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AGRICULTURE AND ECONOMIC GROWTH
THEORY AND MEASUREMENT (Part I)

by

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Abstract

An analytic framework for the analysis of the growth of the agricultural sector within a framework of overall economic growth is presented. The analysis begins with a neoclassical framework within which the consequences of resource accumulation and technical change are evaluated. The process of capital accumulation and population growth are examined. To prepare the model for empirical work, the process of resource allocations and the changes in technology are endogenized.

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CHAPTER 1 - INTRODUCTION

Most of world's population still lives in countries that are largely rural, and the development of such countries is of general interest. The development of a rural economy is largely related to the development of its agricultural sector. The agricultural sector is not isolated; however, it is interdependent with the rest of the conomy through the factor and product markets, and changes in such markets affect all major sectors. This interdependence must be taken into account when important policy questions and measures are considered and evaluated. Specifically, the effects of changes in resource endowment and in supply of final or intermediate products (for instnace, due to foreign aid) cannot be limited to one sector, and the feedback cannot be ignored. That, of course, also holds true with respect to price policies, changes in technologies, and any other important measures.

In dealing with the role of agriculture in the process of economic growth, it is important to have a conceptual as well as an operational framework which takes into account the relationships between agriculture and the rest of the economy. Such a formulation can be constructed at various levels of aggregation. The decision on the level of aggregation should largely depend on the questions asked: the more specific the quesiton, the less aggregated the models. When disaggregation is carried far enough, however, the problem may well be studied within a partial rather than a general framework.

The present framework is general, and the analysis is carried out at the highest level of aggregation, which facilitates concentration on the process of agricultural growth. It should be noted that aggregation is not the invention of the analyst byt is rather a result of the need to simplify.

We talk about agriculture and agricultural policy or food and food policy under the assumption that they convey unambiguous meanings. When we need to deal with a specific product, we refer to that product. Similarly, the present framework could be disaggregated to any desirable, and yet meaningful, degree.

In order to be able to assess the consequences of development policies, it is necessary to understand the process of growth. The distinction between the two concepts is made here in order to emphasize that the concept of development implies some intervention in the process of economic growth. Intervention is generally motivated by the desire to achieve targets, such as higher rates of growth or improvement in the distribution of income or consumption, and it will result in altering the process of growth. A full understanding of these results requires evaluating them within a dynamic framework which allows comparing the growth paths that will occur under different intervention measures in any given economy.

There are various models that deal with sectoral growth in the context of general equilibrium. However, the empirical relevance of such models has not been established. The main flaws of such models are related to the formulations of resource allocation and technology. The formulation of resource allocation is basically that of comparative statics and as such it is detached of real time. The technology is identified with a production function. As such it oversimplifies reality to the extent that does not allow to obtain meaningful results. The alternative is to realize that at any time there are numerous techniques that can be employed and the implemented techniques are chosen in a process of optimization. These two major deviations produce a framework which can yield an empirical comparative dynamic analysis.

The present volume concentrates on the conceptual framework which allows to evaluate the modifications. The empirical analysis of Israel agriculture will be carried out at the second stage of this research.

The starting point is the neoclassical framework adapted to the evaluation of agricultural growth. Chapter 2 discusses the short-run equilibrium of a two sector economy with demand for the final products. Chapter 3 traces the consequences of changes in resource endowments. Chapter 4 analyzes the consequences that technical change has on the equilibrium position of the economy. In this chapter, technical change is defined in the conventional form, as exogenous changes in the parameters of a given production function.

The existence of a steady state solution of such an economy is established in Chapter 5. The chapter discusses only the process of capital accumulation with population growth taken to be exogenous. In the future it will be augmented to include the analysis of endogenous population growth.¹

The first step toward reality is made in Chapter 6 where distortions in the factor market are included. In this chapter the rates of distortion are taken to be exogenous. This may be an appropriate assumption for a short run analysis but not for the analysis of growth where the rate of distortion is determined by the economic system. The first formulation in this direction is taken in Chapter 7. Finally Chapter 8 opens up the question of dealing with the measurement of productivity and technical change.

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¹This subject has been analyzed in an MS dissertation by Javier M. Ekboir

This manuscript is incomplete in three major respects: First, the various chapters, and primarily chapters 5 - 8 do not exhaust their subject matter. Second, additional work has to be done in order to link the theoretical discussion with the empirical analysis. Third, there is no attempt to relate the discussion to the literature. Having said this, it can be added that this volume contains enough new material in most of the chapters to justify its submission in its present form as a final research report.

CHAPTER 2 - THE ECONOMY IN THE SHORT RUN

1. INTRODUCTION

In discussing the interdependence of agriculture and the rest of the economy it is convenient to use the analytic framework of a two-sector economy. This framework is widely used and known. Because of its importance in our discussion, it is summarized here. This chapter deals with the behaviour of the economy constrained by constant resources and technology, often referred to as the short run.

2. SUPPLY

Assumptions. The economy consists of agriculture (sector 1) and non-agriculture (sector 2), and two factors, capital and labor. For the time being land is ignored.

There is full employment so that the total sectoral use of the two factors is equal to their total supply.

$$(1) \quad K = K_1 + K_2$$

$$(2) \quad L = L_1 + L_2$$

Technology: The technology is represented by constant returns to scale (or homogenous of degree 1) production functions.

$$(3) \quad Y_1 = F_1(K_1, L_1)$$

$$(4) \quad Y_2 = F_2(K_2, L_2)$$

It is assumed that the functions are twice differentiable and strongly concave. The assumption of strong concavity simplifies the discussion at this stage.

Competition: Perfect competition exists among firms and consumers within each sector and between sectors. Consequently, the value marginal productivities are equal to the factor prices. Let $p = p_1/p_2$ and write

$$(5) \quad r = p \frac{\partial F_1}{\partial K_1} = \frac{\partial F_2}{\partial K_2}$$

$$(6) \quad w = p \frac{\partial F_1}{\partial L_1} = \frac{\partial F_2}{\partial L_2}$$

It should be noted that r and w are measured in terms of sector 2 output. Part of the discussion requires weaker assumptions.

The Structure of Supply

The eight equations, numbered (1)-(6) summarize the supply conditions under constant technology and fixed factor supply. There are nine endogenous variables in the economy: L_1 , K_1 , Y_1 , w , r and the product price ratio $p = p_1/p_2$. Utilizing equations (1)-(6) these variables can be solved in terms of any one of them, for instance in terms of p , and thus summarize the supply side.

For analytic reasons, it is more convenient to solve the system in terms of the wage-rental ratio. Under the competitive assumptions the wage-rental ratio is equal to the ratio of marginal productivities of labor and capital, and is denoted by ω . Utilizing the assumption of constant returns to scale, the marginal and average productivities are uniquely determined by the capital labor ratios. It may be useful to show this and other results graphically. In so doing we disregard the sectoral indexes. In Figure 2.1 the average labor productivity $Y/L = f(k)$, is drawn as a function of the capital labor ratio $k = K/L$. For the value of k given by OD, the value of $f(k)$ is DA, $MP_K = \tan \alpha$, $MP_L = OB$, and using the triangle similarities, $\omega = OC$. It is now simple to see that ω is monotonically increasing in k . This monotonicity can be established as follows:

Note that $MP_K = f'(k)$, $MP_L = f(k) - kf'(k)$, we have

$$\omega(k) = \frac{f(k)}{f'(k)} - k \text{ and } \omega'(k) = - \frac{f(k)f''(k)}{(f'(k))^2} > 0.$$

By definition of ω , it is well defined for all k , and $f'(k) > 0$.

In the two-sector economy, ω can vary, but its possible values are bounded. The boundaries are established by full employment conditions (1) and (2) which are combined to yield:

$$(7) \quad k = \ell k_1 + (1 - \ell)k_2$$

where $k = K/L$ is the overall capital-labor ratio and $\ell = L_1/L \geq 0$ is the share of agriculture in the total labor force. Under the competitive conditions ω is equal to the wage rental ratio. Since $\omega_1(k_1)$ is monotonic, the inverse function, $k_1(\omega)$ exists. However, not all allocations are admissible. Specifically, allocations which violate the full employment equation (7) are inadmissible. To characterize an admissible allocation let $\underline{k}(\omega) = \min(\cdot)$ and $\bar{k}(\omega) = \max(\cdot)$ where $(\cdot) = (k_1(\omega), k_2(\omega))$.

An allocation $k_1(\omega)$ is admissible if

$$(8) \quad \underline{k}(\omega) \leq k \leq \bar{k}(\omega)$$

An admissible ω is a value of ω which is consistent with an admissible allocation. The determination of the set of admissible ω 's can be illustrated graphically in Figure 2.2. The intersection of the two functions $\omega_1(k_1)$ with the vertical line at $k_1 = k$ determine the boundaries of the admissible values of ω in the economy. Let $(\cdot) = (\omega_1(k), \omega_2(k))$, then $\underline{\omega}(k) = \min(\cdot)$ and $\bar{\omega}(k) = \max(\cdot)$. In what follows we deal only with $\omega \in (\underline{\omega}(k), \bar{\omega}(k))$.

The properties to be derived are related to the sectoral capital intensities. Without a loss in generality sector 2 is assumed to be more capital intensive. That is, for any admissible ω , $k_2(\omega) > k_1(\omega)$, or equivalently, $\omega_1(k) \geq \omega_1(k_1(\omega)) = \omega$ and $\omega_2(k) \leq \omega_2(k_2(\omega)) = \omega$, which establishes the boundaries on the admissible values of ω . Figure 2.2 is drawn in accordance with this assumption.

The solution of the supply side in terms of ω is now easily determined. Using the property of constant returns to scale, the production functions can be rewritten as:

$$(9) \quad y_1 = Y_1/L = \ell f_1(k_1)$$

$$(10) \quad y_2 = Y_2/L = (1 - \ell) f_2(k_2)$$

Note that y_i represents per capita production whereas f_i represents average productivity per worker in the i -th sector. Use (7) to obtain ℓ as a function of ω and k :

$$(11) \quad \ell(\omega, k) = \frac{k_2(\omega) - k}{k_2(\omega) - k_1(\omega)}$$

Combine (11) and (9) :

$$(12) \quad y_1(\omega, k) = (k_2(\omega) - k) \frac{f_1(k_1(\omega))}{k_2(\omega) - k_1(\omega)}$$

The dependence of ℓ and y_1 on k is indicated here for later reference. In the present discussion k is assumed to be constant and therefore ω is the only relevant determinant.

The determination of $y_1(\omega, k)$ is illustrated in the lower panel of Fig. 2.2. Referring to the notation $f_1(k_1(\omega))/(k_2(\omega) - k(\omega)) = \tan \theta_1$, it is noted that

$$y_1(\omega, k) = (k_2(\omega) - k) \tan \theta_1$$

and for any admissible ω , y_1 is read at the intersection of the diagonal from f_1 to k_2 with the vertical line at k . Similarly, y_2 is obtained from $y_2(\omega, k) = (k - k_1(\omega)) \tan \theta_2$.

Having determined $k_1(\omega)$, the marginal productives are determined. Utilizing the competitive conditions, factor prices are determined as well as p :

$$(13) \quad p(\omega) = \frac{f_2'(k_2(\omega))}{f_1'(k_1(\omega))}$$

It is noted that p , r and w are functions of ω only and one independent of k . As shown below $p(\omega)$ is monotonic in ω . Thus, the system can be solved in terms of any one of the endogeneous variables, and particularly in terms of p , utilizing (13).

The Response to Price Variations

The dependence of the system on prices is monotonic. To see this, select a feasible $\omega^* > \omega$ and examine

$$\frac{\ell(\omega^*, k)}{1 - \ell(\omega^*, k)} = \frac{k_2(\omega^*) - k}{k - k_1(\omega^*)}$$

By the monotonicity of $k_1(\omega)$, it is easy to see that

$$\ell(\omega^*, k)/(1 - \ell(\omega^*, k)) > \ell(\omega, k)/(1 - \ell(\omega, k))$$

and hence $\ell(\omega^*, k) > \ell(\omega, k)$. This result follows immediately from the full employment conditions. Both $k_1(\omega)$ can increase with ω only if the labor intensive sector expands and the capital intensive sector shrinks.

