should be larger than in Y because they represent a consensus of two independent sensing systems, albeit this consensus is confined to a smaller portion of the original variance in Y . The only downside of eliminating the zero yield readings is a slight increase in the predicted mean yield, i.e. 60.541 t/ha by Y_{pred} , versus 59.939 t/ha by Y_{plock} .

Figure 3.11. Results of FFT regression with *Fphas* lock option. Image (a) is the modified reference yield map *Ypref*, image (b) is *Ypref* predicted by *Ypred*, shown in plot (c). Plot (d) depicts the residuals.

3.12 Summary of results by multi-spectral imaging

Typically, there are three stages in predictive remote sensing studies, such as yield map predictions: "Training", "Validation" and "Runtime". The training stage usually comprises the "proof of concept", as outlined in the previous sections. In the validation stage, the predictive model is tested versus the ground truth reference over time. Thereby, the validity and robustness of the predictive model is tested. Based on the confidence gained in the validation stage, the predictive model may then be used independently in the runtime stage.

During the 1999 growth season we have conducted six image acquisition flights over the experimental sugarbeets and corn fields. For each field and flight, yield prediction maps *Yreg*, *Yplock* and *Ypred* were computed while using harvester's yield monitor maps as a reference. The results of these experiments comprise the training stage, which should be followed by the validation stage during the next two years.

Table 1. SUGARBEET YIELD MAPS PREDICTED BY NDVI**Field #B1123**

Y.M. 60.1 t/ha		Regression					<i>Yplock</i>				<i>Ypred</i>			
Date	NDVI	R	r	t/ha	s.d.		R	r	t/ha	s.d.	R	r	t/ha	s.d.
7.21.99	0.283	0.95	0.79	59.8	7.34		0.91	0.94	60.0	3.85	0.95	0.78	60.6	3.47
8. 2.99	0.392	1.05	0.79	59.5	7.70		0.96	0.94	59.9	3.84	1.01	0.79	60.5	3.49
8. 5.99	0.372	1.06	0.78	59.5	7.80		0.96	0.94	59.5	3.91	1.03	0.79	60.5	3.56
8.30.99	0.379	0.93	0.82	59.9	6.73		0.92	0.95	59.9	3.68	0.95	0.80	60.6	3.35
9. 9.99	0.559	0.77	0.83	60.2	6.52		0.85	0.95	60.2	3.90	0.84	0.77	60.7	3.43
9.21.99	0.481	0.82	0.80	60.1	6.89		0.86	0.95	60.1	3.88	0.86	0.77	60.7	3.41
Validation: $Y_{reg}=156.3671*NDVI + 0.1203$ $Y_{fft}=157.1756*NDVI + 0.0600$														
8. 2.99	0.392	1.09	0.78	61.3	7.85		0.99	0.94	61.7	3.87	1.12	0.79	62.3	3.54

Field #M0522

Y.M. 66.9 t/ha		Regression					<i>Yplock</i>				<i>Ypred</i>			
Date	NDVI	R	r	t/ha	s.d.		R	r	t/ha	s.d.	R	r	t/ha	s.d.
7.21.99	0.277	0.87	0.71	66.5	10.3		0.90	0.90	66.8	6.00	0.89	0.65	67.7	5.80
8. 2.99	0.413	0.82	0.77	66.8	9.20		0.89	0.92	66.8	5.67	0.88	0.70	67.7	5.40
8. 5.99	0.400	0.85	0.78	66.7	9.15		0.90	0.92	66.8	5.70	0.90	0.69	67.7	5.44
8.30.99	0.414	0.89	0.79	66.8	9.73		0.88	0.90	66.8	6.01	0.83	0.64	67.7	5.74
9. 9.99	0.586	0.75	0.75	66.9	9.61		0.87	0.91	66.9	5.88	0.80	0.65	67.8	5.59
9.21.99	0.518	0.75	0.71	66.8	10.2		0.87	0.91	66.8	6.06	0.79	0.64	67.8	5.74
Validation: $Y_{reg}=162.527*NDVI + 0.2592$ $Y_{fft}=164.6099*NDVI - 0.2886$														
8. 2.99	0.392	0.83	0.77	66.5	9.21		0.90	0.92	67.6	5.66	0.90	0.69	68.5	5.41

The main results on two sugarbeet fields #B1123 and # M0522 are summarized in table 1. By and large, the results in both fields are quite similar to the above detailed example, which was from field #B1123 on 8.2.99, (second row in table 1). In all cases the standard deviation of the residuals by FFT regression is about half of the results by regular regression. This reiterates the advantage of FFT regression for yield distribution predictions.

The accuracy of mean yield prediction is quite good in all cases while Y_{pred} usually overestimates them slightly. In both fields, the mean value of the NDVI images followed the same timely pattern. It increases from mid July and levels off during August. Then it rises further in the beginning of September and drops a little towards the end of September.

The best results in terms of yield prediction accuracy were obtained during August, as can be seen from the figures in table 1. The stable mean NDVI values of the images acquired in August also tend to stabilize the values of the parameters A and B, in the regression model functions. Therefore, the month of August seems to be the ideal time window for sugarbeet yield distribution predictions in eastern Montana.

Since the parameters A and B did not change much during August, we used their mean values as a "Validation Model". The resulting validation functions Y_{reg} and Y_{fft} are listed in table 1, (row before last of each field). These validation functions were then used with the August 2 NDVI images for computing validation maps by Y_{reg} , Y_{plock} and Y_{pred} .

The NDVI images of August 2 were selected because they are farthest from the harvest dates, i.e. October 8-14 and October 10-11, for fields #B1123 and #M0522 respectively.

The results for each of the two fields, are listed in the last rows of table 1. From these it may be seen that the standard deviation of the residuals and the predicted mean yields changed very little. These results need to be validated in subsequent years.

Note that the standard deviations of the residuals s.d. in field #B1123 are significantly lower than in field #M0522. This is mainly because the calibration and adjustment of the yield monitor was poor when the #M0522 field was harvested. Thus, in this case, the remotely acquired yield map is probably a better predictor of the true yield distribution in this field, then the yield monitor map derived two months later!

Table 2 CORN YIELD MAPS PREDICTED BY NDVI**Field #M2922**

Y.M. 7.28 t/ha		Regression				Yplock				Ypred			
Date	ndvi	R	r	t/ha	s.d.	R	r	t/ha	s.d.	R	r	t/ha	s.d.
7.21.99	0.379	0.86	0.86	7.28	0.65	0.92	0.96	7.28	0.36	0.86	0.81	7.34	0.35
8. 2.99	0.454	0.80	0.82	7.28	0.72	0.90	0.95	7.28	0.40	0.79	0.77	7.34	0.39
8. 5.99	0.443	0.80	0.78	7.28	0.79	0.89	0.94	7.27	0.44	0.80	0.70	7.34	0.44
8.30.99	0.271	0.87	0.69	7.24	0.94	0.90	0.92	7.25	0.51	0.89	0.61	7.32	0.50
9. 9.99	0.423	0.81	0.79	7.28	0.78	0.89	0.94	7.27	0.42	0.81	0.73	7.34	0.42
9.21.99	0.080	1.97	0.22	6.42	2.44	1.13	0.77	6.98	0.93	1.71	0.27	7.06	1.04

Field #M0521

Y.M. 8.11 t/ha		Regression				Yplock				Ypred			
Date	ndvi	R	r	t/ha	s.d.	R	r	t/ha	s.d.	R	r	t/ha	s.d.
7.21.99	0.440	0.75	0.71	8.10	1.11	0.84	0.91	8.10	0.65	0.76	0.64	8.19	0.63
8. 2.99	0.442	0.72	0.69	8.10	1.15	0.82	0.90	8.10	0.69	0.76	0.60	8.19	0.66
8. 5.99	0.428	0.73	0.67	8.10	1.17	0.82	0.89	8.10	0.71	0.76	0.59	8.18	0.67
8.30.99	0.355	0.73	0.64	8.10	1.22	0.81	0.89	8.10	0.74	0.76	0.55	8.18	0.69
9. 9.99	0.506	0.72	0.67	8.10	1.17	0.81	0.90	8.10	0.69	0.74	0.61	8.19	0.65
9.21.99	0.082	1.51	0.35	7.55	2.31	1.12	0.78	7.75	1.11	1.40	0.36	7.94	1.12

The main results on two corn fields #M2922 and # M0521 are summarized in table 2.

It may be seen that the results in both corn fields reiterate the main findings in the sugarbeet fields. Namely, in all cases the standard deviation of the residuals by FFT regression is about half of the results by regular regression. The accuracy of mean yield prediction is quite good in all cases while *Ypred* usually overestimates them slightly.

In both fields, the mean value of the NDVI images followed a similar timely pattern, which was quite unlike the one in the sugarbeet fields. The best results in terms of yield prediction accuracy were obtained in the 21 July flight, as can be seen from the figures in table 2.

This is remarkable because the harvest time of field #M2922 was November 3, 1999 while field # M0521 was harvested in two shifts, October 31 and November 2, 1999. The harvest of field # M0521 was not completed on October 31 because of a severe wind storm, which destroyed part of the crop. The 21 July NDVI image obviously does not reflect the wind storm. Consequently, the standard deviations of the residuals s.d. in field #M2922 are significantly lower than in field #M0521. Thus, in this case, the yield map is probably a better predictor of the true yield distribution in this field. From the R , r and s.d. values in table 2, it seems that the time window for corn yield distribution predictions is much shorter than in sugarbeets. The last week of July and the first week of August are probably best.

In that time window the mean NDVI values of the images are quite stable but the parameters A and B , in the regression model functions are not. Since the parameters A and B changed appreciably their mean values could not be used as a "Validation Model".

4. Hyperspectral data analysis

The airborne CASI II hyperspectral imaging system became fully operational during the 2001 summer season. It was operated in the “Spectral Mode” and flown at an altitude of 2750 m, providing image cubes of 48 spectral bands and 2m pixel size.

The main advantage of replacing multi-spectral by hyperspectral imaging is the ability to use characteristic spectral signatures to identify specific materials in the imaged target. This approach is similar to the “chemometrics” methods which are successfully used, to identify compositions of chemical mixtures by their spectral absorption signatures. Chemometrics methods are conducted in controlled laboratory conditions wherein the illumination, environmental conditions and sample configurations are constant and standardized. In contrast, the conditions in airborne image acquisition are highly variable, since the sun’s illumination and atmospheric conditions change constantly. This reduces the reliability and the repeatability of the acquired data, rendering the classical chemometrics data analysis methods inadequate. Consequently, in this research project we developed more powerful spectral data analysis tools, as described in the following sections.