The simultaneous increase of $k_1(\omega)$ and $\ell(\omega, k)$ with ω implies a shift of resources to sector 1, that is L_1 and K_1 increase and L_2 and K_2 decline. Consequently, Y_1 increases and Y_2 declines. The sectoral behavior depends on their factor intensity. The discussion can then be summarized in the following proposition.

Proposition 1: An increase in the wage rental ratio is associated with an increase in employment of both resources and in output of the labor-intensive sector and a decline in resource employment and output of the capital intensive sector.

Since $k_1'(\omega) > 0$, we also have:

Proposition 2: An increase in the wage-rental ratio is associated with an increase in the wage rate (w) and a decline in the rental rate (r).

By repeating the foregoing solutions for all the admissible values of ω we can generate all the combinations of y_1 and y_2 . By plotting these values in the $y_1 - y_2$ plane we obtain the transformation curve. The

transformation curve is the set of efficient production plans in the sense that it is impossible to reallocate resources so as to increase the output of one product without decreasing the output of the other product. The concavity of the transformation curve follows from the concavity of the production functions. Let ω_A and ω_B be two admissible values of ω , and y_{1A} , y_{1B} the corresponding outputs. For any $0 < \lambda < 1$ define y_{1C} as:

$$y_{1C} = \lambda y_{1A} + (1 - \lambda) y_{1B}$$

Points A, B and C are shown in Figure 2.3. To prove concavity it is necessary to show that there exists a feasible production plan, say D, such that $y_{1D} \geq y_{1C}$. Consider the admissible allocation K_1^* , L_1^* given by:

$$K_1^* = \lambda K_{1A} + (1 - \lambda) K_{1B} \quad i = 1, 2$$

$$L_1^* = \lambda L_{1A} + (1 - \lambda) L_{1B}$$

In view of the concavity of the production functions:

$$Y_1^* = F_1(K_1^*, L_1^*) \geq y_{1C}$$

Thus, for any initial points A, B and for any λ there exists an admissible allocation which dominates C. It should be noted that the production plan (Y_1^*) need not be an efficient one. If that is the case, then by definition, there exists an efficient allocation D leading to:

$$Y_{1D} > Y_{1I} > Y_{1C} \quad i = 1, 2$$

There is a monotonic relationship between the product price ratio p and the factor price ratio ω . By the strong concavity of the transformation curve, the derivative $dy_2(\omega)/dy_1(\omega)$ increases in absolute value as y_1 increases. Next, $y_1(\omega)$ is monotonic in ω . Since by the competitive conditions, the derivative is equal to p , we have established the following:

Proposition 3 : The wage rental ratio is monotonically increasing with the price of the labor intensive product relative to that of the capital intensive product.

When agriculture is labor intensive, we can write $p(\omega)$ and $p'(\omega) > 0$. Note that the admissible values of p are bounded by $\bar{p} = \max(\dots)$, $\underline{p} = \min(\dots)$ where $(\dots) \equiv \{p(\bar{\omega}(k)), p(\underline{\omega}(k))\}$.

The supply structure is summarized in Fig. 2.4. Starting with a wage-rental ratio ω_A , $k_1(\omega)$ are determined and thus result in a production plan A as shown by A on the transformation curve, in panel IV. The price p_A corresponding to ω_A is derived in panel II. Panel III presents the supply function for y_1 . It is obtained from $y_1(p) = y_1(\omega(p))$. As sector 1 is the labor intensive and p is its relative price, we have $y_1'(p) > 0$. Knowing p_A , y_{1A} is determined. This corresponds to point A on the transformation curve.

It is now clear that the system can be solved in terms of other variables, rather than w . For instance, knowing y_1 or p makes it possible to solve for all the remaining variables. Similarly, the system can be solved in terms of $y = py_1 + y_2$. Referring to Figure 2.4, given y_A , a tangent line is drawn to the transformation curve and thus p_A , y_{1A} and y_{2A} are derived. If a tangent line does not exist, that implies that the drawn y is inadmissible.

DEMAND

It is assumed that the preferences of consumers in the economy can be summarized by a well behaved utility function. Under such an assumption, the equilibrium position in a closed economy is determined by a tangency of the indifference and the transformation curves. Such a solution is shown by point A in Figure 2.5. Changes in the conditions on the supply side which lead to changes in the transformation curve will result in a new equilibrium point to be determined in a similar way to A. However, the analysis of such changes can be related to some general properties of demand and it is therefore useful to conduct the analysis in terms of demand.

Under the assumption of a well behaved utility function, we can derive the demand functions:

$$(14) \quad x_1 = D_1(p, c)$$

$$(15) \quad x_2 = D_2(p, c)$$

Where x_1 is the per capita demand of the i -th product and c is the per capita consumption:

$$(16) \quad c = px_1 + x_2$$

At the preset it is assumed that there are no savings so that $c = y$.

The assumptions with respect to the demand are:⁽¹⁾

$$\frac{\partial D_1}{\partial p} < 0 \quad \frac{\partial D_2}{\partial p} > 0 \quad \frac{\partial D_i}{\partial c} > 0 \quad i = 1, 2$$

$$\lim_{p \rightarrow \infty} D_1(p, c) = 0 \quad \lim_{p \rightarrow 0} D_1(p, c) = \infty$$

$$\lim_{p \rightarrow \infty} D_2(p, c) = \infty \quad \lim_{p \rightarrow 0} D_2(p, c) = 0$$

In words, the two products are assumed to be normal (positive income coefficients), and the demand functions are asymptotic to the price-quantity axes. The locus of $x_1(p, c)$ with p held constant is the income-consumption-curve, denoted as $x(p, c)$ in Figure 2.5. Since the two products are assumed to be normal, it is possible to combine (14) and (15) to:

Footnote: (1) This assumption was used by Mosenson and Mundlak ( ). The assumption of asymptoticity is equivalent to assume that the two products are desirable.

$$(17) \ x_1 = D(p, x_2)$$

with

$$\frac{\partial D}{\partial p} < 0 \text{ and } \frac{\partial D}{\partial x_2} > 0$$

$$\lim_{p \rightarrow \infty} D(p, x_2) = 0 \quad \lim_{p \rightarrow 0} D(p, x_2) = \infty$$

EQUILIBRIUM

The structure of the model meets the conditions of a competitive equilibrium, and, consequently general results imply existence of competitive equilibrium. The simple structure of the present model makes it possible to show constructively the determination of equilibrium. Such a construction is used in subsequent discussion to analyze the response of the system to various changes. At this stage it is assumed that there are no other demand components beside demand for final consumption as specified in the previous section. Specifically, there is no demand for capital goods. Capital goods are thus assumed to be exogenously given. This assumption simplifies the present exposition and it is relaxed in subsequent discussion. Under that assumption, equilibrium is achieved if there exists a price p which equates production and demand, i.e., $x_1 = y_1$.

To show the determination of the equilibrium solution graphically, we refer to Figure 2.6. Under monotonicity of consumers' preferences, we can limit our examination of equilibria to points of the transformation curve. Consequently, we can draw the demand function (17) for any given price

demand p^d under the assumption that $x_2 = y_2$. Begin with $x_1(p_A^d, y_2)$ which intersects the transformation curve at A. At this point, the supply price is p_A^s . If p_A^d is equal to the (supply) price p_A^s , then A is an equilibrium point. Suppose that $p_A^s > p_A^d$, then draw another demand curve where the demand price is p_A^s . By the properties of (17) the new curve is to the right and below the original curve, intersecting the transformation curve at B. Thus, at p_A^s there is an excess supply of the agricultural product: $y_1(p_A^s) - x_1(p_A^s) > 0$. Alternatively, we can find the supply which corresponds to p_A^d . This will be to the right and below point A. Consequently $x_1(p_A^d) - y_1(p_A^d) > 0$. By the continuity of the demand functions, there exists a p , $p_A^d < p < p_A^s$ such that $x_1(p) = y_1(p)$. By Walras law, this price also clears the non-agricultural market.

The solution can also be illustrated in terms of supply and demand equations. This is shown in Figure 2.7 where the demand curves are drawn for the limits of the admissible prices, $\underline{p}$ and $\bar{p}$. By the assumptions on demand, the quantities demanded at those prices are positive. The supply function $y_1(p)$ drawn in the left panel is derived as in Fig. 2.4. The demand curve reports demand as function of p and $x_2 = y_2(p)$, where $y_2(p)$ is obtained from the transformation curve. Thus, for $p = \underline{p}$, $y_2(\underline{p}) = \bar{y}_2 > 0$ and $x_1(\underline{p}, \bar{y}) > 0$ whereas the supply $y_1(\underline{p}) = 0$. Thus, at $\underline{p}$ excess demand is positive. Alternatively, at $\bar{p}$, $y_1(\bar{p}) = \bar{y}_1 > 0$, $y_2(\bar{p}) = 0$, and by the demand assumption $x_1(\bar{p}, 0) = 0$ and excess demand is negative. The demand and supply are continuous and with opposite slopes, and thus there is a unique and stable equilibrium solution.

The discussion can now be summarized. Let $y_1 = y_1(p, k)$ be the supply function of the 1-th sector. The level of the function is determined by k . As our interest is in the short run equilibrium, the dependence on k is suppressed and the supply functions are written as $y_1(p)$. As $p = p_1/p_2$, we have $y_1'(p) > 0$, $y_2'(p) < 0$. The excess demand function for agriculture is

$$z_1(p) = x_1^d(p, x_2^d) - y_1(p).$$

For any p , $y_1(p)$ and $y_2(p)$ are jointly determined. For p to be an equilibrium price, imposing $x_2^d = y_2(p)$ should result in $z_1(p) = 0$. As can be seen in Figure 2.7, $z_1(p)$ alternates signs. Furthermore, in the present model $z(p)$ is everywhere declining in p :

$$z_1'(p) = \frac{\partial D(\cdot)}{\partial p} + \frac{\partial D}{\partial x_2^d} \frac{\partial x_2^d}{\partial S_2(p)} S_2'(p) - S_1'(p) < 0$$

and consequently, the equilibrium is unique.

Convergence is achieved when excess demand generates an increase in price: $z(p)dp \geq 0$. The equality is achieved when both $z(p)$ and dp are equal to zero.

In some cases it may be more useful to deal with a somewhat different convergence rule which states that whenever the cost of production is

below the demand price output will increase. Define $\pi_1(y_1) = p^d(y_1) - p^s(y_1)$, as the unit profit in sector 1. Equilibrium is achieved by finding y_1 such that $\pi_1 = 0$. As can be seen from Figure 2.7, $\pi_1(y_1)$ alternates signs. Furthermore, in the present model it is ever declining in y_1 . To see this note that $dy_2/dy_1 = S_2'(p)/S_1'(p) < 0$, and evaluate:

$$\pi_1'(y_1) = \frac{\partial p^d}{\partial x_1} + \frac{\partial p^d}{\partial x_2} \frac{dx_2^d}{dy_2} \frac{dy_2}{dy_1} - \frac{\partial p^s}{\partial y_1} - \frac{\partial p^s}{\partial y_2} \frac{dy_2}{dy_1} < 0$$

As there is no contradiction in signs. The convergence rule is $\pi_1(y_1)dy_1 \geq 0$ where equality is achieved only when the two terms are identically zero.

THE CASE OF EQUAL FACTOR INTENSITIES

The foregoing discussion does not properly cover the case of equal factor intensities, that of $k_1(\omega) = k_2(\omega)$. This equality may hold locally where $k_1(\omega) = k$, as shown in Figure 2.8, or in the large as shown in Figure 2.9. In this case equation (7) degenerates and the subsequent discussion which was based on this equation does not hold. Yet, the economy has an equilibrium point. The analysis in this case is by far simpler than the case discussed above.

Under equal factor intensities the transformation curve is a straight line. Since by assumption,

$$(18) \quad k_1(\omega) = k_2(\omega) = k$$

we have

$$(19) \quad \omega = \underline{\omega}(k) = \bar{\omega}(k)$$

That is there is a unique value of ω which is admissible for the economy. Utilizing (9) and (10), the intercepts of the transformation curve are $\bar{y}_1 = f_1(k)$ and $\bar{y}_2(k) = f_2(k)$. Utilizing the marginal productivity conditions, $p = f'_2(k)/f'_1(k)$ and since k is constant, so is p . In other words, the transformation curve represents the combination of outputs $lf_1(k)$ and $(1-l)f_2(k)$. Consequently:

$$dy_2/dy_1 = f_2(k)d(1-l)/f_1(k)dl = -f_2(k)/f_1(k) \equiv -\bar{y}_2/\bar{y}_1.$$

All this is shown in Figure 2.10. Thus, in addition to (19) we can also indicate that $p(k)$ is uniquely determined.