4.1 Conversion of Radiance to Reflectance

Hyperspectral imaging systems provide reflected radiation imagery which changes with atmospheric conditions and solar illumination of the monitored targets. Thus, temporal changes can only be detected and quantified by “reflectance”, i.e. the ratio of the reflected radiation and the incidence radiation of each pixel at each wavelength.

Theoretically, this can be accomplished by measuring the incident and reflected radiations while assuming that the sun’s illumination is perfectly diffuse, the target is a perfect Lambertian reflector and the atmosphere is a perfect transmitter of radiation of all wavelengths in all directions. In practice these conditions never exist. Nevertheless, it may be feasible to obtain approximations of true reflectance by models and empirical verification of the typical reflectance patterns of the inspected targets, the atmospheric attenuation and dispersion of the incident and reflected radiation. Many model based, programs were proposed for converting radiance imagery acquired by space satellites to reflectance.

Figure 4.1 Hydraulically operated tractor boom with hinged cage for housing and suspending the ASD spectroradiometer.

Figure 4.2 Electrically operated platform for periodically presenting a "Spectralon" disc to the fiberoptics sensor of the ASD spectroradiometer, including the GPS antenna.

These are dubbed: “atmospheric corrections programs” of which the 6S, LOWTRAN, MODTRAN and ATREM software packages are typical examples. The 6S program was developed by NASA and the French Space agency. The MODTRAN program was developed for atmospheric correction in aerial photography, by the University of Arizona, the US Air Force and NASA. ATREM was developed at the University of Colorado for converting radiance to reflectance of AVIRIS imagery. These programs are platform and application dependent and neither is directly applicable for relatively low altitude airborne hyperspectral imaging. Most important, their results are dominated by assumptions or measurements of numerous parameter values, which can not be determined with sufficient accuracy in precision agriculture applications. In this project, we developed an empirical procedure for obtaining close approximations of reflectance images from hyperspectral apparent radiation images.

The procedure relies on a two-tone reference target of known reflectance, located at or close by the imaged targets. Specifically, in this project we used a reference target located at the local airport, which is just a few km. away from the farthest of our precision agriculture experimental fields. The low-tone part of the reference target was a piece of dark asphalt tarmac while the high-tone was a bright cement helicopter landing pad. The reflectance of many spots in these reference targets was measured by the ASD spectroradiometer.

A hydraulically operated tractor boom with a hinged cage was designed and fabricated, wherein the ASD spectroradiometer could be mounted and suspended over the reference target at a height of 3m, as depicted in Figure 4.1. An electrically operated small platform was used to periodically present a “Spectralon” disc to the fiberoptics sensor, whereby the ASD spectroradiometer was referenced to a flat spectrum every 5 readings, as required for accurate reflectance measurements. This arrangement is depicted in the close-up picture of figure 4.2, which also shows the GPS antenna.

Neither the asphalt of the tarmac nor the cement of the helipad are truly homogeneous materials. This results in the relatively wide range of the spectral reflectance curves obtained at different spots, as depicted in the bottom graphs of figure 4.3. However, repeated measurements during the 2000 and 2001 seasons indicated that the mean reflectance curves are very stable, so they can be used as a reference.

We adopted a simple linear model $\rho = \alpha(I - \beta)$ due to (Conel et.al. 1987), for quantifying the functional relationship between the spectral radiance recorded by the airborne sensor I , and the reflectance ρ , of the reference target as measured by the ASD spectroradiometer. The

multiplicative parameter α and the subtractive parameter β can be readily found by applying the model to the two-tone reference target:

$$\rho_1 = \alpha(I_1 - \beta) \quad \rho_2 = \alpha(I_2 - \beta) \quad (36)$$

Where ρ_1 and ρ_2 are the mean reflectance of the low-tone and high-tone parts of the reference target respectively, as measured by the ASD spectroradiometer, and exemplified by the bottom graphs in Figure 4.3.

Similarly, the mean radiances I_1 and I_2 , of the low-tone and high-tone parts of the reference target, are computed by averaging their pixel values, as recorded by the airborne sensor.

Figure 4.3 Top - helipad and tarmac radiance, middle - multiplicative and subtractive parameters spectra, bottom - Spectral reflectance of helipad and tarmac.

Proceeding so, for each and every spectral band, we obtain the mean radiance spectra of the reference target, as exemplified by the top graphs in Figure 4.3.

Solving equations (36) for α and β we get:

$$\alpha = \frac{\rho_1 - \rho_2}{I_1 - I_2} \quad (37)$$

$$\beta = \frac{\rho_1 I_2 - \rho_2 I_1}{\rho_1 - \rho_2} \quad (38)$$

By applying equations (37) and (38) to each spectral band of the two-tone target image cube we obtain the multiplicative and subtractive parameters spectra, as exemplified in the middle graphs of Figure 4.3. These spectra can then be used to convert all the voxels of an image cube from radiance to pseudo reflectance, as exemplified in figure 4.4.

Figure 4.4. Pseudo reflectance spectra of entire reference target which includes dark asphalt, a bright cement helipad and adjacent grass. Derived by equation 36 using the parameters from 37 and 38.

The wavelength scales of the CASI II and the ASD spectroradiometer are different. Therefore, the practical implementation of this procedure requires preliminary interpolation of the ASD data to the CASI II spectral range and scale.

Only a small section of the tarmac in the reference target and the helipad were used for deriving the multiplicative and subtractive parameter spectra. A demonstrative example of using these spectra for computing pseudo reflectance curves of all the voxels in the image cube of the entire reference target is depicted in figure 4.4. The reflectance spectra of the dark asphalt and the bright cement of the

helipad are clearly visible, similarly to the respective spectra in the bottom images of figure 4.3. Additionally, figure 4.4 also includes typical vegetation spectra, due to the grass adjacent to the tarmac.

Note that the underlying assumption of this simple model is that the sun's illumination and the atmospheric conditions are the same at the reference target and at the imaged fields. This is only true if the reference target is located in the imaged field proper, which is seldom the case. The farther the reference target is from the imaged fields, the longer it takes for the airplane to travel between them. This reduces the accuracy and repeatability of the resulting conversion from radiance to reflectance. The repeatability can be improved by flying over the reference target along the sun's azimuth trajectory, as described in the next section.

4.2. Repeatability of remotely sensed imagery

A hyperspectral remote sensing imaging system is effectively a measuring instrument whereby the spectrum of each pixel in the target image may be used to quantify or identify its properties. A key performance feature of any measurement system is the basic requirement that repeated measurements of the same property should provide the same results, if the environmental conditions have not changed. Similarly, hyperspectral image cubes acquired in consecutive flights over the same target, should ideally be identical. In practice, two consecutive flights over the same target usually yield significant differences between the image cubes. These differences are due to:

1. Variations in solar illumination.
2. Changes in atmospheric conditions.
3. Measurement errors of the imaging system proper.

Manufacturers of remote sensing imaging systems use sophisticated equipment to spectrally and spatially calibrate their instruments and reduce the measurement errors as much as possible.

Nevertheless, even in the laboratory, when two image cubes of a test target are compared voxel by voxel, (3D pixel), one finds significant differences. Clearly, when the non-repeatability variance of image cubes is similar in magnitude to the variance of the spectral information of interest, it would be impossible to use it for classification or prediction modeling. Therefore, users of remote sensing imaging systems should be able to quantify the repeatability of image cubes. Furthermore, it is important to quantify separately, the non-repeatability contribution of solar illumination and atmospheric variation, as well as the non-repeatability of the imaging system proper. Analysts of

hyperspectral imagery could benefit greatly by separately quantifying the repeatability of spectral bands. Less repeatable bands may be excluded from the model or averaged for improving repeatability. Most important, by quantifying repeatability, the analyst may assess the feasibility of creating an efficient regression or classification model for a given target. For if the repeatability of all the bands is too poor then no model will work without over fitting or producing a model that can only be used for that dataset.

4.2.1 Objectives

Our main objective was to quantify the differences between image cubes of the same target, whereby the repeatability performance of a remote sensing imaging system may be objectively assessed. To this end, we conducted extensive experiments wherein the specific objectives were:

1. Assess "**Spectral Repeatability**", which is quantified by the differences between the spectral curves of respective voxels, in two image cubes of the same target.
2. Assess "**Spatial Repeatability**" which is quantified by the differences between the pixels of respective spectral bands, in two image cubes of the same target.
3. Assess the "**Repeatability of Image Cubes**" which is quantified by the differences between respective voxels of two image cubes of the same target. Effectively, this is the combined Spectral and Spatial Repeatability.
4. Detect the coordinates of defective cells in the CCD array of the imager, if any.
5. Compare spectral curves of the hyperspectral imaging system to curves generated by a lab spectroradiometer, (e.g. ASD).

4.2.2 Laboratory tests protocol

The imaging head of the CASI II was suspended above a white (Barium Sulphate) panel of about 60x60 cm. dimensions, while using two types of illumination methods.

1. One experiment was conducted in a "dark room", illuminating the panel by a 150w Halogen lamp. The objective is to use a relatively large aperture and long integration times, simulating low illumination conditions and slow flying speeds.
2. Another experiment was conducted using sunlight illumination, by setting up the experimental system outside, reducing the integration time and using a small aperture. The objective is to simulate high illumination conditions and fast flying speeds.

In each experiment, two image cubes were acquired, each about one thousand scans long. The sensor of the ASD spectroradiometer was pointed at the center of the Barium Sulphate panel in all the experiments. Spectral curves were recorded by the spectroradiometer simultaneously with the hyperspectral image acquisitions.

4.2.3 Flight tests protocol

Seven test flights were conducted over the reference target at the local airport, as described in section 4.1. The flights and consecutive image acquisitions were conducted on August 31 2001, close to solar noon, with as little time spacing between them as permitted by the airplane's maneuverability. All flights were flown at approximately the same altitude, about 1400 m, while varying the heading. The first two flights were flown along the sun's azimuth trajectory, i.e. the sun airplane and the target were lined up while flying away from the sun, (sun lineup). The next four flights were flown along a South, North, East and West heading while flight seven was another sun lineup flight. The six images in figure 4.5 depict one band of the reference target (band 16 = 596.5 nm). The top two images are from the first two sun lineup flights, acquired at 12:01 and 12:07 hours local time respectively. The middle two images, South and North headings were acquired at 13:01 and 13:04 hours respectively. The bottom two images, East and West headings were acquired at 13:08 and 13:08 hours respectively. The last image (not shown in figure 4.5), was acquired at 13:26 hours. The precision agriculture experimental fields were imaged during the time between the first two flights and the next five flights, i.e. between 12:07 and 13:01 hours. The corresponding changes in the sun's azimuth and zenith angles are shown in the respective images of figure 4.5. It may be seen that during the first six flights (about an hour), the sun's azimuth angle changed between 158 to 186 degrees while the zenith angle varied only from 41.1 to 39.3 degrees. Although the zenith angle changed very little close to solar noon, the azimuth angle varied by 28 degrees! Such change in the direction of target illumination has a profound effect on the reflected radiation, as exemplified by the considerable visual differences between the six images in figure 4.5. In the next section we shall quantify such differences in terms of repeatability indexes.