The equilibrium point is determined graphically by drawing the demand curve corresponding to $p(k)$. As in the general case, the intersection of this demand curve with the transformation line determines the equilibrium quantities.

Income Distribution

Income generated by production is distributed as wages to labor and rental to the owners of the capital good. Variations in factor prices affect the income distribution between labor and capital. When the demand depends only on the level of income and not on its source, the income distribution is determined by the equilibrium position but does not affect it. A change in factor prices may increase or decrease the share of labor in total income (labor share), depending on the ease of factor substitution, to be measured by the elasticity of substitution (ES).

For the case of two-factor production function the ES is defined by:

$$(20) \quad \sigma = d \ln k / d \ln \omega .$$

For a given sector, the share of labor in sectoral income is given by $S_{iL}(\omega, k) = wL_i / p_i Y_i$. We emphasize the dependence of the share on the position of the economy as indicated by (ω, k) . Instead of dealing with S_{iL} , we examine $\theta_i \equiv S_{iL} / (1 - S_{iL}) = wL_i / rK_i = \omega / k_i$. Thus, $\text{sign } \partial S_{iL} / \partial \omega = \text{sign } \partial \theta_i / \partial \omega$. But $\text{sign } \partial \theta_i / \partial \omega = \text{sign } \partial \ln \theta_i / \partial \ln \omega = \text{sign } (1 - \sigma_i)$. Consequently, the share of labor in total income of sector 1 increases (decreases) with the wage-rental ratio when $\sigma_1 < 1$ ($\sigma_1 > 1$). It is unaffactive by the wage rental ratio when $\sigma_1 = 1$. If $\sigma_1 = 1$ everywhere, then the production function is Cobb-Douglas.

The relationship between σ_i and the concavity of ω_i is illustrated in Fig. 28. θ is drawn for the point k, ω . As ω increases, θ decreases for sector 1 ($\sigma_1 > 1$) and increases for sector 2 ($\sigma_2 < 1$). An elasticity of substitution larger than 1 describes a situation where the increase in ω causes more than a proportional substitution of capital for labor.

It is clear from the foregoing discussion that a given change in ω may affect differently the labor shares in the two sectors when $(\sigma_1 - 1)(\sigma_2 - 1) < 0$. In this case, the net affect of such a change on the labor share in the economy at large depends on the relative importance of the sectors and the change in the sectoral composition called for by the change in ω .

Figure 2.1

Figure 2.2

Figure 2.3

Figure 2.4

Figure 2.5

Figure 2.6

FIGURE 2.7

FIGURE 2.8

FIGURE 2.9

FIGURE 2.10

APPENDIX TO CHAPTER 2

In the following notes the results of Chapter 2 are derived analytically. These results also serve as a basis for derivations in subsequent chapters.

Displacement of Supply

The discussion is conducted under the assumption of unequal factor intensities.. The case of equal factor intensities is discussed separately in the text.

Resource Allocation

Let $\hat{l}_1 = L_1/L$ and $\hat{\rho} = k_1/k$, and differentiate the full employment conditions (2.1) and (2.2):

$$(A.2.1) \quad \rho_1 \hat{\rho}_1 + \rho_2 \hat{\rho}_2 = 0$$

$$(A.2.2) \quad l_1 \hat{l}_1 + l_2 \hat{l}_2 = 0$$

Let $\sigma_1 = \partial \ln k_1 / \partial \ln \omega = \bar{k}_1 / \hat{\omega}$ where $\hat{\omega} = d \ln \omega$, etc.

Utilizing $\hat{k}_1 = \hat{K}_1 - \hat{L}_1 = \hat{\rho}_1 - \hat{l}_1 + \hat{k}$ we can rewrite (A.2.1) and (A.2.2):

$$(A.2.3) \quad \Delta_1 \hat{l}_1 + \Delta_2 \hat{l}_2 = \sigma_{\rho} \hat{\omega} - \hat{k} \quad \Delta_1 \neq 0$$

$$(A.2.4) \quad \Delta_1 \hat{\rho}_1 + \Delta_2 \hat{\rho}_2 = \sigma_{\hat{l}} \hat{\omega} - \hat{k}$$

where $\Delta_1 = l_1 - \rho_1$, $\sigma_\rho = \rho_1\sigma_1 + l_2\sigma_2$, $\sigma_l = l_1\sigma_1 + l_2\sigma_2$, σ_ρ and σ_l are positive and $\Delta_1 + \Delta_2 = 0$. It then follows that $\Delta_1\Delta_2 < 0$, $\hat{l}_1\hat{l}_2 < 0$ and $\hat{\rho}_1\hat{\rho}_2 < 0$. Consequently, $\Delta_1\hat{l}_1 > 0$, $\Delta_1\hat{\rho}_1 > 0$.

Using the foregoing relationships, it becomes evident that in the short run ($\hat{k} = 0$), $\text{sign } (\hat{l}_1/\hat{\omega})|_k = \text{sign } (\hat{\rho}_1/\hat{\omega})|_k = \text{sign } \Delta_1 \cdot \Delta_1 > 0$ implies that sector 1 is the labor intensive, thus the employment of labor and capital in this sector grows with the wage rental rates.

The system (A.2.1) - (A.2.4) can be solved for $\hat{l}_1$ and $\hat{\rho}_1$ to yield:

$$(A.2.5) \quad (\hat{l}_1/\hat{\omega})|_k = l_2\sigma_\rho/\Delta_1 \quad (\hat{l}_1/\hat{k})|_\omega = l_2/\Delta_2$$

$$(\hat{l}_2/\hat{\omega})|_k = l_1\sigma_\rho/\Delta_2 \quad (\hat{l}_2/\hat{k})|_\omega = l_1/\Delta_1$$

$$(\hat{\rho}_1/\hat{\omega})|_k = \rho_2\sigma_l/\Delta_1 \quad (\hat{\rho}_1/\hat{k})|_\omega = \rho_2/\Delta_2$$

$$(\hat{\rho}_2/\hat{\omega})|_k = \rho_1\sigma_l/\Delta_2 \quad (\hat{\rho}_2/\hat{k})|_\omega = \rho_1/\Delta_1$$

Outputs

Differentiate the production functions (2.10) and (2.11):

$$(A.2.6) \quad \hat{y}_1 = \alpha_1\hat{l}_1 + (1 - \alpha_1)(\hat{\rho}_1 + \hat{k})$$

$$(A.2.7) \quad \hat{y}_2 = \alpha_2 \hat{\ell}_2 + (1 - \alpha_2)(\hat{\rho}_2 + \hat{k})$$

Where $\alpha_1 = \frac{\partial \ln y_1}{\partial \ln L_1}$, the production elasticity with respect to labor which, under the competitive conditions, is simply the labor share. Substituting the results of (A.2.5) in (A.2.6), (A.2.7) to obtain:

$$(A.2.8) \quad \hat{y}_1 = e_1 \hat{\omega} + (\ell_2 / \Delta_2) \hat{k}$$

$$(A.2.9) \quad \hat{y}_2 = e_2 \hat{\omega} + (\ell_1 / \Delta_1) \hat{k}$$

where

$$e_1 = \{\alpha_1 \ell_2 \sigma_\rho + (1 - \alpha_1) \rho_2 \sigma_\ell\} / \Delta_1$$

$$e_2 = \{\alpha_2 \ell_1 \sigma_\rho + (1 - \alpha_2) \rho_1 \sigma_\ell\} / \Delta_2$$

$$\text{sign } e_1 = \text{sign } \Delta_1 .$$

Thus the short run elasticity $(\hat{y}_1 / \hat{\omega})|_k$ is positive for the labor intensive sector and negative for the capital intensive sector.

Product Price Ratio Utilizing the Euler condition for a constant returns to scale and the competitive assumptions, we can write -

$$Pf_1 = w + rk$$

Differentiating and noting that $\hat{r}_1 = (1 - \alpha_1)\hat{k}_1$ we get

$$\hat{p} = \alpha_1 \hat{w} + (1 - \alpha_1) \hat{r}$$

$$0 = \alpha_2 \hat{w} + (1 - \alpha_2) \hat{r}$$

By subtraction and rearrangement:

$$(A.2.10) \quad \hat{p} = I \hat{w}$$

where $I = \alpha_1 - \alpha_2$

Equation (A.2.10) indicates that the (relative) price of the labor intensive product is monotonically increasing with the wage rental ratio.

Demand In the appendix we deal with the demand equations (2.14) and (2.15) in a form that allows immediate generalization to more than two sectors. Consequently we start by assuming explicit prices, p_1 and p_2 for the two products respectively. By differentiating the equations we obtain:

$$(A.2.11) \quad \hat{x}_1 = E_{11} \hat{p}_1 + E_{12} \hat{p}_2 + E_{1c} \hat{c}$$

$$(A.2.12) \quad \hat{x}_2 = E_{21} \hat{p}_1 + E_{22} \hat{p}_2 + E_{2c} \hat{c}$$

where $E_{1j} = \partial \ln D_1 / \partial \ln p_j$, $E_{1c} = \partial \ln x_1 / \partial \ln c$ are respectively price and income elasticities. As it is well known these elasticities are

subject to some constraints. Rewriting the budget constraint (2.16) as $c = p_1 x_1 + p_2 x_2$ and differentiating it holding p_i constant for all i , we obtain

$$(A.2.13) \quad \pi_1 E_{1c} + \pi_2 E_{2c} = 1$$

$$\pi_1 = p_1 x_1 / c, \quad \pi_1 + \pi_2 = 1$$

Next differentiate the budget constraint with respect to p_i holding $p_j \neq p_i$ and c constant to obtain:

$$\pi_1 E_{11} + \pi_2 E_{21} = -\pi_1$$

(A.2.14)

$$\pi_1 E_{12} + \pi_2 E_{22} = -\pi_2$$

Finally, imposing the constraint of homogeneity of degree zero in c and p_i , we obtain from (A.2.12):

$$E_{11} + E_{12} = -E_{1c}$$

(A.2.15)

$$E_{21} + E_{22} = -E_{2c}$$

Next, in order to differentiate between the substitution and income effects, define the substitution elasticities.

$$(A.2.16) \quad \epsilon_{ij} = \partial \ln x_i / \partial \ln p_j |_u$$

$$= E_{ij} + E_{ic} \pi_j$$

Combining (A.2.16) and (A.2.15) we obtain:

$$\epsilon_{11} + \epsilon_{12} = 0$$

(A.2.17)

$$\epsilon_{21} + \epsilon_{22} = 0$$

Similarly combining (A.2.16) and (A.2.14) and utilizing (A.2.13):

$$\pi_1 \epsilon_{11} + \pi_2 \epsilon_{21} = 0$$

(A.2.18)

$$\pi_1 \epsilon_{12} + \pi_2 \epsilon_{22} = 0$$

Using the budget constraint it is possible to eliminate one of the demand equations. This is convenient for the graphical presentation of the two sector case analyzed in the first part of the manuscript. Thus, to express the parameters of the combined demand equation (2.17) in terms of the parameters of the individual demand functions (2.14) and (2.15), we combine (A.2.11) and (A.2.12) to eliminate $\hat{c}$:

$$E_{2c} \hat{x}_1 - E_{1c} \hat{x}_2 = [E_{2c} E_{11} - E_{1c} E_{21}] \hat{p}_1 + [E_{2c} E_{12} - E_{1c} E_{22}] \hat{p}_2$$

Using (A.2.16) the bracketed terms can also be written in terms of the compensated elasticities, (ϵ_{ij} 's). Then utilizing (A.2.17) we obtain:

$$(A.2.19) \quad \hat{x}_1 = \frac{E_{1c}}{E_{2c}} \hat{x}_2 + (\epsilon_{11} - \frac{E_{1c}}{E_{2c}} \epsilon_{21}) \hat{p}$$

$$= \hat{\eta}x_2 - \sigma_D \hat{p}$$

where

$$(A.2.20) \quad \eta = E_{1c}/E_{2c}$$

$$(A.2.21) \quad -\sigma_D = \epsilon_{11} (1 - \pi_1 E_{1c})$$

Thus, $\eta = 1$ if and only if $E_{1c} \gtrless 1$

The expression for σ_D in (A.2.21) is obtained by using (A.2.18) and (A.2.13). Note that $\sigma_D = 0$ if and only if $\epsilon_{11} = 0$. Also $\sigma_D = 1$ if and only if $\epsilon_{11} - \pi_1 E_{1c} = -1$ or simply when the uncompensated demand elasticity is unity.

CHAPTER 3 - THE ECONOMY IN THE LONG RUN UNDER CONSTANT TECHNOLOGY1. INTRODUCTION

A change in the resource endowment has various consequences on the economy which we now turn to examine. Continuing to work with per capita variables, it is noted that the initial resource supply is represented by k alone. The question is then what effect does a change in k have on the economy. As before, we start with the discussion of the supply conditions. We then continue with the analysis of the long run equilibrium using the demand conditions specified in Chapter 2. The term long run indicates that resources are allowed to change. Such a terminology serves analytic convenience of isolating effects rather than reality. It is rather artificial to isolate resource change from technological change.

THE RESPONSE OF OUTPUTS TO ACCUMULATION UNDER CONSTANT PRICES

The main result here is due to Rybczynski (1955) which states that an increase in k , holding p constant, increases the output of the capital intensive sector and decreases the output of the labor intensive sector. This result can be obtained from Figure 2.2 by drawing a vertical line for the new value of k . This is illustrated in Figure 3.1. Increasing k to k^* does not change the sectoral capital labor ratios $k_1(\omega)$ for any given ω . Holding ω constant, increasing k results in the growth of the capital intensive sector. In fact, this is the only way that the economy can absorb a larger overall k while holding k_1 constant, as it is indicated by (2.11). The resources that are diverted from the labor intensive sector are of low capital labor ratio. This ratio is augmented by utilizing the

new resources which by assumption have a larger capital labor ratio. Keeping $k_1(\omega)$ constant implies that mobility of resources from the labor intensive sector cannot be restricted to labor alone. Consequently, the total employment of resources in the labor intensive sector declines and so does its output. The opposite is true for the capital intensive sector. Referring to Figure (3.1) it is seen that $y_2^* \equiv y_2(\omega, k^*) > y_2(\omega, k) \equiv y_2$ and $y_1^* \equiv y_1(\omega, k^*) < y_1(\omega, k) \equiv y_1$ for $k > k^*$.

ADMISSIBLE PRICES

As a result of the monotonicity of the $\omega_1(k_1)$ functions and the full employment conditions, the boundaries on the admissible ω , $\underline{\omega}(k)$ and $\bar{\omega}(k)$ are monotonically increasing with k . Furthermore, if we assume that $\omega_1(k_1) \rightarrow \infty$ as $k_1 \rightarrow \infty$, then for any arbitrary ω , there exist values of k , $\underline{k}(\omega)$ and $\bar{k}(\omega)$, such that for all $k > \bar{k}(\omega)$ and $k < \underline{k}(\omega)$, ω is non-admissible. That is, accumulation of capital eventually makes any given wage rental ratio to be too low to be admissible.

For convenience, and without a loss in generality, let the capital intensive sector be the numeraire, as we did in the foregoing discussion, then $\underline{p}(k) = p(\underline{\omega}(k))$, and as k increases, so does $\underline{p}(k)$. Consequently, by the process of accumulation, any arbitrary price $\hat{p}$ becomes eventually non-admissible. That is, the process of capital accumulation increases the wage relative to rental on capital. Consequently, the cost of production of the labor intensive product increases relative to that of the capital intensive product. This is a strong result to which we return

subsequently.

ACCUMULATION AND THE EQUILIBRIUM POSITION

It is assumed that the economy is in a short run equilibrium. A change in resources produces a move to another equilibrium point that we now try to characterize. Without a loss in generality the discussion deals with an increase in k . From the Rybczynski theorem it is clear that prices cannot remain constant for if the economy was initially in equilibrium an increase in the production of the capital intensive product generates, under constant prices, an increase in the demand for the labor intensive product and a decline in its production and therefore a positive excess demand. Consequently, the relative price of the labor intensive product should increase. Since the relative price of the labor intensive product is monotonically increasing in the wage rental ratio, we thus established:

Proposition 1. Along the long run equilibrium path generated by an increase in the capital labor ratio, the wage rental ratio and the relative price of the labor intensive product are monotonically increasing.

This argument can be repeated by reference to Figure 3.1. The new y_1^* achieved under constant prices generates positive excess demand for y_1 . Consequently, the production of y_1 should increase. A new ω should be found that will result in a larger value for y_1 , and a smaller value for y_2 . Note that the search for such a combination is conducted for given

resources, as given by the new bundle, and therefore the results of Chapter 2 are immediately applicable. A possible solution is shown in Figure 3.2 with the wage rental ratio going up from ω_A to ω_B . The solution is also described in terms of the transformation curves with the product price ratio going up from p_A to p_B in Figure 3.3.

While the change in prices caused by accumulation is uniquely determined, this is not the case with respect to outputs and therefore resource allocation. Referring to Figure 3.3 the question is what can be said with respect to outputs at B as compared to those at A.

As k increases, holding p constant, there is an expansion effect on production shown by a move from point A to A^* , and in per capita income. The latter is shown by the tangent line to A^* . Given this new income and the old price, consumption is chosen at point C. The movement from A to C can be referred to as the expansion effect of demand caused by an expansion of k . Note that points C and A are on the same demand curve.

It is obvious that C is not an equilibrium point since it is not within the feasible production set of the economy. Thus, with p_A there is now a positive excess demand for agricultural product given by $x_{1C} - y_{1A^*}$ and (not independently) negative excess demand for the industrial product given by $x_{2C} - y_{2A^*}$. Consequently, the price goes up until the excess demand disappears, as shown by points B with $p_B > p_A$. The movement from A to B can be described in terms of income or expansion effect and a substitution effect. The income effect, by assumption, increases the

demand for the two products whereas the substitution effect encourages the consumption of the capital intensive product whose relative price declines and discourages the consumption of the labor intensive product. In terms of Figure 3.3, point B is to the right of point A but it can be above or below A. In other words, B is in either the first or fourth quadrants with respect to A. This discussion is now summarized in the following:

Proposition 2. Along the long run equilibrium path generated by an increase in the capital labor ratio the per capita output of the capital intensive sector increases whereas that of the labor intensive sector may increase or decrease depending on the relative strength of the income and substitution effects.¹

Next, we turn to examine the changes in factor employment along the equilibrium path. The increase in the wage rental ratio encourages substitution of capital for labor. However, by assumption, all the available labor has to be employed. Therefore, this substitution is restricted by the full employment condition as well as by the need to satisfy the demand for the final products. In what follows we examine employment under these conditions, trying to separate the induced changes to various effects.

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¹ See the appendix for an expression of this result in terms of income and demand elasticities.

Referring to Figure 3.4, it is assumed that point A represents the initial equilibrium position where Y_{iA} was produced with L_{iA} units of labor and K_{iA} units of capital, yielding a capital labor ratio of k_{iA} and a wage rental ratio ω_A . Assume that the new equilibrium point results in ω_B . Point B* in Figure 3.4 indicates the corresponding least cost resource combination that produces the initial output Y_{iA} under ω_B . We can refer to the decline in sectoral employment $L_{iA} - L_{iB*}$ as the factor substitution effect on employment. Similarly, $K_{iA} - K_{iB*}$ represents the factor substitution effect on capital utilization.

The actual sectoral employment that corresponds to the equilibrium point B may be smaller or larger depending on the location of the final equilibrium point. In terms of Figure 3.4, if the equilibrium point B, located on the ray through N, is below N, employment in sector 1 declines whereas it goes up when B is above N. The situation is different with respect to capital since here the factor substitution effect calls for a larger employment of capital. We can then summarize the discussion in the following proposition:

Proposition 3: Along the long run equilibrium path generated by an increase in the capital labor ratio we observe the following effects on the sectoral employment of labor and capital:

E f f e c t	Labor Intensive Sector		Capital Intensive Sector	
	Labor	Capital	Labor	Capital
Expansion	+	+	+	+
Product Substitution	-	-	+	+
Factor Substitution	-	+	-	+

Of all the four cases, only one is uniquely signed. The employment of the factor whose intensity increases in the economy increases in the sector which is more intensive in this factor. That is, increasing capital intensity results in an increase in the employment of capital in the capital intensive sector. In all other cases, the final outcome depends on the relative strength of the various effects. If however, we assume a decline in y , then $0 > \hat{y}_1 = \hat{\ell}_1 + s_{12} \hat{k}_1 \rightarrow \hat{\ell}_1 < 0 \rightarrow L_1 \downarrow \rightarrow L_2 \uparrow$. Thus, the only ambiguous term is that of $\hat{k}_1$.

It should however, be noted that the capital-labor ratios increase in both sectors, that is $(\hat{k}_1/\hat{k})|_{\text{equil.}} > 0$. This simply follows from the fact that $(\hat{w}/\hat{k})|_{\text{equil.}} > 0$.

FIGURE 3.1

FIGURE 3.2

FIGURE 3.3

FIGURE 3.4

APPENDIX TO CHAPTER 3
THE EFFECT OF CAPITAL ACCUMULATION

Supply Side: The effect of a change in resources on supply can be obtained directly from equations (A.2.8) - (A.2.10). Since the function $p(w)$ is independent of k , holding p constant is the same as holding w constant. Therefore

$$(A.3.1) \quad (\hat{y}_1/\hat{k})|_p = \ell_j/\Delta_j \quad 1 \neq j = 1, 2$$

This is the supply expansion effect. It is positive (negative) for the labor (capital) intensive sector (Rybczynski).

Aggregate Output: Aggregate percapita output is given by:

$$(A.3.2) \quad y = py_1 + y_2$$

and therefore

$$(A.3.3) \quad \hat{y} = \pi_1(\hat{p} + \hat{y}_1) + \pi_2\hat{y}_2$$

where $\pi_1 = p y_1/y$, $\pi_2 = 1 - \pi_1$

Substitute (A.2.8) - (A.2.10) in (A.3.3) to obtain

$$(A.3.4) \quad \hat{y} = \left(\pi_1 + \frac{\pi_1 \ell_1}{I} + \frac{\pi_2 \ell_2}{I} \right) \hat{p} + \left(\frac{\pi_1 \ell_2}{\Delta_2} + \frac{\pi_2 \ell_1}{\Delta_1} \right) \hat{k}$$

Note that

$$\pi_1 = p y_1/y = \frac{rK_1 + wL_1}{Y} = \frac{rK}{Y} \frac{K_1}{K} + \frac{Lw}{Y} \frac{L_1}{L} = S_L \ell_1 + S_K \rho_1$$

where S_L and S_K are the shares of labor and capital in aggregate output, respectively.

Consequently,
$$\frac{1}{\Delta_2} \pi_1 \ell_2 + \frac{\pi_2 \ell_1}{\Delta_1} = S_K$$

Next we note that $\pi_1 \ell_1 + \pi_2 \ell_2 = 0$. This follows from the fact that it is evaluated at the equilibrium point, so that for fixed p and k , y is maximized.

Consequently we can write:

$$(A.3.5) \quad \hat{y} = \pi_1 \hat{p} + S_K \hat{k}$$

Thus, S_K can be interpreted as the elasticity of aggregate output with respect to capital evaluated at constant p .

Resource Allocation: The change in the overall capital labor ratio leads to changes in the intersectoral allocation of resources with prices held constant.

The result is given by (A.2.5). That is:

$$(A.3.6) \quad \left. \left(\hat{\ell}_1 / \hat{k} \right) \right|_{\omega} = (\ell_j / \Delta_j) \quad i \neq j$$

$$\left. \left(\hat{\rho}_1 / \hat{k} \right) \right|_{\omega} = (\rho_j / \Delta_j)$$

It is noted that the expansion effect given by (A.3.1) is obtained as a result of a change in resource allocation with prices held constant. We refer to such a change in output as an allocation effect. In Chapter 4 we discuss some other effects.

$$\frac{1}{\Delta_2} \pi_1 \ell_2 + \frac{\pi_2 \ell_1}{\Delta_1} = \frac{1}{\Delta_1} [\ell_1 (1 - \pi_1) - \pi_1 (1 - \ell_1)] = \frac{1}{\Delta_1} (\ell_1 - \pi_1). \quad \text{Substitute}$$

for π_1 : $\frac{1}{\Delta_1} (\ell_1 - \ell_1 S_L - \rho_1 S_K) = \frac{1}{\Delta_1} (\ell_1 (1 - S_L) - \rho_1 S_K) = S_K.$

Displacement of Equilibrium: As indicated above, a change in k causes a shift of the supply functions $y_1(p, k)$. Consequently, a new equilibrium is achieved by allowing p to change so that $\hat{x}_1 = \hat{y}_1$. In order to find the change in p we first find the change in w associated with a change in k . Combine (A.2.19) with (A.2.8 - A.2.10) to obtain after a simplification:

$$(A.3.7) \quad \hat{w} = \frac{1}{\sigma} \hat{k}$$

$$\sigma = \Delta_1(e_1 + I\sigma_D - \eta e_2/\ell).$$

$$\ell = \eta \ell_1 + \ell_2$$

The various variables in σ are defined in Appendix 2.