Figure 4.5. Six images (band 16 = 596.5 nm) acquired over the reference target while flying in different directions, at the same altitude (about 1400 m).

4.2.4 Repeatability Indexes

In this subsection we define several repeatability indexes for quantifying differences between image cubes. Each of these indexes has some advantages and disadvantages in terms of emphasizing different aspect of repeatability.

Repeatability Error:
$$RE = \text{RMS}[\text{signal1} - \text{signal2}] \quad (39)$$

This index quantifies the difference between two signals, by the Root Mean Square (RMS) of their differences. The "signals" in equation (39), are one dimensional vectors. When computing the RE of monochromatic images, e.g. images of single spectral bands, the columns or rows of the respective matrices have to be "chained" i.e. converted to vectors.

It is similarly possible to compute the RE of entire Hyperspectral Image Cube pairs. To this end, each matrix of each spectral band has to be first converted into a vector, by chaining as above. The resulting vectors are then chained to form 2 long vectors, each containing all the data points (Voxels), of an entire image cube. Note that for relatively large image cubes, the memory size of the computer may be a limiting factor.

Ideally, the Repeatability Error should be zero but in practice $RE > 0$ so it may be viewed as “Repeatability Noise”. A noise to signal ratio may then be defined by normalizing RE by the RMS of the union of the two signals:

$$\text{Noise to Signal Ratio:} \quad N/S = \frac{\text{RMS}[\text{signal1} - \text{signal2}]}{\text{RMS}[\text{signal1} \cap \text{signal2}]} \quad (40)$$

Whereby the familiar Signal to Noise ratio is simply:

$$\text{Signal to Noise ratio:} \quad S/N = \frac{\text{RMS}[\text{signal1} \cap \text{signal2}]}{\text{RMS}[\text{signal1} - \text{signal2}]} \quad (41)$$

The Repeatability Error may be similarly normalized by the Standard Deviation (STD), of the union of the two signals, which shall be defined as the “Cross Noise Index” (CN):

$$\text{Cross Noise Index} \quad CN = \frac{\text{RMS}[\text{signal1} - \text{signal2}]}{\text{STD}[\text{signal1} \cap \text{signal2}]} \quad (42)$$

Whereby a Repeatability Index” (RI) shall be defined as:

$$\text{Repeatability Index:} \quad RI = \frac{1}{CN} = \frac{\text{STD}[\text{signal1} \cap \text{signal2}]}{\text{RMS}[\text{signal1} - \text{signal2}]} \quad (43)$$

Another useful index is defined as Percent Absolute Relative Difference (RD):

$$\text{Relative Difference (RD)\%:} \quad RD = \frac{\text{absolute}[\text{signal1} - \text{signal2}]100}{[\text{mean}(\text{signal1}) + \text{mean}(\text{signal2})]/2} \% \quad (44)$$

Wherein, the numerator is the absolute difference between the repeated signals, while the denominator is the mean of the repeated signals.

The N/S and S/N, as well as CN and the RI pairs, convey the same information. Therefore, we shall only use the S/N, RI. and the RD% indexes. Each of these three indexes has some advantages and disadvantages. Therefore they should be used selectively, depending on the target at hand.

Additionally, the mean Relative Difference ($RD_{\text{mean}} \%$), and the maximal values of RD% ($RD_{\text{max}} \%$), were also plotted. The spectral and spatial repeatability plots of $RD_{\text{max}} \%$, are particularly useful for detecting defective CCD cells, if any, wherein some of the values in one or both signals may be very large or close to zero.

The advantage of the signal to noise ratio (S/N), is that it is a familiar concept, well documented in the signal processing literature. However, it is “target independent”. For a given repeatability error

(RE), two targets of widely different spectral content may have the same S/N rating, if their albedo is the same i.e., their mean reflectance, quantified by the $\text{RMS}[\text{signal1} \cap \text{signal2}]$ term in equation (41) is the same.

On the other hand, the repeatability index (RI) is “target dependent” because the spectral variance of the target is quantified by the $\text{STD}[\text{signal1} \cap \text{signal2}]$ term in equation (43). This is advantageous in remote sensing. Generally, the objective is to detect and distinguish between different features or objects in the target image. If the spectral signatures of the different objects in the target image differ widely, they can hopefully be efficiently classified despite a relatively large repeatability error (RE). Conversely, a similar RE may preclude efficient classification, if the spectral signatures of the different objects in the target image are quite similar, as for example in a field crop.

Although the S/N may be similar in both cases, the latter example may not be able to tolerate the same RE without adversely affecting the classification efficiency.

It is instructive to compare the S/N ratio to the RI, in light of the difference between the RMS and the STD:

$$\text{STD}^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 = \frac{1}{n-1} \left[\sum_{i=1}^n X_i^2 - 2\bar{X} \sum_{i=1}^n X_i + \sum_{i=1}^n \bar{X}^2 \right] \quad (45)$$

$$\text{RMS}^2 = \frac{1}{n} \sum_{i=1}^n X_i^2 \quad (46)$$

Where n is the number of elements in the signal vector X_i and $\bar{X}$ is its mean value.

Typically, $\frac{1}{n-1} \approx \frac{1}{n}$ because n is quite large so STD^2 can be closely approximated by

$$\text{STD}^2 \approx \frac{1}{n} \sum_{i=1}^n X_i^2 - 2\bar{X} \frac{1}{n} \sum_{i=1}^n X_i + \frac{1}{n} \sum_{i=1}^n \bar{X}^2 = \frac{1}{n} \sum_{i=1}^n X_i^2 - 2\bar{X}^2 + \bar{X}^2 = \text{RMS}^2 - \bar{X}^2 \quad (47)$$

Assuming that for a given pair of image cubes:

$$\text{STD}[\text{signal1} \cap \text{signal2}] \approx \text{STD}(\text{signal1}) \approx \text{STD}(\text{signal2}) \quad (48)$$

$$\text{RMS}[\text{signal1} \cap \text{signal2}] \approx \text{RMS}(\text{signal1}) \approx \text{RMS}(\text{signal2}) \quad (49)$$

$$\frac{\text{S/N}}{\text{RI}} \approx \frac{\text{RMS}(\text{signal1})}{\text{STD}(\text{signal1})} \approx \frac{[\text{RMS}(\text{signal1})]^2}{\sqrt{[\text{RMS}(\text{signal1})]^2 - \bar{X}^2}} \approx \frac{[\text{RMS}(\text{signal2})]^2}{\sqrt{[\text{RMS}(\text{signal2})]^2 - \bar{X}^2}} \quad (50)$$

Equation (50) quantifies approximately the relationship between S/N and the RI. When $\bar{x}=0$, $S/N=RI$, as may be also seen from equation (45). From equation (50) it is clear that $S/N \gg RI$ and, smaller differences between the RMS and the mean $\bar{x}$, will result in larger $\frac{S/N}{RI}$ values.

Consider an image cube of a uniform target, such as a uniformly illuminated Barium Sulphate panel. In this case, the spectra of all the voxels are approximately the same, i.e. the RMS and the mean $\bar{x}$ off all voxel columns and rows are approximately the same. Therefore $\frac{S/N}{RI}$ is approximately constant so the shapes of the spectral and spatial repeatability curves, quantified by either S/N or RI, shall be very similar. Conversely, in an image cube of a normal target, the spectra of the voxel columns and rows will vary so the ratio $\frac{S/N}{RI}$ will vary, and the spectral and spatial repeatability curves quantified by S/N and RI, shall be quite different. In this case, the RI has a distinct advantage over the S/N ratio.

Suppose for example, that at 950 nm the $S/N=20$ while $RI=1.5$. The value $S/N=20$ means that the average magnitude of the signal is 20 times larger than the average magnitude of the noise. This may or may not be adequate for detecting or distinguishing between the different features or objects in this target image. In contrast, the value of $RI=1.5$ means that the range of the pixel values in the 950 nm band image, is only about 1.5 times larger than the average noise. Thus, on a pixel to pixel basis, the “signal” is practically undistinguishable from the noise, in spite of the $S/N=20$ value.

4.3 Presentation of repeatability tests results

Neither one of the above indexes can adequately represent all the aspects of image cube repeatability, while some of them convey redundant information. Therefore, the presentation of spectral and spatial repeatability shall be summarized by plots of three indexes:

1. Mean (RD_{mean} %) and maximal (RD_{max} %), of the Relative Difference ($RD\%$), defined in equation (44). The plots of RD_{mean} shall depict the mean spectral and spatial repeatability error. The plots of RD_{max} shall depict the maximal spatial and spectral repeatability errors, as well as indicate locations of defective CCD cells, if any.
2. The plots of signal to noise ratios shall depict spectral and spatial repeatability in terms of the “target independent” S/N index defined by equation (41).

3. Plots of spectral and spatial repeatability in terms of the “target dependent” Repeatability Index (RI) defined by equation (43). These plots are particularly useful for selecting the spectral bands, which have the potential for detecting and distinguishing between different features or objects in the target image at hand, with relatively good repeatability.
4. Scatterplots of all the voxels in one image cube versus the other shall depict the Combined Spectral and Spatial Repeatability. The overall value of S/N and RI between the compared image cubes shall accompany these Scatterplots.

4.3.1 Representative examples of repeatability tests results

For the sake of brevity we shall include herein only a small portion of the repeatability tests results. The intent is to demonstrate the problematics of spectral, spatial and overall repeatability, while using an artificial Barium Sulphate target and different modes of illumination in the laboratory tests. Some representative examples of repeatability results of test flights over real targets shall also be presented.

The CASI II imager is a “Push Broom” line scan sensor, incorporating a matrix of Charge Coupled Devices (CCD’s). The image acquisition proper is effected by a portion of this matrix, incorporating a CCD array of 288 rows by 511 columns. In the 48 spectral bands configuration mode, each 6 rows are summed and averaged, so effectively each scan yields a frame of 48 rows (spectral values by 511 columns (spatial coverage). As the airplane flies over the target, the imager scans the target line by line, generating one frame in each scan. The image cube is then composed of any desired number frames along track X (X=number of scans), while each frame is composed of Y=511 columns across track and each column is Z=48 rows deep.

4.3.2 Spectral Repeatability - Barium Sulphate target illuminated by Halogen lamp

In each scan along track, each column of the CCD matrix produces one spectral curve. As the scanning progresses, these curves are "chained" into a long vector that repeats every 48 values. Figure 4.6 depicts two examples of such chained vectors, after 5 scans of the Barium Sulphate target illuminated by the halogen lamp. The top one was generated by CCD column 1 and the bottom by CCD column 256, which is the central column, looking nadir.