Equation (A.3.7) defines σ . It is a quasi overall elasticity of substitution in that it shows the relative change in w associated with such changes in k , evaluated along the equilibrium path of the economy. It cannot, however, be derived from an aggregate production function as elasticity of substitution is derived ordinarily. The discussion in Appendix 2, shows that $\Delta_1 e_1 > 0$, $\Delta_1 I > 0$, $\Delta_1 e_2 < 0$. Consequently $\sigma > 0$. It thus appears that an increase in the overall capital labor ratio increases the wage rental ratio along the equilibrium path.

Combining (A.2.10) with (A.3.7) :

$$(A.3.8) \quad \hat{p} = \frac{1}{\sigma} \hat{k}$$

As $I = \alpha_1 - \alpha_2$ and p is the relative price of sector 1, we obtain that the price of the labor intensive sector increases with k , as stated in proposition 3.1.

To obtain the changes in outputs we substitute (A.3.7) in (A.2.8) and (A.2.9) to obtain:

$$(A.3.9) \quad \hat{y}_1 = \left(\frac{e_1}{\sigma} + \frac{l_2}{\Delta_2} \right) \hat{k}$$

$$(A.3.10) \quad \hat{y}_2 = \left(\frac{e_2}{\sigma} + \frac{l_1}{\Delta_1} \right) \hat{k}$$

The terms on the r.h.s. represent the price and expansion effects respectively. Thus, in terms of Figure (3.3) the movement from A to A* is represented by l_2/Δ_2 and l_1/Δ_1 for y_1 and y_2 respectively. With sector 1 being labor intensive, $\Delta_1 > 0$ and $\Delta_2 < 0$. The movement from A* to B is represented by e_1/σ and e_2/σ with signs opposite to those of the respective expansion effects.

Alternatively, the two effects can be shown with reference to the demand functions. To show this we return to the individual demand functions, and rewrite the rates of change in (A.2.11) and (A.2.12) in terms of the

compensated elasticities. That facilitates a reduction of the price effect to a pure substitution effect. For instance, for $\hat{x}_1$ we have:

$$\hat{x}_1 = \epsilon_{11}\hat{p}_1 + \epsilon_{12}\hat{p}_2 - E_{1c}(\pi_1\hat{p}_1 + \pi_2\hat{p}_2) + E_{1c}\hat{c}.$$

In the present model consumption is equal income, and therefore $\hat{c} = \hat{y}$ and also $\pi_1 = \frac{p_1 y_1}{y} = \frac{p_1 x_1}{c}$. Also we can express $\hat{c} = (\pi_1\hat{p}_1 + \pi_2\hat{p}_2) + \hat{c}|_p$ and using (A.3.5) $\hat{c}|_p = S_k \hat{k}$. Consequently,

$$(A.3.11) \quad \hat{x}_1 = \epsilon_{11}\hat{p} + E_{1c}S_k \hat{k}$$

and similarly,

$$(A.3.12) \quad \hat{x}_2 = \epsilon_{21}\hat{p} + E_{2c}S_k \hat{k}$$

Substitute (A.3.8):

$$(A.3.13) \quad \hat{x}_1 = (\epsilon_{11}I/\sigma + E_{1c}S_k)\hat{k}$$

$$(A.3.14) \quad \hat{x}_2 = (\epsilon_{21}I/\sigma + E_{2c}S_k)\hat{k}.$$

With $\hat{p} = \hat{p}_1 - \hat{p}_2$ $\epsilon_{11} < 0$ and $\epsilon_{21} > 0$. Thus, in terms of Figure 3.3, the consumption expansion effect is a movement from A to C, and is represented by $E_{1c}S_k$. This effect is positive for the two products.

The substitution effect is a movement from C* to B and is given by

$\epsilon_{11}I/\sigma < 0$ for x_1 and $\epsilon_{21}I/\sigma > 0$ for x_2 . It is possible to compare the two effects. Note that we can write $\epsilon_{11} = -\sigma_D \pi_2 E_{2c}$.¹ Then, for x_1 we

¹From (A.2.19) $-\sigma_D = (\epsilon_{11} - \frac{E_{1c}}{E_{2c}} \epsilon_{21})$. Using (A.2.18) and (A.2.13), the result is obtained.

obtain that the expansion effect is dominating if and only if:

$S_k > \frac{\pi_2 E_{2c} I \sigma_D}{E_{1c} \sigma} = \frac{\sigma_D \pi_2 I}{\eta \sigma}$. The smaller is the difference in factor intensity, as measured by I the smaller is the substitution effect.

Resource Allocation

Following the discussion in the chapter, the changes in resource allocation represent in addition to product expansion and price effects, a factor substitution effect. To isolate the latter differentiate the production functions of (2.9) and (2.10):

$$(A.3.15) \quad \hat{y}_1 = \hat{l}_1 + (1 - \alpha_1) \hat{k}_1.$$

Write $\hat{k}_1 = \sigma_1 \hat{\omega}$ and note that the pure factor substitution effect is obtained for constant y_1 :

$$(A.3.16) \quad \left. \hat{l}_1 / \hat{\omega} \right|_{y_1} = -(1 - \alpha_1) \sigma_1 < 0.$$

To obtain a similar expression for ρ_1 we write $\rho_1 = k_1 l_1 / k$ and differentiate for a constant k :

$$(A.3.17): \quad \left. \hat{\rho}_1 / \hat{\omega} \right|_{y_1} = \hat{k}_1 + \hat{l}_1 \Big|_{y_1} = \alpha_1 \sigma_1 > 0.$$

The total change in labor allocation is obtained from:

$$\hat{l}_1 = \hat{y}_1 - (1 - \alpha_1) \sigma_1 \hat{\omega}$$

where $\hat{y}_1$ can be evaluated either from (A.3.9), (A.3.10) or by imposing $\hat{y}_1 = \hat{x}_1$ and using (A.3.13) and (A.3.14).

CHAPTER 4 - CONSEQUENCES OF TECHNICAL CHANGE1. INTRODUCTION

In economic thinking, the performance of the economy depends to a large extent on the nature of the implemented technology. However, the empirical determination of the role of technology is not a simple matter as one can learn from the voluminous work on that subject. Basically, the difficulty arises from the fact that technology is not an observed quantity but rather an abstract concept. It is fair to say that the dominant approach in economics has been to identify technology with a production function and technological change has been associated with shifts in the production functions. The concept of a production function is borrowed from the sciences where input-output relationships are generated by what is thought to be well described processes. As such, the production functions describe some laws of nature. Empirical applications which are close in spirit to such a concept are given by agricultural production functions (cf. Heady and Dillon, 1962). Examples from agriculture had a general flavor in the early days of economic theory when agriculture accounted for a large part of total output. However, that attribute diminished with time and it became desirable to have similar applications to non-biological processes, leading to "engineering production functions" (Chenery, 1949). Thus, in the words of Solow (1967, p. 26) "The pure theory of production is fundamentally micro-economic in character; it deals with physically identifiable inputs and outputs. In the classroom one usually says that the economic theory of production takes for granted the 'engineering' relationships between inputs and outputs and goes on from there. By contrast, much (though not quite all)

of the recent interest in the theory of production has been macroeconomic in character. Since the 'inputs' and 'outputs' are statistical aggregates like 'labor', 'plant', 'equipment', 'durable manufactures', there is no possibility of finding engineering relationships. Econometric methods have to do duty instead. Still, it remains an intriguing idea to deduce economically useful production functions from raw technological information."

The role attributed to econometric analysis in this statement is rather narrow and basically an impossible one. For one thing, aggregation of inputs and outputs depends on prices and to do it right requires a knowledge of the production functions, the very thing one wants to estimate with the aggregated variables. Second, the micro "engineering functions" change with time and as such affect the relationships between the aggregates. Thus there is no pure engineering type relationships that indeed can be revealed by econometrics. In fact, Solow does not really believe in it himself when he says that "If aggregation is inevitable. relax and enjoy it." (Ibid, p. 27).

The work on production functions has progressed in several directions, determined largely by the objective. The empirically oriented work has searched for algebraic relationships which make economic sense and at the same time can be fitted to the data. The economic sense is judged by requirements that are imposed on the micro functions. The estimation is conducted under the basic, but tacit, assumptions that all the observations were generated from the same aggregate relationship,

which by itself is allowed to change with time in one way or another. That work alone is very rich as, for instance, can be learned from the survey by Fuss, McFadden and Mundlak (FMM, 1978). New developments which have taken place since, largely fall within the same conceptual framework.

Alternatively, theoretical work on technical change concentrated on formulations which could be conveniently analyzed. The outcome of such analysis is not the same as outcome of the empirically oriented literature. The difference between the two approaches can be attributed to the complexity of the subject matter and more work is needed to integrate the various approaches.

Our interest in the subject covers both aspects. We want to trace the effect of technical change on the behavior of the economy. However, we want to do it in order to be able to come up with empirically verifiable conclusions. In this we follow the procedure we have followed thus far. We begin with this chapter where we trace the effect of technical change on the behavior of our simple neoclassical economy. For that matter, we adopt the convenient conventional formulations of technical change. Later on, in chapter xxx we take up a somewhat different view of technology which is expected to be more realistic and therefore more pertinent empirically. It is more difficult to trace the consequences of that approach analytically and it is therefore evaluated using numerical methods. Basically, that approach recognizes that at any point in time, the economy employs many techniques, each identified with a production function. We thus identify technology, with the collection of all techniques. This is in contrast to

the standard approach, to be followed in this chapter which assumes that the technology of each sector can be represented by a well defined production function.

A general formulation of technical change is to write the production function with an explicit representation of a technology shifter, t . For a single product function with two inputs, this is written as:

$$(1) Y = F(K, L, t)$$

where $\frac{\partial F}{\partial t} > 0$.

The essence of technological change is that a given set of inputs gives a larger output, or alternatively, the same output can be obtained with less inputs. The formulation in (1) is too general to allow specific results. A common simplification is that of factor augmenting technical changes (FATC):

$$(2) Y = F(A_L(t)L, A_K(t)K)$$

where $A_L(t)$ and $A_K(t)$ are positive functions of the technology index t . As it is assumed that technology improves with time. we usually identify t with time. Actually, A_L and A_K can be functions of various variables. This possibility is further discussed in Chapter xxxx below. However, for the present discussions, they are assumed to be determined uniquely by the technology which in turn is monotonic increasing with time. As such, the

identification of t with time is just a question of language. The essence of (2) is that the new technology expresses itself only as a result of an interaction with the inputs. Consequently, the products $A_L(t)L$ and $A_K(t)K$ are viewed as effective inputs measured in efficiency units. The functions can be standardized so that for any arbitrary technology, designated as $t = 0$, we have $A_L(0) = A_K(0) = 1$. $A_L(t)$ and $A_K(t)$ are referred to as labor and capital augmenting functions respectively.

A special case of this formulation is that of equal augmentation, $A_L(t) = A_K(t) \equiv A(t)$ for all t . Since F is homogeneous of degree one in the effective inputs $A(t)$ can be factored out so that

$$(3) Y = A(t)F(K, L)$$

In this case, the technical change is referred to as Hicks neutral (NTC). The neutrality of this concept is related to the invariance of the marginal rate of factor substitution to $A(t)$.

Other cases of interest are when only one input is affected by the TC. The case of $A_L(t) > 1$ and $A_K(t) = 1$ is identical with what is known as Harrod neutral TC (HNTC) whereas the opposite case of $A_L(t) = 1$ and $A_K(t) > 1$ is known as Solow neutral TC (SNTC). These concepts of neutrality are discussed in Section 3.6 below.