Figure 4.6 Example of 5 "chained" spectral curves, top CCD column #1, bottom CCD column #256, each curve is composed of 48 bands readings.

Each of the five repeated curves represents the reflected radiation spectrum from the same voxel across track on the stationary Barium Sulphate target. Since the reflectance spectrum of Barium Sulphate is approximately flat in the spectral range at hand, the shape of these curves is similar to the illumination spectrum of the Halogen lamp.

By visual inspection, it seems that the 5 spectral curves repeat quite well. The question is how well they repeat after a very large numbers of scans, from image to image cube, and is the repeatability the same in all 511 columns of the CCD matrix?

The two top graphs in Figure 4.7 depict an example of spectral repeatability of the leftmost voxel layer along track in two CASI II image cubes, generated by 400 scans.

Figure 4.7 Spectral repeatability of the leftmost voxel layer along track in two image cubes generated by 400 scans of Barium Sulphate target illuminated by Halogen lamp, X=1-400, Y=1, Z=1-48

The coordinates of this voxel layer are: X=1-400, Y=1, Z=1-48, i.e. first line along track with 48 spectral bands. Each scan generates a vector of 48 spectral bands so in each of the 2 compared voxel layers the number of superimposed spectra is 400 and the lengths of the "chained" signals, is $48 \times 400 = 19200$. The differences between the spectra in the compared voxel layers as well as a plot of one voxel layer versus the other are shown in the bottom of Figure 4.7. In this case $S/N=58.35$, $RI=27.13$ and the maximal relative difference between the two voxel layers is $RD=20\%$, as indicated in the figure's caption.

Figure 4.8. similarly depicts the spectral repeatability of the central voxel layer along track, from the same 2 image cubes. The coordinates of this voxel layer are: X=1-400, Y=256, Z=1-48, i.e. this is the nadir central line that runs along track. In this case $S/N=120.8$, $RI=54.69$ and the maximal relative difference between the two voxel layers is $RD=6.9\%$, which is significantly better. Figure 4.9. similarly depicts the spectral repeatability of the last voxel layer, from the same 2 image cubes. The coordinates of this voxel layer are: X=1-400, Y=511, Z=1-48, i.e. this is the rightmost line that runs along track. In this case $S/N=92.39$, $RI=41.98$ and $RD=9.9\%$, which is worse than the nadir line but better than the first line.

Figure 4.8 Spectral repeatability of the central voxel layer along track in two image cubes generated by 400 scans of stationary Barium Sulphate target illuminated by Halogen lamp, X=1-400, Y=256, Z=1-48

Figure 4.9 Spectral repeatability of the last voxel layer along track, in two image cubes generated by 400 scans of stationary Barium Sulphate target illuminated by Halogen lamp, X=1-400, Y=511, Z=1-48

Figure 4.10. Spectral repeatability in two image cubes (511 CCD array columns), generated by 400 scans of the stationary Barium Sulphate target illuminated by the Halogen lamp.

Comparative inspection of Figures 4.7, 4.8, and 4.9, indicates that the spectral repeatability of the nadir voxel layer is much better than the rightmost and leftmost voxel layers.

Since the spectral repeatability of all the 511 columns in the CCD array is not equal, it may be interesting to investigate its variation across and along track. Figure 4.10. shows the spectral repeatability of all the 511 CCD columns from the same two image cubes. It may be seen that the spectral repeatability of the 200-300 central CCD columns, is significantly better than the left and right edge columns. Apart from the outliers, which indicate defective CCD cells, to be considered later, the expected mean RI values at the edges of the CCD are about 45 and about 50-55 in the middle part of the CCD array. Similarly, the signal to noise ratio in the central part of the CCD array is 120 and about 95 at the edges. Similar trends can also be observed in the mean and maximal values of the Relative Difference. This edge effect phenomenon of about 8% drop in all the repeatability indexes at the edges of the CCD array is probably due to aberrations in the optical system. Practically, this means that the reliability of the data at the edges of the imaging swath is significantly lower than at the center, whence the imager is looking nadir. When the imaged target is too big to fit into a single imaging swath it is necessary to mosaic two or more of them.

For best results, the overlap between swathes should be as large as practically possible, to minimize the influence of the edge phenomenon.

4.3.3 Spatial Repeatability - Barium Sulphate target illuminated by Halogen lamp

In the previous section we quantified Spectral Repeatability by looking at the differences between the spectral curves, which are generated by individual columns of the CCD matrix.

In this section we shall quantify "Spatial Repeatability", at every individual spectral band, from scan to scan. In each spectral band, a vector of 511 readings is produced per scan. Since the imaged target is stationary, these vectors should be identical. Their lack of repeatability can be quantified by RI, S/N and RD as in the previous section.

The false color images in Figures 4.11, 4.12 and 4.13, of spectral bands 1, 24 and 48 respectively, clearly exemplify the lack of Spatial Repeatability. The image cube in this example is just 5 scans long, 511 lines wide and 48 bands high. The scaled false colors quantify the reflectance at each image point.

The 5 scans along track are clearly visible in the top two images of these figures, which indicates that there is a definite lack of repeatability from scan to scan. If all the scans were equal, there would be no differences between them so they would be indistinguishable. The images would simply be composed of continuous lines, each of the same color. Colors variations between lines can be the result of uneven illumination or local surface variations of the imaged panel. However, repeated scans along each line should produce the same values. Differences from scan to scan at each and every spectral band create the dissimilarity between consecutive image cubes.

The difference images in the lower left parts of Figures 4.11, 4.12, and 4.13 depict the distribution of the differences within each scan line and between the 511 columns, which is also quantified by the RD and RDmax values in the figure captions.

The scatter plots in the lower right parts of these figures further emphasize the differences between the images of the respective bands. As may be expected, from the results of the previous section, the repeatability is best in the central band 24, worst in the first band and somewhat better in the last band 48. As the number of scans increases, the probability for mismatches between scans increases, which further degrades spatial repeatability. This may be observed in the scatter plots of figures 4.14, 4.15 and 4.16, which are similar to figures 4.11, 4.12 and 4.13, only here each image includes 400 scan lines instead of 5.

Figure 4.11 Five scans of Barium Sulphate target using Halogen lamp illumination (band 1)

Figure 4.12 Five scans of Barium Sulphate target using Halogen lamp illumination (band 24)

Figure 4.13 Five scans of Barium Sulphate target using Halogen lamp illumination (band 48)

The presentation format in figures 4.11, 4.12 and 4.13 is preferable when the number of scans is small, while the presentation format in figures 4.14, 4.15 and 4.16 is preferable when the number of scans is large.

The difference images in the lower left parts of Figures 4.11, 4.12, and 4.13 depict the distribution of the differences within each scan line and between the 511 columns, which is also quantified by the RD and RDmax values in the figure captions. The presentation format in figures 4.11, 4.12 and 4.13 is preferable when the number of scans is small, while the presentation format in figures 4.14, 4.15 and 4.16 is preferable when the number of scans is large.

Figure 4.14. 400 scans of Barium Sulphate target using Halogen lamp illumination (band 1).

Figure 4.15. 400 scans of Barium Sulphate target using Halogen lamp illumination (band 24).

Figure 4.16. 400 scans of Barium Sulphate target using Halogen lamp illumination (band 48).

Figure 4.17. 400 scans of Barium Sulphate target using Halogen lamp illumination (entire spectrum).

When $RI \leq 1$, and $S/N \sim 10$ as in figure 4.14, the variance of the signal is about equal to or larger than the RMS of the noise. Thus, if the RI value of a spectral band is close to 1, this band is too noisy for any kind of predictive modeling.

The best repeatability performance was recorded in the central spectral band (426.7 nm) as exemplified in figure 4.15, wherein the values of $RI=5.8$, $S/N=155$ and $RD_{max}=2.8\%$ are adequate for prediction modeling. Generally, our experience indicates that for reasonably good prediction the repeatability indexes should be approximately: $RI \sim 5$, $S/N \sim 100$ and $RD_{max} \sim 3\%$. Bands with lesser repeatability should be either summed and averaged for improved repeatability or excluded from the analysis.

Figure 4.17 depicts the repeatability indexes over the entire spectrum of the same 400 scans of the Barium Sulphate target using Halogen lamp illumination. Using the above limiting guidelines for required repeatability indexes it seems that for adequate prediction modeling the usable bands range is about 550 – 930 nm. Bands outside this range should be either summed and averaged for improved repeatability or excluded from the predictive model.

Comparing the Spectral and Spatial Repeatability indexes, in figures 4.10 and 4.17, one observes that the mean values are $RI=50.8$ and $RI=6.375$ while $S/N=112$ and $S/N=116.1$ respectively.

One may ask why the S/N values are about the same while RI is much larger in spectral repeatability?

The answer to this question is in the different definitions of S/N and RI in equations 41 and 43 respectively wherein the denominators are the same RE from equation 39. The mean value the image signal quantified by the numerator $RMS[\text{signal1} \cap \text{signal2}]$ is approximately the same in spectral and spatial repeatability so the value of S/N remains about the same. Conversely, the standard deviation of the image signal $STD[\text{signal1} \cap \text{signal2}]$ is quite different in spectral and spatial repeatability because the images we are looking at are different. In this example, the spectral repeatability image signal is effectively composed of the chained spectra of the Halogen lamp while in spatial repeatability it is composed of the essentially uniform Barium Sulphate target wherein the STD at each spectral level is much smaller. Had we used a different target with diverse features who's mean reflectance is similar to the Barium Sulphate target but the STD is larger, then the S/N would remain the same while the differences between RI in spectral and spatial repeatability would decrease. Therein lies the main advantage of the RI which is dependent on the diversity of the imaged target while the S/N ratio is only dependent on its mean reflectance.

Field models are usually image based, wherein we are primarily interested in Spatial Repeatability. Spatial Repeatability is of lesser interest but it is helpful in assessing the overall performance of an imaging system. Additionally, we may use a scatter plot of all voxels of one image cube versus the other, as exemplified in figure 4.18. This plot quantifies the overall repeatability of entire image cubes and is effectively a combined spectral and spatial repeatability. The mean values of $RI=51.01$, $S/N=111.9$ and $DR_{max}=109.5$ are similar to their counterparts in Spectral Repeatability, (figure 4.10). In this case the overall repeatability is dominated by the variance of the illumination spectra while the variance of the reflectance from the uniform Barium Sulphate target was much smaller.

Figure 4.18. Repeatability of entire image cubes, 400 scans of Barium Sulphate target using Halogen lamp illumination. Effectively, this is the combined Spectral and Spatial Repeatability.