There is analytic convenience in dealing separately with NTC and one factor augmentation. The function in (2) can be brought to such a form by factoring out one of the A 's. For instance:

$$(4) Y = A_L(t)F(a_K(t)K, L)$$

where $a_K(t) = A_K(t)/A_L(t)$.

Note that $a_K(t)$ can now be viewed as a capital augmenting function. For that, however, $a_K(t)$ has to be larger than 1. If $a_K < 1$ then the term $a_K(t)K$ will decline with time with K held constant. In this case it is more meaningful to factor out A_K . Thus, for $F(.)$ to increase with t factor out $\underline{A}(t) = \min(A_L(t), A_K(t))$. The factored function $\underline{A}(t)$ is the common rate of augmentation of the two factors. This is the neutral component.

When we now turn to the two sector economy, we have two production functions. The TC in each sector is decomposed according to (4). The analysis of the effect of TC on the economy is first conducted by considering only NTC in the two sectors. This is followed by analyzing only one factor augmentation in each sector. The final outcome of course is obtained by combining the two cases.

Technical change affects the supply side of the economy and consequently the equilibrium position. In tracing the effect on the supply side, we differentiate between various effects: allocation, efficiency and cost to be explained below.

2. HICKS NEUTRAL TECHNICAL CHANGE

2.1 General Results

In this section we examine the effect of NTC in the two sectors. The sectoral production functions have the form of (4). They can therefore be written as follows to replace (2.9) and (2.10):

$$(5) \quad y_1 = \ell A_1(t) f_1(k_1)$$

$$(6) \quad y_2 = (1 - \ell) A_2(t) f_2(k_2)$$

The supply of the economy now consists of (2.1), (2.2), (2.5), (2.6), (5) and (6). Writing the competitive conditions under the present formulation:

$$(7) \quad r/p_1(\omega, t) = A_1(t) f_1'(k_1)$$

$$(8) \quad w/p_1(\omega, t) = A_1(t) f_1(k_1) - k_1 A_1(t) f_1'(k_1)$$

and hence the wage rental functions are:

$$(9) \quad \omega_1(k_1) = \frac{f_1(k_1)}{f_1'(k_1)} - k_1$$

Since $\omega_1(k_1)$ is unaffected by $A_1(t)$, so is the resource allocation corresponding to a given ω and (2.12) still determines the proportion of agriculture in the labor force, as a function of ω .

On the other hand, $\bar{p}(\omega)$ depends on the technology and therefore may

generaally be affected by the technical change. It is therefore denoted as $p(\omega, t)$. Similarly, the supply functions are also affected and will be denoted by $y_1(p, k, t)$ or $y_1(p, t)$ when the dependence on k is suppressed.

To derive $p(\omega, t)$ we use (7) to obtain:

$$\begin{aligned} (10) \quad p(\omega, t) &= \frac{A_2(t) f_2'(k_2(\omega))}{A_1(t) f_1'(k_1(\omega))} \\ &= \frac{A_2(t)}{A_1(t)} p(\omega, 0) \end{aligned}$$

where $p(\omega, 0)$ is p evaluated in terms of the base technology.

Thus, the relative price of the product which undergoes a larger rate of technical change declines. In the special case of $A_2(t) = A_1(t)$, $P(\omega, t) = P(\omega, 0)$.

Next we turn to examine the changes in the outputs corresponding to a given allocation. We begin with the case of $A_1(t) = A_2(t)$.

2.2 Neutral Technical Change - Equal Rates.

In this section we deal with the case of $A_1(t) = A_2(t) = A(t)$. As the $\omega_1(k_1)$ functions are unaffected by the NTC, there is no change in resource allocation for any given ω . The change in outputs corresponding to a given resource allocation is determined by $A(t)$. This is defined as the efficiency effect.

Definition: The efficiency effect is the increase in output derived from a constant bundle of resources.

By assumption, we deal here with the case where the efficiency effect is the same for both sectors. It can be expressed in terms of the transformation curve. Referring to Figure 4.1, point A is the initial point, and point B is the output in time t representing the efficiency effect. It is located on a ray through A and its coordinates are $A(t)$ times the coordinates of A. Equal rates of NTC leaves the $p(\omega)$ function unchanged, as $p(\omega, t) = p(\omega)$ for all t . Thus, in terms of Figure 4.1 $p_B = p_A$. The constancy of $p(\omega)$ is referred to as no cost effect to be defined below. Therefore, in this case, the shift of the supply function, $y_1(p, t)$ represents only the efficiency effect.

Technical change increases real income in that it expands the consumption possibilities of the economy. This affects the demand function $x_1(p, y_2(p, t))$ defined on all the feasible $y_2(p, t)$. Thus, a shift in $y_2(p, t)$ causes a shift in the demand x_1 . This shift is shown in panel III of Figure 4.1.

The new equilibrium point E is obtained at the intersection of the new supply and demand functions. The new equilibrium values can be read in the various panels of Figure 4.1. Thus, p declines from p_A to p_E . Moving up to panel II, it is seen that ω declines to ω_E . Panel I indicates the corresponding declines in k_1 . With k held constant, that implies a shift

of resources to the capital-intensive sector. Since the efficiency effect is the same in the two sectors, the change in resource allocation implies a larger relative increase in the production of sector 2 which is both, the capital intensive and the high income elasticity sector.

The dependence of the solution on income elasticity is underlined when the equilibrium solution is obtained in panel IV of Figure 4.1. Note that $p_A = p_B$. With p_A , the demand curve intersects the new transformation curve at point C with $p_C < p_B = p_A$. Following the analysis of Chapter 2, we find the new equilibrium point E, located somewhere between B and C, with $p_E < p_B$. Having determined y_{1E} , p_E is read from panel III and the analysis proceeds as before.

In generalizing the finding two points must be kept in mind. First, that the equilibrium change in ω depends on the capital intensity assumption. Second, that if the income elasticity of the agricultural product were larger than 1, then point E would be to the left of B, say E^* , with $p_{E^*} > p_B = p_A$ and consequently $\omega_{E^*} > \omega_A$. With these observations, we can now state:

Proposition 1: (Equilibrium effects of neutral technical change of equal rates).

Equal rates of neutral technical change in the two sector produce efficiency effects. As a result, outputs increase in the two sectors.

The relative price of the product whose income elasticity is smaller than 1 declines. The wage rental ratio increases when the low income elasticity sector is capital intensive and decreases when that sector is labor intensive. Prices are unaffected when both income elasticities are unitary or when factor intensities are the same.

2.3 Neutral Technical Change - Unequal Rates.

Next we turn to unequal rates of NTC. In dealing with this case it suffices to consider TC only in one sector. That is, any NTC in the two sectors can be decomposed as follows: Let

$$(.) \equiv (A_1(t), A_2(t)), \underline{A}(t) = \min(.), \bar{A}(t) = \max(.).$$

The equal rate is given by $\underline{A}(t)$ and the differential rate is $\bar{A}(t) - \underline{A}(t)$.

In what follows we let agriculture be the active sector in that $A_1(t) > 1$ and $A_2(t) = 0$. Consequently, the efficiency effect is limited to agriculture. It is shown in panel IV of Fig. 4.2 as a movement from A to B. Both of these points are obtained with the resource allocation corresponding to the wage-rental ratio w_A .

Unequal rates of technical change affect the relative costs of production corresponding to given factor prices. The function $p(w, t)$ is now different from $p(w, 0)$ as can be seen from equation (10). This is

shown in panel II of Fig. 4.2, with $p_B < p_A$. The initial price, p_A , can be restored by shifting resources to the active sector, to obtain point A^* , with $p_{A^*} = p_A$. The movement from B to A^* is referred to as the cost effect. More formally, let $y_i(p, t)$ $i=1,2$ be the supply schedule under technology t and $y_i(l_i, k_i, t)$ the percapita output obtained with resource allocation l_i, k_i and technology t .

Definition: The shift of supply from $y_i(p, 0)$ to $y_i(p, t)$ caused by technical change in one sector represents the cost effect in the passive sector and efficiency and cost effect in the active sector. The cost effect in the active sector can be obtained by subtracting the efficiency effect. That is, the cost effect in the active sector is:

$$y_1(p, t) - y_1(p, 0) - (y_1(l_1, k_1, t) - y_1(l_1, k_1, 0)) .$$

Proposition 2. (cost effect): The cost effect of technical change is positive for the active sector and negative for the passive sector.

Thus, in the active sector, the cost effect supplements the efficiency effect. In the passive sector, the cost effect reduces output whereas the efficiency effect leaves the output of that sector unchanged. The shift in the supply function of the active sector is illustrated in panel III of Figure 4.2.

The shift in $y_2(p, t)$ affects $x_1(p, y_2(p, t))$, the quantity demanded at a given p now declines. With sector 2 being the passive sector $y_2(p, t)$

declines, and thus causes a decline in the demand. The new equilibrium is then determined by an intersection of a larger supply and smaller demand resulting in a lower equilibrium price. Thus, the income and price effects supplement each other and the equilibrium quantity of the active sector will increase. For the passive sector the two effects contradict each other and the direction of the change is indeterminant. That can be illustrated in terms of Fig. 4.2. The demand $x_1(p_A, y_2)$ intersects the new transformation curve at C. Thus at p_A , the excess supply is given by $z_1(p_A) = y_{1A*} - x_{1C} > 0$, and the equilibrium price will be lower than p_A . The output of the active sector will increase. The equilibrium output of the passive sector depends on the price elasticity of demand. To see this we can obtain from (10) that $\hat{p}(\omega) \big|_{\omega, A_2(t)} = -\hat{A}_1(t)$. That is, with ω and industrial technology held constant, the rate of change in p is equal to the rate of technical change (with sign reversed). Consequently, $p_B = p_A(1 - \hat{A}_1)$. On the other hand, $y_{1B} = y_{1A}(1 + \hat{A}_1)$. Thus, the new demand curve will pass through B when the price elasticity (2.17) is unitary. It will intersect to the left of B when the price elasticity is larger than 1 and to the right of B when the elasticity is smaller than 1.

To trace the implications of the shift in equilibrium for resource allocation we note that $\omega_B = \omega_A$. Let sector 1, the active sector, be labor intensive and p be its relative price. Then, points on the transformation curve to the right of B correspond to values of ω which are lower than ω_A and conversely for points to the left of B. In other words, points to the right of B imply a shift of resources to the passive sector, which in our case is capital intensive, whereas points to the left of B imply a shift of resources away from the passive sector. We can now summarize the

discussion in proposition 3.

Proposition 3: (Equilibrium effects of neutral technical change of unequal rates). Assume neutral technical change in one sector (the active sector) and constant technology in the other sector (passive sector). Then

- (1) The efficiency effect is positive for the active sector and zero for the passive sector. The cost effect is positive for the active sector and negative for the passive sector.
- (2) The equilibrium (relative) price of the active product declines and its output increases. The equilibrium output of the passive sector declines, remains constant or increases when the absolute value of the demand elasticity for the active product is larger, equal or smaller than 1 respectively.
- (3) Allocation of resources changes in the same direction as the changes in the output of the passive sector.
- (4) The change in w is in the same direction as that of the allocation of resources to the labor intensive sector.

Propositions (1) and (3) can now be combined to give the net effect of the neutral technical change in the two sectors. A basic result developed in the Appendix can be written as:

$$\text{sign } (\hat{w}|_{k, \text{equil.}}) = -\text{sign } \Delta_1 E_0$$

where

$$\Delta_1 = \hat{\lambda}_1 - \rho_1$$

$$\begin{aligned} E_0 &= \hat{A}_1(1 - \sigma_D) - \hat{A}_2(\eta - \sigma_D) \\ &= \hat{A}_1(1 - \eta) + (\hat{A}_1 - \hat{A}_2)(\eta - \sigma_D) \end{aligned}$$

Consequently, for $\hat{A}_1 = \hat{A}_2$, $E_0 = (1 - \eta) \hat{A}_1$ and only the income elasticity matters. On the other hand, for $\hat{A}_2 = 0$, $\hat{A}_1 > 0$, $E_0 = \hat{A}_1(1 - \sigma_D)$ and this is the case discussed in proposition (2).

3. FACTOR AUGMENTING TECHNICAL CHANGE

3.1 The Response of the $w(k)$ Function

We begin by examining some general properties related to factor augmenting technical change. For that purpose there is no need at this point to differentiate between the sectors. Consider the production function:

$$(11) \quad Y = F(K^e, L^e)$$

where $K^e = A_K K$ and $L^e = A_L L$ are capital and labor measured in terms of efficiency units, respectively. F is constant returns to scale, concave and twice differentiable function in K^e and L^e . We can thus write the average labor productivity for physical labor, $y = Y/L$.

$$(12) \quad y = A_L f(k^e)$$

where k^e is the efficiency capital-labor ratio. $k^e = a_k k$, $a_k = A_K/A_L$. As the function is defined in terms of efficiency units, the marginal productivities and the competitive conditions can be immediately written as:

$$(13) \quad F_K^e = f'(k^e) = r^e/p$$

$$(14) \quad F_L^e = f(k^e) - k^e f'(k^e) = w^e/p.$$

Under the competitive conditions, factor prices in efficiency units are equal to the marginal productivities.

The marginal productivities and the competitive conditions can also be stated in terms of physical units: $F_K = \frac{\partial Y}{\partial K} = \frac{dK^e}{dK} F_K^e = F_K^e A_K$ and similarly, $F_L = F_L^e A_L$. By the competitive conditions we have $r/p = F_K$ and $w/p = F_L$. Combining this with (13) and (14), we obtain the relationships between factor prices in physical and efficiency units:

$$(15) \quad r = A_K r^e$$

$$(16) \quad w = A_L w^e$$

and consequently,

$$(17) \quad \omega = w/r = \omega^e / a_k$$

and in equilibrium we have

$$(18) \quad \omega^e(k^e) = \frac{f(k^e)}{f'(k^e)} - k^e$$

Note that the production function is formulated in terms of efficiency inputs. On the other hand, economic data are reported in terms of physical inputs. It is therefore of interest to examine the implication of the FATC on the relationships pertaining to physical inputs. We start by examining the effect of augmentation on $\omega(k)$. This result is then used for the evaluation of the effect of FATC on the supply side and the equilibrium position of the economy.

We start with a graphical exposition where we use the function $\omega^e(k^e)$ drawn in Figure 4.3 to determine how $\omega(k)$ is affected by the FATC. Select an arbitrary point k_0 . For the initial technology we set $a_k(0) = 1$. Thus $\omega(k_0 | a_k = 1) = \omega^e(k_0^e | a_k = 1)$, as shown by point C. As a result of the FATC we now have, without a loss in generality, $a_k(t) > 1$, and the value of k_0 in terms of efficiency units become $k_1^e = a_k k_0$. Then, by (17), we have $\omega(k_0 | a_k) = \frac{1}{a_k} \omega^e(k_1^e | k_1^e = a_k k_0)$. We can add to the right hand side of this equation $(1 - \frac{1}{a_k})0$ and thus $\omega(k_0 | a_k(t))$ is considered to be a weighted average of $\omega^e(k_1^e)$ and the origin. Consequently, by definition of concavity, $\omega(k_0 | a_k(t))$ will be below point C, say point N, when $\omega^e(k^e)$ is concave. We can now repeat the process for other initial values instead of k_0 and thereby generate the function $\omega(k | a_k)$. The graph of this function will go through points like N and thus will be below and to the right of $\omega^e(k^e)$ when the function is

strictly concave, above and to the left of $\omega^e(k^e)$ when the function is strictly convex and will be identical with $\omega^e(k^e)$ when that function is a straight line through the origin. But those three possibilities correspond to σ larger, smaller or equal to 1, respectively.

Having established the geometry, we can turn to the economic explanation. For that, assume only capital ATC so that $a_k = A_K$. Increasing the efficiency of capital when σ is large allows its absorption with little change in the marginal rate of substitution of the efficiency factors or simply in ω^e . Consequently, $\omega = \omega^e/a_k$ decreases. The situation is reversed for small σ , in which case ω^e has to increase considerably in order to absorb the change in k^e . This change in ω^e is larger than the change in A_K and therefore ω increases. The two effects are the same for $\sigma = 1$.

Finally, we can also derive this result analytically. As the production function is defined in terms of the efficiency factors, σ is the elasticity of substitution of that function. Consequently we can write:

$$d \ln \omega^e = \frac{1}{\sigma} d \ln k^e$$

But by definition, $d \ln \omega^e = d \ln \omega + d \ln a_k$ and $d \ln k^e = d \ln k + d \ln a_k$, hence

$$(19) \quad \left. \frac{\partial \ln \omega}{\partial \ln a_k} \right|_k = \frac{1}{\sigma} - 1$$

and

$$(20) \quad \frac{\partial \ln k}{\partial \ln a_k} \bigg|_{(u)} = \sigma - 1$$

The discussion can then be concluded with the following proposition:

Proposition (4): Factor augmenting technical change (FATC) in one factor generates the following changes in the factor intensity of that factor under constant wage-rental ratio: the intensity increases, remains constant, declines if and only if σ is larger, equal or smaller than 1, respectively.

A change in the function $\omega(k)$ leads also to a change in the income distribution. Let the ratio of the two factor shares be written as $\frac{wL}{rK} = \omega/k$. In terms of Figure 4.3 this ratio is equal to $\tan \theta$. Consequently, the movement from C to N reduces the labor share, and can be viewed as labor saving. Note that this outcome is obtained by capital augmenting technical change and elasticity of substitution larger than 1. The logic of it is clear; an increase in efficiency capital and ease in its absorption results in a substitution of capital for labor to the extent that the share of labor in total output is reduced. It should be emphasized that the classification of a particular factor augmentation as a factor saver depends not only on the factor which is augmented but also on the elasticity of substitution. We can then have the following corollary to proposition (4).

Corollary: The share of the augmented factor in total output, with the wage-rental ratio held constant, increases, remains constant or declines if and only if σ is larger, equal, or smaller than 1, respectively. The augmentation is referred to as factor using, neutral and factor saving respectively.

3.2 Allocation and Efficiency Effects

In a full employment economy, a change in factor intensity in one sector, with resources held constant, requires a change in resource allocation, otherwise the full employment conditions will be violated. An increase in $k_1(\omega)$ in either sector, regardless of capital intensity, can take place only if the shares of the capital intensive sector in the employment of both resources decline. The opposite is true if $k_1(\omega)$ declines. This follows from the full employment condition as stated in equation (2.7) which can be rewritten as:

$$l(\omega) = \frac{k_2(\omega, t) - k}{k_2(\omega, t) - k_1(\omega, t)}$$

Thus, $l(\omega)$, which in this case is interpreted as the share of the labor intensive sector in total employment, is positively related to either $k_1(\omega)$ or $k_2(\omega)$. We deal with a FATC in one sector only, thus, one $k_1(\omega)$ remains constant. For this ratio to remain constant, the change in the share of this sector in the total use of capital should follow the same pattern as that of labor. That is, $\rho(\omega)$ behaves similarly to $l(\omega)$ in its

response to FATC. We thus have:

Proposition 5. With w held constant, factor saving technical change in either sector increases the relative share in factor employment of the sector which is intensive in the saved factor.

The change in factor allocation which takes place at a constant w causes a change in sectoral output not realized under NATC.

Definition: The change in outputs which results from resource allocation necessary to restore the wage-rental ratio under FATC is defined as allocation effect.

The allocation effect may supplement or contradict the efficiency effect. The possibilities are summarized in proposition 6.

Proposition 6. The allocation effect is positive for the active sector when that sector is intensive in the factor saved, and negative when it is extensive in the saved factor. The sign of the allocation effect of the passive sector is opposite to that of the active sector.

The allocation and efficiency effects of a FATC in sector 1 are illustrated in Figure 4.4. Panel I shows the change in $\omega_1(k_1)$:

$$\omega_B = \omega_1(k_{1A}, t) \neq \omega_1(k_{1A}, 0) = \omega_2(k_{2A}).$$

Consequently, the initial allocation represented by point A, becomes inefficient. With the new technology, the capital labor ratio in the active sector evaluated at ω_A is $k_1(\omega_A, t) = k_{1C}$, and consequently, the shares of this sector in total labor is $\ell(\omega_A, t) > \ell(\omega_A, 0)$. This is simply an application of proposition 5, to the case where the technical change is labor saving. The implied impact of this change on outputs is given by Point C on the transformation curve in panel IV of Fig. 4.4. This represents the combined efficiency and allocation effects. The efficiency effect alone is represented by B, which is now an internal point under the new technology. It is dominated by points like B^* corresponding to $\omega_B^* < \omega_A$.

3.3 The Cost Effect

To obtain the total shift in the supply, we have to add the cost effect. The cost effect was derived above from (10), which should now be generalized to:

$$(21) \quad p(\omega, t) = \frac{A_2(t)f'_2(k_2(\omega, t))}{A_1(t)f'_1(k_1(\omega, t))}$$

In the present case $A_2(t) = 1$, $A_1(t) > 1$, $k_2(\omega, t) = k_2(\omega, 0)$ and $k_1(\omega, t) > k_1(\omega, 0)$. which implies $f'_1(k_1(\omega, t)) < f'_1(k_1(\omega, 0))$. Thus, the change in technology affects the two terms in the denominator in different directions. The net effect should be a reduction in the price of the active sector, but as we have just seen, this is not immediately obvious from (21). The result, however, can be obtained by using the cost function. The competitive assumption forces producers to produce at zero profits, implying that p is equal to the ratio of the average costs of production in the two sectors. The cost functions, being

dual to the production functions are also expressed in terms of efficiency units. Thus, for sector i we have for the average cost function: $c_i(w^e, r^e) = c_i(\frac{w}{A_{iL}}, \frac{r}{A_{iK}})$ and consequently,

$$(22) \quad p(\omega, t) = c_1(w^e, r^e) / c_2(w^e, r^e) .$$

The cost function is increasing in factor prices. Therefore, keeping technology in sector 2 constant, any increase in A_{1L} or A_{1K} or both, decrease the price of that sector.

Returning to the case under consideration, $p(\omega, t) < p(\omega, 0)$ and, therefore, the change in output necessary to restore the original price calls for a shift of resources to the active sector. Thus, the cost effect supplements the efficiency and allocation effects. In terms of Fig. 4.4, the cost effect is represented by the move from C to H.

3.4 Equilibrium

Finally, the new equilibrium point for the economy will be located between the following two points: (1) The intersection of the demand curve, corresponding to the initial price, with the new transformation curve. That point gives the quantity demanded at the original price and this represents the income effect alone; (2) The new product mix at the original price. The income effect is represented by a movement from B to D in figure 4.4. The location of the supply at p_A depends on the relationships between the allocation and the other two effects. When they supplement each other, the supply is given by H, to the left of C. Consequently, the excess demand at p_A is $z_1(p_A) = x_{1D} - y_{1H} < 0$, the price will decline and the equilibrium point will be located between H and D.

With the equilibrium price of the active sector declining, its output increases. The question is what happens to the output of the passive sector, does it increase as well? The answer depends on the relative strength of the price and income effects generated by the technical change and on the price and income elasticities.

The dependence of the solution on the demand elasticities is intuitively clear. If the income and price elasticities for the product of the active sector were very high, then the technical change might cause a shift of demand in the direction of the active sector to the extent that will cause a decline in the output of the passive sector. In fact, it is sufficient that the demand will be perfectly elastic, as it is the case for the small country open economy, that the output of the passive sector will shrink. In that case, H will represent the equilibrium output. The consumption will be determined by the intersection of demand functions with the income corresponding to output H.

3.5 Factor augmenting technical change in the two sectors.

The foregoing analysis was conducted for the special case where the FATC occurred in one sector. With two sectors and two factors, there are various combinations that might occur. Those, however, can be reduced to the situations analyzed above plus an additional one to be outlined here: FATC in two sectors.

Our foregoing discussion dealt with four factor augmenting functions; $\hat{A}_{1L}, \hat{A}_{1K}, \hat{A}_{2L}, \hat{A}_{2K}$. For each sector this is reduced to $\hat{A}_i, \hat{a}_i$, where $\hat{A}_i = \min(\hat{A}_{iL}, \hat{A}_{iK})$, and $\hat{a}_i = \max(\hat{A}_{iL} - \hat{A}_i, \hat{A}_{iK} - \hat{A}_i)$. The case of $\hat{a}_i = 0$ for $i = 1, 2$ were dealt with under the NTC. The case of $\hat{a}_i \neq 0$ for one sector was dealt above. Thus, to complete the discussion we have to consider the case of $\hat{a}_1 \hat{a}_2 \neq 0$.