Another useful piece of information that may be gleaned from figure 4.18 is that the repeatability of the extremely smaller and larger reflectance values is significantly less than in the midrange. Lesser repeatability of the low reflectance values is understandable because there the signal is low. Why the high reflectance values are less repeatable is not clear.

Table 3 summarizes the repeatability indexes of the above two image cubes created by 400 scans of the stationary Barium Sulphate target illuminated by a halogen lamp. Similar results were obtained in the second test wherein sunlight was used for illuminating the Barium Sulphate target. Table 4 summarizes the repeatability indexes of the resulting image cubes, which were created by 400 scans

as before. The third test was conducted in the same way as the second test except a time delay of 5 minutes was introduced between the acquisitions of the two image cubes.

Comparing the values in tables 4 and 5 one observes that the repeatability is significantly degraded by the 5 minutes delay between the acquisitions of the two image cubes. Since the imager and the target were stationary and the atmospheric conditions hardly changed in 5 minutes, the increased difference between the two image cubes is only due to the change in the sun's illumination. These test were conducted at about 10 AM local time, whence the sun's zenith and azimuth angles change quite rapidly. Therefore, in the 5 minutes delay between the image acquisitions, the illumination of the target changed considerably as demonstrated by the difference between the two image cubes. Similar changes were observed in the test flights as described in the next section.

With sunlight illumination, the RI was always much larger in spectral repeatability than in spatial repeatability, as may be observed by comparing the corresponding RI values in tables 4 and 5. The same trend was observed with the Halogen lamp illumination in table 3. This is due to the nature of the flat reflectance characteristics of the uniform Barium Sulphate target wherein the variance is much smaller than the variance of the sunlight illumination spectrum. In contrast, the S/N ratio, is approximately the same in spectral and in spatial repeatability. This is because on the average, the RMS magnitudes of the signals and the noise are about the same in spectral and spatial repeatability, hence so is their S/N ratio.

Note that the signals of vastly dissimilar images with different variances may have the same mean values which are quantified by their RMS. Since the S/N ratio is insensitive to the images variances, which quantifies their spectral and spatial composition, it is effectively "Image Independent". In hyperspectral image analysis this is a distinct disadvantage.

From tables 3, 4 and 5, it may be seen that both the RI and S/N ratio are generally in agreement with regard to the shapes of the spectral and spatial repeatability curves, as may also be observed in figures 4.10 and 4.17. They both indicate that the repeatability in the central part of the CCD matrix is much better than on its spectral and spatial edges.

It is interesting to observe that the combined spectral and spatial repeatability indexes of the entire image cubes created by the Halogen lamp illumination were $RI=51.01$, $S/N=111.9$ while by sunlight they were $RI=29.08$, $S/N=139.08$.

Table 3. Summary of repeatability indexes of two image cubes created by 400 scans of stationary Barium Sulphate target illuminated by halogen lamp

Spectral Repeatability				Spatial Repeatability			
CCD #	RI	S/N	RDmax%	W nm	RI	S/N	RDmax %
1	27.13	58.35	20.0	426.7	1.08	11.7	46.0
255	54.69	120.08	6.9	687.1	8.11	155.0	2.8
511	41.98	92.39	9.9	965.1	3.78	63.4	11.0
ALL	50.80	112.00	103.0	ALL	6.37	116.1	103.0
Repeatability of image cubes RI=51.01, S/N=111.9, RDmax=109.5%							

Table 4. Summary of repeatability indexes of two image cubes created by 400 scans of stationary Barium Sulphate target illuminated by sunlight.

Spectral Repeatability				Spatial Repeatability			
CCD #	RI	S/N	RDmax%	W nm	RI	S/N	RDmax %
1	15.80	79.7	9.7	426.7	3.63	70.8	9.8
255	30.85	153.1	5.0	687.1	12.20	199.0	3.3
511	22.02	104.7	5.8	965.1	3.48	53.5	15.0
ALL	28.40	141.0	7.5	ALL	9.13	155.6	15.0
Repeatability of image cubes RI=29.08, S/N=139.08, RDmax=10.37%							

Table 5. Summary of repeatability indexes of two image cubes created by 400 scans of stationary Barium Sulphate target illuminated by sunlight, (5 minutes delay between image acquisitions).

Spectral Repeatability				Spatial Repeatability			
CCD #	RI	S/N	RDmax%	W nm	RI	S/N	RDmax %
1	6.66	33.5	12.0	426.7	2.62	46.8	11.0
255	26.45	131.4	4.4	687.1	4.79	71.9	5.3
511	8.31	39.5	9.6	965.1	3.06	42.8	15.0
ALL	16.00	79.8	12.0	ALL	4.09	63.4	14.6
Repeatability of image cubes RI=13.22, S/N=62.99, RDmax=13.27 %							

The RI decreased because the sun's illumination of the target was much more uniform than by the Halogen lamp. The Halogen lamp illuminated the center of the target much more than its edges. So the variance of the resulting image cubes and the RI was much larger than by sunlight illumination. The S/N ratio increased because the sunlight illumination was much brighter than by the halogen lamp, which increased the RMS of the signal. This example reiterates the main difference between the RI and the S/N ratio.

4.3.4 Detection of Defective CCD cells.

As a byproduct of the above procedures, we can also detect the coordinates of defective cells in the CCD array. The maximal values of the Relative Difference index RD_{max} are particularly sensitive in detecting extreme differences between single values in long vectors of chained spectra. This is exemplified in Figure 4.17, wherein outlier values of over 60% and 100% were detected at spectral bands 3 and 22, (450.2 and 665.2 nm) respectively. Note that such outliers do not appear in every scan but rather randomly. In figure 4.17, the first one appeared before scan 230 while the second appeared after scan 230. Similar outliers are also found in the Spectral Repeatability graphs, as exemplified in figure 4.10. In this case the outliers were detected in CCD columns 291, 295, 299 and 303, and all of them are located on CCD line 22, as shown in figure 4.17. These defective CCD cells are located close to the nadir column and the central spectrum line, which is particularly bothersome.

At each scan, our program checks for such outliers and records their columns and rows. Apart from the above, we also found defective CCD cells in columns 1 and 2 at row 44, as well as column 1 at row 2. The locations of these defective pixels, is less detrimental because we can discard CCD columns 1 and 2 from the image without much loss.

It is quite possible that additional defective CCD cells exist in this array, cells that were not detected. To detect all defective CCD cells one must acquire and analyze much longer images, perhaps several thousand scans long.

4.3.5 Results of repeatability in tests flights

Consecutive flights were conducted over the reference target, which includes a helipad, a piece of tarmac and adjacent grass on the edge, as described in section 4.1 and in figure 4.4. The results of these flights are summarized in figure 4.19. The flight's times were close to solar noon and in clear skies, while flying at the same altitude, speed and using the same CASI II parameters. The times in the captions indicate the acquisition times of the plotted image cubes. The flight's direction headings are indicated on the respective axis.

Each one of the 4 graphs in figure 4.19 is a plot of 2 consecutively acquired image cubes, i.e. all the pixels in all the spectral bands of the 2 image cubes are plotted, one versus the other. If the values of all the pair-wise plotted pixels were the same, the scatter plots would converge to a 45 degree

diagonal line. Clearly, the amount of deviation from the 45 degree line is proportional to the lack of repeatability.

Observe that this deviation was smaller, when both image cubes were acquired while flying on a "Sun Lineup Heading" i.e. flying away from the sun along the sun's azimuth trajectory, wherein the sun, airplane and the target are lined up. This is due to the directional nature of the solar illumination.

When the flight direction is not the same, the illumination of the target by the sun, in relation to the position of the airborne imager is changed, so the recorded reflectance patterns of the image cube are different.

Figure 4.19. Combined Spectral and Spatial Repeatability of entire image cubes from test flights over the reference target, flying on various headings but at the same altitude, speed and using the same CASI II parameters.

The time intervals between the 4 consecutive flight pairs summarized in figure 4.19, were 6, 3, 4 and 85 minutes respectively. The corresponding repeatability indexes were RI(4.06, 2.83, 2.47, 3.78) and S/N(11.5, 8.0, 7.08, 10.7) respectively. The first and last flight pairs were along the sun lineup heading wherein the repeatability indexes were significantly better. Note the very large time delay between the last flights pair, yet the repeatability indexes there are better than in the very short time delays in flight pairs 2 and 3, where the headings were not along the sun's azimuth line.

Figure 4.20. Repeatability indexes from sun lineup headings flights over sugarbeet field B1123.

Figure 4.21. Repeatability indexes from North South headings flights over sugarbeet field B1123.

However, even in the sun lineup headings flights the repeatability indexes were much lower than in the stationary image cube acquisitions of the Barium Sulphate target summarized in tables 4 and 5. This is primarily due to the perturbations in the airplanes flight parameters, as well as the scattering and attenuation influence of the atmosphere.

Significantly better results were obtained in consecutive image cube acquisitions of a sugarbeet field (B1123), as summarized in figures 4.20 and 4.21. In the sun lineup headings the combined repeatability indexes were better than in the North South headings as expected, $RI=15.3$, $S/N=27.8$ and $RI=8.93$, $S/N=16.3$ respectively.

Though the time gaps between the image cube acquisitions were 5 and 4 minutes respectively, similar to the test flights over the reference target, in both sugarbeet test flights there is a definite repeatability improvement. This is mainly due to the difference between the spectral characteristics of the reference target and the sugarbeet field.

The main component of the reference target was dark asphalt and a relatively small area of bright cement with some vegetation on the edges, e.g. see figure 4.5. The spectral variance and signal mean of the asphalt and cement is quite small in comparison to the spectral variance and signal mean of the green vegetation, e.g. see figure 4.4. In contrast, the spectral content of the sugarbeet field is of pure vegetation, hence the significant improvement in repeatability indexes.

Note that the Spatial Repeatability curves drop off at the low and high ends of the spectrum, similarly to their counterparts in the stationary target imaging tests, e.g. figure 4.17. However, here the Spatial Repeatability curves do not peak in the central part of the spectrum. Instead, there is a big dip around 700 nm. This is due to the characteristically steep “Red Edge” slope in the spectral reflectance curves of green vegetation. The repeatability of the steep Red Edge slope is typically quite low, so even small perturbations cause large variations in the reflectance values at these wavelengths. The practical significance and *modus operandi* to overcome this difficulty shall be discussed in the next section.

5. Field maps of growth attributes by hyperspectral imaging.

In this section we describe the methodology of using hyperspectral images from remote sensing, for deriving predicted field maps of growth attributes that are of interest in precision cropping. Although the line taken is quite general it is best explained by an example. To this end we shall use one of the

sugarbeet fields of this project wherein predicted maps were derived for beet yield, leaf nitrate, sugar content in the beets and biomass.

The ground reference data gathered for the crop's growth attributes came from hand samples and a yield monitor mounted on a commercial sugarbeet harvester. In a 23 hectare field hand sample locations were determined by using an offset grid with no sample points within 25 meters of the edge of the field to eliminate any effect due to overlap in tillage, spraying or fertilizing operations. There were approximately 2 sample locations per hectare. At each one of the sample locations a five foot (1.52m) section of two adjacent rows was hand harvested. The above ground biomass was separated from the tubers in this operation. Weeds were kept to a minimum for commercial fields but if any were present in the sample area, they were included in the above ground biomass. The row spacing was 2 feet (0.61m), so the total area sampled was 20 ft² (1.86 m²). This sample area was chosen because the local sugar factory that did some of the sample analysis had used this sample size for many years.

Biomass is the weight of the above ground plant material after being dried in an oven and is reported in tons per hectare. It includes all above ground biomass including beet tops, the small amount of crown tissue to the lowest leaf scar and any weeds if they are in the sample area.

Leaf nitrate is determined by drying and then grinding the above ground biomass and analyzing the dried material for the total Nitrogen content. The values of N reported by the LECHO analyzer are expressed in parts per million (PPM) of dry matter.

The sugar content in the beets at each sampling location was determined by the local sugar factory's analysis lab. The sample is weighed, washed, weighed, hand crowned to remove any tissue above the last leaf scar and then weighed again. The cleaned and crowned sample is diced and analyzed for sugar content. The result is expressed as a percent sugar (sucrose) of the clean sample weight on an "as is" moisture content.

Two methods were used for determining the beets yield, i.e. by hand sampling and by a yield monitor on the harvester. The yield monitor provided a value in t/ha for each pixel in the yield monitor map. The yield monitor data is derived from a pair of load cells located under the truck conveyor that weigh all of the material going into the truck. This included the defoliated sugarbeet tubers as well as a small amount of dirt that is not removed by the cleaning action of the harvester. The beets were scalped by the defoliator that preceded the harvester but some additional crown tissue will be removed before the samples were analyzed. This crown tissue and the remaining dirt that is washed

off before analysis averaged about 7.5% of the gross weight recorded by the yield monitor. This is in contrast to the hand sample weight which was the net weight of the cleaned beets.

The yield monitor references the weight recorded by the load cells every second to the location in the field by using a real-time differentially corrected Global Positioning System (GPS). The GPS system is capable of sub-meter positional accuracy. The width of the harvested area is 8 feet (2.44m) and at a speed of 3.5mph (5.63 km/hr) the distance covered in one second is 5.13 feet (1.56m). Therefore the minimum area discernable by the yield monitor is 41 ft² (3.81m²). It should also be remembered that only the material on the load cells at the reading instant is recorded. That is, not all the beets from the entire 41ft² area are collected and weighed. There is a time lag of about 10 seconds from the time the beets are dug from the ground until the time they reach the truck conveyor where they are weighed. This can introduce some positional error, especially at the ends of the field where speeds change. The software that processes this information fits the data into pixels aligned with the X and Y axis. This pixel size is typically 5m.

The yield results from the hand samples do not compare well to the yield monitor values. Several factors contribute to this discrepancy. The first and foremost issue is that the hand samples are taken from an area of 1.86m² which may be too small an area to accurately capture the location's average yield. This area for sample size was chosen because we wanted our sample size to match that of the sugar factory which analyzed our samples. The root sample from this area would often weigh 10 to 14kg. Sugarbeet crops are subject to many pests which can be very localized. A spring cutworm infestation can reduce the plant population significantly when it migrates down the seed trench destroying several seedling beets in a row. Its actions do not affect the rows on either side, so the yield of the adjoining rows may be 50 t/ha while a 0.5m area within the sample area may have a yield of 0.0 t/ha. Another sugarbeet production problem is *Rhizoctonia* crown and root rot. It is caused by the fungus, *Rhizoctonia solani* AG 2-2 and is a disease which affects both seedlings and older plants. It is typically very spotty and is highly localized, again giving the potential for a normal yield in one row and yet 1 meter away no harvestable yield at all. Other reasons for the differences between hand sample and mechanical harvest is that the hand samples possibly retain more small beets. When digging the hand samples the people taking the samples have to make a judgment call and discard the beets that they feel are too small to be retained in the mechanical harvester. In normal yielding areas there are not a significant number of these small beets, but in poor yielding areas the number of beets of marginal size can be 40% of the entire sample. It is impossible to accurately predict exactly how

many of these will be harvested because variables such as tip breakage due to soil conditions and where they enter the harvester will influence how many beets actually make it across the load cells for the yield monitor. These reasons help explain that while neither the hand sample or the yield monitor is perfect the very large number of points gathered by the yield monitor and the integrating effect of the larger pixel size leads to a truer representation of yield than the hand sample data. It was therefore decided to prefer the yield monitor data for generating the reference beets yield distribution maps, as described next in section 5.1.

The objective is to develop regression models that may be used for generating predicted field maps of the above growth attributes, from image cubes derived by hyperspectral remote sensing.

We shall use the interpolated prototype plot maps of these growth attributes derived in sections 2.4 and 2.5, as “ground truth reference”. Recall that the yield reference map was derived from the yield monitor of the sugarbeet harvester while the reference maps of Leaf Nitrate, Percent Sugar Content and Biomass were interpolated from rather sparse hand sampling. Therefore, the definition “ground truth reference” should not be taken literally. It is just the best available estimate of these quantities, while their true distribution in the field was not known.

Each voxel of a hyperspectral image cube has a unique reflectance spectrum (“spectral signature”), which ideally should identify its physical content. This idea is borrowed from “Chemometrics” methodology, whereby the chemical composition of a solution in a test tube may be modeled from spectroscopy data. While chemometrics models employ transmitted light absorption spectra, we proposed to use reflected light spectra derived by hyperspectral imaging. The most popular regression model in chemometrics, which we adopted, was termed Partial Least Squares PLS. The PLS model was particularly suitable in our case because it can deal with noisy and redundant multi variable data while maximizing the covariance between the reference and predicted datasets. It was also convenient because it is integrated in Matlab via the PLS toolbox (Wise and Gallagher 1998).

The usage of PLS in chemometrics methodology was simplified by using a constant precisely known illumination light spectrum while the image acquisition was conducted in a controlled laboratory environment. In contrast, the reflected spectra in remote sensing are acquired under highly variable illumination and environmental conditions. This renders the acquired image cubes quite non repeatable, as described in the previous section.

To alleviate this problem we adopted a double-flight procedure whereby two image cubes were acquired, by over flying the same field twice in rapid succession. This afforded computation of the

spatial repeatability index RI whereby highly non-repeatable spectral bands were discarded.

Additionally, we developed a “Genetic Algorithm” (GA) based feature selection procedure, which was integrated with PLS into a combined PLS-GA model.

The process of non-repeatable feature elimination by the RI and the feature selection by the PLS-GA model proceeds as follows:

1. The first, second and third derivatives are computed from each voxel spectrum, in each of the two consecutive flights. This produced two vectors of 186 spectral features, one from each flight for each voxel, (e.g. 48 spectral bands + 47 first derivatives + 46 second derivatives + 45 third derivatives = 186 spectral features).

2. The RI of all 186 spectral features is then computed and highly non-repeatable features are discarded, (say when $RI < 5$).

3. One of the two remaining spectral features vectors is used as a “Training Set” in the PLS-GA procedure while the other (“Validation Set”) is used as “new data”, for validating the model.

The training stage comprises “generations”, whereby subset features vectors are iteratively selected by the GA. The cross-validation error of each selected features vector is then computed by the PLS regression model. The resulting cross validation errors, termed “fitness”, are used to determine which features vectors are discarded and which survive and continue to the next generation. At each generation, the surviving features vectors are mated by the cross-over and mutation processes, similar to genes in chromosomes of plants and living organisms, (e.g. Holland 1992). After about 500 generations the process produced the parameters for a PLS model, which significantly reduced the number of features and associated cross validation error.

In the validation stage, the features and regression model parameters, which were determined by the PLS-GA procedure in the training stage, are applied to the validation data set.

This yields an estimated value for each pixel in the predicted growth attributes map. The prediction errors are then computed by the differences between the pixels of the reference and predicted growth attributes maps.

Recall that a reference map is just an estimate of the true distribution of a growth attribute in the field. Therefore, minimizing the difference between the reference and predicted growth attributes maps is not necessarily the most optimal solution. In some cases, it may be assumed that both maps are gradual excursions about a mean uniform map, which may be represented by the field mask. In such cases,

the FFT regression method with the “phase lock option” may be beneficially employed, as described in section 3.7.

Rather than minimizing the difference between a reference and predicted growth attributes map, this approach produces a “Close Consensus Map”, (CCMap) which is akin to a compromise between the reference and predicted maps. In some cases the CCMap is probably closer to the true distribution of the modeled growth property than the reference map. Only extended experience can indicate if this is indeed the case.

Some practical examples are presented in the next sections. These examples demonstrate the above *modus operandi*, by deriving predicted growth attributes maps in a sugarbeet field.

5.1 Predicted sugarbeets yield distribution map.

The false color images in figure 5.1 depict yield distribution maps in a sugarbeets field. The top left image depicts the raw yield distribution map derived from the yield monitor of the sugarbeets harvester. Due to some technical problems in the operation of the yield monitor, this raw yield distribution map contains some small area patches wherein the yield data was missing. Accordingly, the missing data pixels had to be filled in by inverse distance interpolation, which produced the corrected yield monitor image, top right. Applying the pixel block transform to the corrected yield monitor image, as described in section 3.5, stratified the yield map into prototype plots, depicted in the lower left image.

The final stage removes the fragmented pieces of prototype plots around the field’s periphery, by setting seven consecutive peripheral pixel rings to zero. This produced a yield map of 203 whole prototype plots while removing from the map a peripheral field area of about 15 meters. This was done because the yield monitor data is error prone at the edges of a field due to the repeated overlapping turns of the harvester machine. Each prototype plot was composed of a 15x15 pixels block (2m pixels), i.e. its size was 30x30 meters, as depicted in the lower right image.

The objective was to use the spectral information which was captured in this field’s image cube, for deriving a predicted sugarbeet yield map that closely resembles the reference map. In the multi-spectral stage of the project it was determined that the optimal image acquisition time window for sugarbeet yield prediction was August, (e.g. section 3.12). Accordingly, in this example we use an image cube that was acquired on August 31 2001.

Figure 5.2 depicts the maximal, average and minimal spectra of reflectance and its first, second and third derivatives in the image cube of the above sugarbeet field. The reflectance spectra (top left graphs) were derived from the raw radiance by the procedure described in section 4.1. Note that when the radiance values are close to zero the procedure may produce slightly negative reflectance values, as seen around 680 nm. Strictly speaking, negative reflectance has no physical meaning, so one may want to set such negative reflectance values to zero.

Figure 5.1. Sugarbeets harvester yield monitor maps, tons/hectare.

Figure 5.2. Maximal, average and minimal spectra of reflectance and its first, second and third derivative in image cube of sugarbeet field.

Figure 5.3. Repeatability Index spectra of reflectance and its first, second and third derivative in image cube of sugarbeet field.

This was not done, because we want to use the reflectance as a feature in a regression model, wherein its physical meaning is immaterial. Therefore, as long as these negative values are sufficiently repeatable, they may be included in the model. Apart from that, the general shape of these spectra is quite typical of green vegetation.

From the differences between the maximal and minimal values of these spectra, it may be seen that the range of their variation in the entire image cube is quite small. This was expected since it comes from an image cube of an essentially a uniform green vegetation field; however, it severely limited the available information (variance) for modeling. Including the first second and third derivatives in the feature set may hopefully enrich the scope of potentially pertinent information that could be useful for prediction modeling. However, adding to the model noisy information may do more harm than good. Therefore, spectral bands of relatively low repeatability must be excluded from the model, as demonstrated in figure 5.3. The horizontal lines depict the limiting thresholds that were used to discard non repeatable bands. Thus, using the indicated thresholds, only 8, 6 and 3 bands from the first, second and third derivatives respectively, were included in the model. Similarly, the first and the last 3 reflectance bands were excluded from the model. Thus the initial total number of features presented to the PLS-GA model, was 61. This is indicated by the variable number in the lower right bar graph in figure 5.4, which reports the progress of the PLS-GA algorithm at generation 600.

The upper left graph in figure 5.4 depicts the various recorded levels of “fitness” (cross validation error) at generation 600, while using 15, 16, 17 and 18 variables. It may be seen that the best fitness was 3.7, which was achieved by using 17 variables. Thus, from the initial 61 variables only 17 were selected, by the PLS-GA algorithm. The selected variables are shown by the tall bars (lower right bar graph), which indicates that these variables were included in about 30 of the considered PLS models at generation 600. The short bars indicate the variables that were included in just a few models and absence of bars indicates the variables that were not included at all.

The mean and best fitness graphs in the upper right of figure 5.4 indicate a rapid improvement in fitness until generation 70 approximately.

After that the fitness fluctuates rather randomly between 3.7 and 3.8 while the number of variables in the model hovers between 24 and 25, as shown in the lower left graph of figure 5.4. At about generation 380 the number of variables dropped to about 15 – 18 while the best solution was 17 variables and fitness 3.7.

Figure 5.4. Progress report of the PLS-GA algorithm at generation 600.

Figure 5.5. Sugarbeets yield - Repeatability of training and validation features (top bar-graph), multipliers of features selected by the PLS-GA algorithm (bottom bar-graph).

Figure 5.6. Sugarbeets yield reference and prediction maps - training flight.

Figure 5.7. Sugarbeets yield reference and prediction maps – validation flight.

The fitness value, i.e. the root mean square error of cross validation RMSE_{cv} , should be viewed in terms of the range of the modeled property. In this case it is the yield, which ranged from zero to 55 tons/hectare, whence the cross validation error was $100(3.7/55) = 6.7\%$ of the maximal yield.

The top bar-graph in figure 5.5 shows the repeatability of all the features, while the bottom bar-graph quantifies the regression model's multipliers, for the 17 yield predicting features selected by the PLS-GA algorithm. It may be seen that most of the selected 17 features have relatively high repeatability. Thus, in addition to minimizing the cross validation error, the PLS-GA algorithm also prefers features with higher repeatability. This is so because in operation, the cross validation algorithm randomly constructs many sub-training and sub-validation sets. On the average, features of higher repeatability will tend to produce lower cross validation errors hence they are preferred.

Figures 5.6 and 5.7 depict the resulting sugarbeet yield reference and prediction maps in the training and validation flights respectively. The scatter plots in the lower left parts of the figures depict the closeness of the predicted readings in the 203 prototype plots, to the 45 degrees line. If all the predicted readings were exactly equal to the reference readings all points would fall on the 45 degrees line exactly. The excursions of the readings about this line give a measure of the errors. The errors are also quantified by the listed correlation coefficients r and the coefficients of determination R , e.g. $r=0.85$, $R=0.85$ and $r=0.73$, $R=0.88$, in the training and validation flights respectively.

Additionally, the scatter plots in the lower right parts of the figures depict the distribution of the errors amongst the 203 prototype plots. It may be seen that in both the training and the validation flights, the errors are randomly distributed about the zero line, as required. The listed root mean square values of the residual errors RMS_{res} were 3.4 and 4.5 in the training and validation flight respectively.

As in the case of the cross validation error the RMS_{res} , should be viewed in terms of the range of the modeled property. In this case it was the yield, which ranged from zero to 55 tons/hectare, whence $100(3.4/55)=6.2\%$, $100(4.5/55)=8.2\%$, of the maximal yield in the training and validation flights respectively. Note that the red lines delimit a $\pm 2\text{RMS}$ range which includes about 95% of the errors population, assuming that the errors are normally distributed, e.g. $\pm 6.2 \times 2 = \pm 12.4\%$ $\pm 8.2 \times 2 = \pm 16.4\%$, in the training and validation flights respectively. This result is quite good, given the rather low repeatability of the features, as depicted by the top bar-graph in figure 5.5.

Visually, the two predicted yield maps from the training and validation flights appear to be similar to the reference map, while the small differences between them are quantified by the above RMS_{res}

values. The underlying assumption in computing these RMSres values is that the reference yield map is absolutely correct and represents the true yield distribution in the given field. Factually, the reference map is just an estimate of the true yield distribution in the field. Assuming that both maps are gradual excursions about a mean uniform yield map, which may be represented by the field mask, the FFT regression method with the “phase lock option” may be beneficially employed, as described in section 3.7.

Proceeding so we obtained the “Close Consensus Maps”, (CCMap) for the training and validation flights, as depicted in figures 5.8 and 5.9, which are akin to a compromise between the reference and predicted maps. The derivation of the CCMap effectively comprises a sensor fusion approach wherein the agreement between the Yield monitor and remote sensing data is amplified while the disagreement is attenuated. Therefore it is probably closer to the true yield distribution than the reference map. However, only extended experience can verify if this is indeed the case at hand. By visual comparison between the figure pairs 5.6, 5.8 and 5.7, 5.9 it is quite obvious that the similarity between the reference and predicted maps is closer in the latter. This is also evident from the much lower values of RMSres=1.9 and of RMSres=2.2, as listed in figures 5.8 and 5.9, for the training and validation flights respectively.

Note that the price paid for lowering the RMSres is a reduction of the readings range, e.g. from about 20-55 tons/hectare to about 30-50 tons/hectare.

The corresponding error percentages are $100(1.9/50)=3.8\%$, $100(2.2/50)=4.4\%$ for the training and validation flights respectively. These should be compared to the above $100(3.4/55)=6.2\%$, $100(4.5/55)=8.2\%$ values, that were obtained without sensor fusion.

Note that these results are similar to the results obtained in section 3.7 wherein similar yield maps were derived by NDVI from a 3 camera (R, G, NIR) system on August 30.1999.

There we had an error percentage of $100(7.8/75)=10.4\%$ by regular regression and $100(3.68/70)=5.3\%$ by FFT regression with the phase lock option.

In both cases FFT regression with the phase the lock option halved the error percentage, but in all cases hyperspectral imaging produced significantly lower error percentages than by the NDVI.

Figure 5.9 Sugarbeets yield reference and prediction maps of “Close Consensus” obtained by FFT regression with phase lock option – training flight.

Figure 5.9 Sugarbeets yield reference and prediction maps of “Close Consensus” obtained by FFT regression with phase lock option – validation flight

Figure 5.10 Reference Leaf Nitrate distribution maps in sugarbeets field

Figure 5.11 Leaf Nitrate - Repeatability of training and validation features (top bar-graph), multipliers of features selected by the PLS-GA algorithm (bottom bar-graph).

5.2 Predicted Leaf Nitrate distribution map.

The procedure for deriving the predicted Leaf Nitrate distribution map is essentially the same as for the yield distribution map. The only difference is that the Leaf Nitrate reference map was derived by hand sampling and minimal curvature interpolation, while the yield reference map was obtained from the yield monitor of the sugarbeets harvester.

The false color images in figure 5.10 depict the Leaf Nitrate distribution maps in the above sugarbeets field. The top left image depicts the hand sampling locations in the field. The false colors scale the Leaf Nitrate in Parts Per Million (PPM) at the indicated locations. The number of sampling points was 71, wherein the Leaf Nitrate PPM was determined as described in section 2.3. The top right image depicts the interpolated Leaf Nitrate map derived by minimal curvature interpolation, as described in section 2.5. Applying the pixel block transform to this image, as described in section 3.5, stratified the Leaf Nitrate map into prototype plots, depicted in the lower left image. The final stage removes the fragmented pieces of prototype plots around the field's periphery, by setting seven consecutive peripheral pixel rings to zero. This produced the reference Leaf Nitrate map of 233 whole prototype plots while removing from the map a peripheral field area of about 15 meters. Each prototype plot was composed of a 15x15 pixels block (2m pixels), i.e. its size was 30x30 meters, as depicted in the lower right image. The objective is to use the spectral information which was captured in this field's image cube acquired on August 31 2001, for deriving a predicted Leaf Nitrate map that closely resembles the reference map.

The top bar-graph in figure 5.11 shows the repeatability of all the features, which is the same as in figure 5.5. The bottom bar-graph quantifies the regression model's multipliers, for the 28 Leaf Nitrate predicting features selected by the PLS-GA algorithm.

Figures 5.12 and 5.13 depict the resulting Leaf Nitrate reference and prediction maps in the training and validation flights respectively. The scatter plots in the lower left parts of the figures depict the closeness of the predicted readings in the 233 prototype plots, to the 45 degrees line.

If all the predicted readings were exactly equal to the reference readings all points would fall on the 45 degrees line exactly. The excursions of the readings about this line give a measure of the errors. The errors are also quantified by the listed correlation coefficients r and the coefficients of determination R , e.g. $r=0.77$, $R=0.77$ and $r=0.62$, $R=0.84$, in the training and validation flights respectively.

Figure 5.12 Sugarbeets Leaf Nitrate reference and prediction maps - training flight.

Figure 5.13 Sugarbeets Leaf Nitrate reference and prediction maps - validation flight

Additionally, the scatter plots in the lower right parts of the figures depict the distribution of the errors amongst the 233 prototype plots. It may be seen that in both the training and the validation flights, the errors are randomly distributed about the zero line, as required. The listed root mean square values of the residual errors RMS_{res} were 0.14 and 0.19 PPM in the training and validation flight respectively. The range of the recorded reference Leaf Nitrate values is about 1.5 to 2.5 PPM whence the RMS error rate was $100(0.14/3.1)=4.3\%$, $100(0.19/3.1)=6.1\%$, of the maximal yield in the training and validation flights respectively. Note that the red lines delimit a $\pm 2RMS$ range which includes about 95% of the errors population, assuming that the errors are normally distributed, e.g. $\pm 4.3 \times 2 = \pm 8.6\%$ $\pm 6.1 \times 2 = \pm 12.2\%$ in the training and validation flights respectively. This result is quite good, but not as good as in the yield distribution prediction. There are two main reasons for the reduction in prediction accuracy.

First, the minimal number of features, required for predicting Leaf Nitrate was 28 while for yield prediction only 17 features were needed. Given the rather low repeatability of the features, as depicted by the top bar-graph in figure 5.11, more features means increased probability of error. Second, the yield reference map derived from the yield monitor data is much less error prone than the interpolated Leaf Nitrate reference map, which was derived from just 71 isolated hand samples.

5.3 Summary of predicted growth attributes maps.

The procedure for deriving the predicted Biomass and Sugar Content distribution maps is exactly the same as for the Yield and Leaf Nitrate distribution. For the sake of brevity, we shall not repeat the entire detailed derivation of the predicted distribution maps for Biomass and Sugar Content. The main results and performance parameters in predicting the distribution maps of Biomass and Sugar Content, as well as Yield and Leaf Nitrate, are summarized in table 6.

The **Feat.** column lists the number of features selected by the PLS-GA algorithm, for deriving the predicted map of each of the 4 growths attributes.

The table is divided into 4 prediction mode sections and each section contains 4 columns headed by **r**, **R**, **max** and **2e%** respectively. The columns headed by **r** list the correlation coefficient between the reference and the predicted quantity. The columns headed by **R** similarly list the coefficient of determination, which quantifies the part of the growth attribute variance that is captured or explained by the model. While $r < 1$, the value of **R** may be bigger than one, whence the model overestimates the true variance of the modeled quantity. The columns headed by **max** list the maximal value of the

modeled attribute in the reference map while $e\%$ is computed as a percentage ratio of the root mean square error of the residuals normalized by $\max$:

$$e\% = 100(RMS_{res}/\max) \quad (51)$$

Under a normal distribution assumption, about 95% of the errors are included within the range of $\pm 2e\%$, which is a conservative estimate of the maximal errors that may ever occur. A less conservative but more practical error range estimate is $\pm e\%$ which includes about 85% of the errors, at normal distribution conditions.

The performance parameters r , R and $2e\%$ in the training flight columns pertain to the first flight's image cube while their values in the validation flight columns were derived from the second flight's validation image cube.

Table 6. Summary of main performance parameters in predicting distribution maps of growth attributes

Prediction Mode		Training Flight				Validation Flight				FFT&Plock Reference				FFT&Plock Remote			
Attribute	Feat.	r	R	max	2e%	r	R	max	2e%	R	R	max	2e%	r	R	max	e%
Yield t/ha	17	0.85	0.85	55	12.4	0.73	0.88	55	16.4	0.84	0.84	50	7.6	0.79	0.90	50	8.8
Leaf N ppm	28	0.77	0.77	3.1	8.6	0.62	0.84	3.1	12.2	0.61	0.80	2.5	6.2	0.38	0.95	2.5	8.0
Biomass t/ha	20	0.68	0.68	8.9	14.2	0.61	1.00	8.9	20.0	0.60	0.64	7.0	10.0	0.53	0.79	7.0	11.3
Sugar%	23	0.61	0.82	20.5	5.4	0.44	1.5	20.5	10.7	0.47	0.94	19.8	3.0	0.35	1.10	19.8	3.5

Similarly, under the FFT&Plock Reference and the Remote headings, the table lists the corresponding performance parameters which pertain to "Close Consensus Maps" (CCMap), between the reference and predicted growth attributes distribution maps. These values were obtained by using the FFT regression method with the phase lock option, as described in sections 3.7 and 5.1. The compared images are the reference growth attributes maps versus the PLS-GA predicted maps, from the first and second flight. The justification for using the FFT&Plock prediction mode stems from the observation that the reference maps are just estimates of the actual distribution of the modeled growth attributes. From table 6 it may be seen that in all prediction modes the performance parameters are best in Yield prediction, next best in Leaf Nitrate prediction and further reduced in Biomass and Sugar Content predictions respectively. Generally the prediction performance in the validation flight is lower than in the training flight, as expected.

The usage of the FFT&Plock Reference and the Remote modes generally halved the values of $2e\%$. However, there is a relatively large reduction in r especially in the FFT&Plock Remote prediction

mode. The only exception is yield prediction, wherein r is only slightly reduced. This eludes to the presence of “Nugget Effects” (e.g. Marcotte 1991) in the reference maps of Leaf Nitrate, Biomass and Sugar Content. That is, presence of localized exceptionally very high or very low values, which are generally not present in the image cube data. The nugget effects are probably introduced by the sparse hand sampling of these attributes. They are less prominent in the yield reference maps because they were derived by the yield monitor. However, when the yield reference maps are derived from hand samples (not summarized in Table 6), nugget effects are noticeable in yield reference maps as well.

6. Project summary and recommendations

A major goal of remote sensing in agriculture is to be able to establish areas of the field that differ from the rest of the field and share a common characteristic. One of the most frequently sought commonalities is that of nitrate-N fertility. This is often established by taking many soil samples in a grid pattern and grouping clusters of similar value samples in a zone, as detailed in section 2.

Determining the soil nitrate-N values by remote sensing may not be possible, but it has been shown that following sugarbeets the pre-plant levels of soil nitrate-N may not be the dominant factor in predicting the likelihood of N-fertilizer response due to the considerable amount of nitrogen present in the sugarbeet top biomass (Moraghan, J, Sims A, Smith L 1999). While remote sensing may not work for determining soil nitrate-N fertility values, it shows a good deal of promise for not only differentiating zones but quantifying the values of nitrogen present in the above ground biomass within those zones.

Table 7. Maximal ranges of prediction errors of nitrogen contained in biomass kg/ha.

Leaf nitrate Biomass	RMSres=0.19 PPM RMSres=0.381 t/ha	Reference Average	95% confidence limit values from validation flights			
			LN + 2 RMSres	LN - 2 RMSres	LN + 2 RMSres	LN - 2 RMSres
Leaf nitrate (LN)(%)		1.84	2.22	1.46	2.22	1.46
Biomass (BioM)(t/ha)		5.87	6.64	6.64	5.11	5.11
Kg of N per hectare		108.07	147.30	96.90	113.46	74.63
Deviation from average (108.07=100%)		0%	+36.3%	-10.3%	+5.0%	-30.9%

The amount of Nitrogen in kg/ha contained in the above ground biomass is calculated from the product of the above ground biomass and the leaf nitrate.

This quantity is important because the beet tops in most fields are shredded prior to the harvesting operation which generally covers the shredded tops with dirt, so much of this nitrogen will be available to the following crop.

In order to accurately assess the N requirements for the following crop, both the N residual from the soil and the N from the crop residue must be considered. Table 7 explores the possible extreme error limits in predicting the amount of N in the crop residue. The calculation is based on the Leaf Nitrate and Biomass reference averages $\pm 2\text{RMSres}$. The predicted values were derived from image cube data of the validation flight, without the FFT&Plock option. The first column displays the measured reference average for the field. The next column is the average plus two times the RMSres. The following 2 columns show the results if one parameter is at the upper limit of the 95% confidence level and the other at the lower limit. The last column shows the lower limit of the estimate with both parameters at the minimum. The last row lists the percentage deviations in predicting the true average, wherein the maximal error in predicting the amount of N in the crop residue is +36.3% or -30.9%.

Table 8. Maximal ranges of prediction errors of sugar yield kg/ha.

Sugar yield Beets Yield	RMSres=0.35% RMSres=2200 kg/ha	Reference Average	95% confidence limit values from validation flights			
			%Sug + 2 RMSres Yield + 2 RMSres	%Sug - 2 RMSres Yield - 2 RMSres	%Sug + 2 RMSres Yield + 2 RMSres	%Sug - 2 RMSres Yield - 2 RMSres
Sugar content in beets %		18.16%	18.86%	17.46%	18.86%	17.46%
Beets Yield (kg/ha)		55400	59800	59800	51000	51000
Sugar yield kg/ha		10060.64	11278.28	10441.08	9618.60	8904.60
Deviation from average (10060.64=100%)		0%	+12.1%	+3.8%	-4.4%	-11.5%

Table 8 follows a similar format to table 7 only the variables explored are the sugar percentages and beets yields. The product of those two is the amount of sugar yield per hectare, which is what determines how much the growers are paid.

The last row in table 8 lists the percentage deviations in predicting the true average, wherein the maximal error in predicting the sugar yield is +12.1% or -11.5%.

Finer resolution would be desirable but this would be adequate to point out gross differences in areas. It would be inadequate to point out varietal differences, however. The difference between the top variety and the lowest producing variety of the 39 varieties tested in 2000 was only 2290 kg/ha. The least significant difference in this trial was 823 kg/ha.

Note however that the error in predicting the sugar yield is much lower than in predicting the amount of N in the crop residue. Although the RMSres=0.35% in sugar yield prediction the RMSres=2200

kg/ha in the beets yield prediction, which is only $100(2200/55400) = 3.9711\%$. Consequently, in their product the small error in predicting the beets yield overshadows the larger error in predicting sugar yield.

In summary, we developed *modus operandi* and algorithms, for deriving important plant growth attributes maps from remote sensing hyperspectral imagery, with sufficient accuracy for precision cropping applications. This achievement is remarkable, given the inherently noisy and highly non-repeatable nature of remote sensing hyperspectral imagery. We have been able to overcome this problem by over-flying each field twice in rapid succession and developing the Repeatability Index (RI). The RI quantifies the repeatability of each spectral band in the hyperspectral image cube.

Thereby, it is possible to select the bands of higher repeatability for inclusion in the prediction model while bands of low repeatability are excluded. Further segregation of high and low repeatability bands takes place in the prediction model algorithm, which is based on a combination of a “Genetic Algorithm” and “Partial Least Squares”, (PLS-GA).

A common technique in measuring devices is to take multiple measurements and use their mean value for improving the repeatability or the signal to noise ratio. Similarly, we effectively used two measurements (e.g. image cubes) for improving the overall signal to noise ratio of our system, which generates predicted plant growth attributes maps. Although the prediction accuracy by 2 over-flights appears to be adequate for precision cropping, we recommend 3 over-flights. The additional cost of a third flight should be well worth the improvement in accuracy and prediction robustness of plant growth attributes maps.

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