Efficiency effects: By definition, the efficiency effects in both sectors are positive. The strength of the effects depends on the strength of the $\hat{a}_i$ and the factor share of the augmented factor.

Allocation effect: If the technical change in both sectors is saving the same factor, then the allocation effect is uniquely determined - it increases the output of the sector which is intensive in the saved factor. This situation covered by proposition 5, is of a particular interest. As we will see in Chapter 8, there is a reason to expect technical change to be on the whole labor saving. Thus, this process has the partial effect of transferring resources to the labor intensive sector.

The relative strength of this effect depends on the rates of factor augmentation and on the values of σ_i . Specifically, recall that if $\sigma_i = 1$ for $i = 1, 2$, there is no allocation effects.

Cost effects: The behavior of $p(\omega, t)$ in t depends on what factors were augmented and their sectoral factor shares. To concentrate on the case under consideration, assume $\hat{A}_{1K} > 0$, $\hat{A}_{2K} > 0$, $\hat{A}_{1L} = \hat{A}_{2L} = 0$. For this to be labor intensive we need $\sigma_1 > 1$. Now, in the special case where the rate of factor augmentation is the same ($\hat{A}_{1K} = \hat{A}_{2K}$), we obtain

$$\hat{P}(\omega, t) \Big|_{\omega} = (S_{1L} - S_{2L}) \hat{A}_{1K} .$$

That is, capital augmenting technical change of equal rates in both sectors increases the price of the labor intensive products corresponding to any given wage rental ratio.

3.6 Income Distribution

The factor augmenting technical change discussed above leads to changes in the factoral distribution of income, as measured by the ratio of factor shares. As shown in Section 3.1, $\hat{\omega}^e = \frac{1}{\sigma} \hat{k}^e$ and by substituting $\hat{\omega}^e = \hat{\omega} + \hat{a}_k$ $\hat{k}^e = \hat{k} + \hat{a}_k$ and rearranging terms to obtain

$$(\omega/k) = (\frac{1}{\sigma} - 1)(\hat{k} + \hat{a}_k)$$

There are three conditions under which ω/k remains constant: (1) $\hat{a}_k = 0$ (2) $\sigma = 1$ and, (3) $\hat{k} = -\hat{a}_k$. The case of $\hat{a}_k = 0$ is that of Hicks neutral technical change. In this case $(\omega/k) = 0$ if and only if k remains constant. The second case is that of $\sigma = 1$ which is equivalent to a Cobb Douglas production function where the shares are everywhere constant.

In examining the third case we ask what implications does condition (3) have on the production function. To do this, fully differentiate the production function:

$$\hat{Y} = S_L(\hat{L} + \hat{A}_L) + (1 - S_L)(\hat{K} + \hat{A}_K)$$

First, assume $\hat{A}_K = 0$ so that $\hat{a}_k = -\hat{A}_L$. Rearrange terms to obtain:

$$\hat{Y} - \hat{K} = S_L(\hat{L} + \hat{A}_L - \hat{K})$$

Under condition 3, the bracketed term is zero. We then have that with labor augmenting technical change the factor shares remain constant along expansion paths which maintain Y/K constant. This is the Harrod neutrality.^{1/}

^{1/}For a proof that Harrod neutrality implies labor augmenting technical change see Burmeister, E. and R. Dobell, "Disembodied Technological Change With Several Factors," Journal of Economic Theory 1, (June 1969) pp. 1-8.

A similar argument, holding A_L constant and allowing A_K to vary indicates that factor shares are constant along expansion paths which maintain Y/L constant.

Figure 4.1

Neutral technical change of equal rates
in both sectors

Figure 4.2

Neutral technical change in Sector 1
 Constant technology in Sector 2

Figure 4.3 The Effect of Factor Augmenting Technical Change on w .
The Case of $\sigma > 1$.

Figure 4.4 FATC in sector I

APPENDIX TO CHAPTER 4CONSEQUENCE OF TECHNICAL CHANGEINTRODUCTION

The appendix is written to follow the order of the discussion in the text, starting with NTC. However, whenever it does not add complexity, the initial development is presented in more general terms, that is, for the FATC, and then the conditions of neutrality are imposed.

The discussion begins with the introduction of the efficiency and cost effects. The allocation effect is specific to FATC and is introduced in the section dealing with that change.

In order to isolate the effect of technical change it is assumed throughout that resources (k) are held constant. The economy is always in short run equilibrium.

The Efficiency Effect Under NTC

We start the analysis by differentiating sectoral production functions of the form (4.2):

$$(A.4.1) \quad \hat{Y}_1 = S_{1K}(\hat{A} + \hat{K}_1) + S_{1L}(\hat{A}_{1L} + \hat{L}_1)$$

which after rearrangement, can be written in the form of (A.2.6) and (A.2.7):

$$(A.4.2) \quad \hat{y}_1 = S_{1L} \hat{\ell}_1 + S_{1K} (\hat{p}_1 + \hat{k}) + T_1$$

$$(A.4.3) \quad T_1 = S_{1K} \hat{A}_{1K} + S_{1L} \hat{A}_{1L}$$

The efficiency effect is defined for constant resources:

$$\hat{y}_1 \Big|_{\ell, p, k} = T_1$$

Under NTC, resource allocation for fixed w and k is unaffected. Consequently, following the derivation of (A.2.8) and (A.2.9) we now derive using (A.4.2):

$$(A.4.4) \quad \hat{y}_1 = e_1^w + (\ell_j / \Lambda_j) \hat{k} + T_1 \quad j \neq 1 = 1, 2$$

The Cost Effect

Any technical change in a given sector reduces the cost of production in that sector. Under competition we have the condition of zero profits and therefore a decline in cost of production means a decline in the product price. The cost can be obtained from the cost function, dual to the production function. Write the average cost function (cost per unit of output): $c_1(w^e, r^e)$ and note that $\frac{\partial c_1}{\partial w^e} = \frac{L_1^e}{Y^e}$ ( ), which is the demand for efficiency per unit output and similarly $\frac{\partial c_1}{\partial r^e} = \frac{K_1^e}{Y^e}$ ( ) is the demand for efficiency capital per unit output. Consequently, differentiating c_1 logarithmically and setting c_1 to be equal to p_1 :

$$\hat{p}_1 = S_{1L} \hat{w} + S_{1K} \hat{r} - [S_{1L} \hat{A}_{1L} + S_{1K} \hat{A}_{1K}]$$

Taking the difference $\hat{p} = \hat{p}_1 - \hat{p}_2$,

$$(A.4.5) \quad \hat{p} = I\hat{\omega} - T$$

$$T \equiv T_1 - T_2, \quad I \equiv S_{1L} - S_{2L} = 0 \iff \lambda = \rho$$

Thus, I is a measure of factor intensity; it is positive when sector 1 is labor-intensive. It is clear that the efficiency effect on outputs changes p as $\hat{p}|_{\omega} = -T$. The cost effect measures the changes in output that restore p . To evaluate it we obtain from (A.4.5):

$$(A.4.6) \quad \hat{\omega}|_p = T/I.$$

Combining (A.4.6) and (A.4.4)

$$(A.4.7) \quad \hat{y}_1| = \frac{e_1 T}{I} + T_1.$$

Thus the cost effect on output is

$$(A.4.8) \quad \hat{y}_1|_{k,p} = \frac{e_1 T}{I}.$$

Now recall that $e_1 I > 0$ and $e_2 I < 0$. Under equal rates of NTC, $T = 0$, and there is no cost effect. Now, suppose $T_1 > 0$, $T_2 = 0$, then $\text{sign}(e_1 T/I) = \text{sign } T_1 > 0$ and $\text{sign}(e_2 T/I) = -\text{sign } T_1 < 0$. This finding is summarized as proposition 4.1. When the two sectors are active in the

sense that $T_i > 0$, $i = 1, 2$, then the sign of the cost effect is determined by sign T and factor intensity. Thus, $T < 0$ implies a positive cost effect for y_1 and negative for y_2 and conversely for the case of $T > 0$.

EQUILIBRIUM UNDER NEUTRAL TECHNICAL CHANGE

The equilibrium position is affected by TC. If nothing else, the efficiency effect changes the supply. To evaluate the displacement in the equilibrium position, we utilize (A.4.4) and (A.4.5) in the displacement of demand as written in (A.2.19) and impose $\hat{x}_1 = \hat{y}_1$.

$$(A.4.9) \quad \hat{\omega}|_{k,eq.} = \frac{\Delta_1}{\bar{\omega}} (T_1(\sigma_D - 1) + T_2(\eta - \sigma_D))$$

$$= \frac{\Delta_1}{\bar{\omega}} (T_1(\eta - 1) + (T_2 - T_1)(\eta - \sigma_D))$$

where σ is defined in (A.3.7). Substituting (A.4.9) in (A.4.4) and (A.4.5) yields the changes in the equilibrium positions of y_1 and p . To simplify matters, we examine separately the two leading cases examined in the text--equal rates and alternatively, one passive sector.

Equal Rates of NTC ($T_1 = T_2 \equiv T^*$):

Under this assumption (A.4.9) is reduced to:

$$(A.4.10) \quad \hat{\omega}|_{k,eq.} = \frac{\Delta_1 T^*(\eta - 1)}{\bar{\omega}}$$

Thus,

$$(A.4.11) \quad \text{sign } \hat{\omega} \Big|_{k,eq.} = \text{sign } \Delta_1(\eta - 1).$$

Implying that ω increases when the low income elasticity sector is capital intensive.

Under equal rates of NTC, p changes with ω alone and there is no cost effect. Thus,

$$(A.4.12) \quad \hat{p} \Big|_{k,eq.} = I\hat{\omega} \Big|_{k,eq.}$$

Substituting (A.4.10) in (A.4.12) and recalling that $I\Delta_1 > 0$,

$$(A.4.13) \quad \text{sign } \hat{p} \Big|_{k,eq.} = \text{sign } (\eta - 1).$$

A change in ω causes a change in resource allocation. To evaluate the change, we utilize (A.2.3) - (A.2.5):

$$(A.4.14) \quad \hat{\ell}_1 \Big|_{k,eq.} = \frac{\ell_2^{\sigma} \rho}{\Delta_1} \hat{\omega} \Big|_{k,eq.}$$

and hence

$$(A.4.15) \quad \text{sign } \hat{\ell}_1 \Big|_{k,eq.} = \text{sign } (\eta - 1).$$

From (A.2.5) it is clear that $(\hat{\rho}_1/\hat{\omega})|_k (\hat{\ell}_1/\hat{\omega})|_k > 0$. Thus, both resources

move in the same direction which is determined by sign $(\eta - 1)$ alone.

In view of (A.4.15) it is clear that the change in resource allocation causes an increase in output of one sector and a decrease in the other. However, even in the sector that loses resources, output will increase because the efficiency effect is dominant. Rewrite (A.4.4) under the equilibrium constraint:

$$(A.4.16) \quad \hat{y}_i|_{k,eq.} = e_i \hat{\omega}|_{k,eq.} + T_i$$

To evaluate (A.4.16), substitute (A.4.10), the definition of θ and $T^* = T_1$:

$$(A.4.17) \quad \hat{y}_i|_{k,eq.} = \frac{[e_i \Delta_1 (\eta - 1) + \theta \sigma] T^*}{\theta \sigma}$$

$$= \frac{\Delta_1}{\theta \sigma} [e_i (\eta - 1) + e_i + \theta \sigma_D - \eta e_2] T^* > 0$$

With $0 < \eta$, this expression is positive for $i = 1, 2$.

Unequal Rates of NTC

The analysis is conducted for $T_1 > 0$ and $T_2 = 0$. It immediately follows from (A.4.9) that:

$$(A.4.18) \quad \hat{\omega}|_{k,eq.} = \frac{\Delta_1}{\theta \sigma} (\sigma_D - 1)$$

and therefore

$$(A.4.19) \quad \text{sign } \hat{w} \Big|_{k,eq.} = \text{sign } \Delta_1 (\sigma_D - 1) .$$

To see the implication of this, we first examine the change in resource allocation. This can be done by combining (A.4.14) and (A.4.18) to obtain:

$$(A.4.20) \quad \text{sign } \hat{\ell}_1 \Big|_{k,eq.} = \text{sign } (\sigma_D - 1) .$$

Thus, resources shift away from the price inelastic sector. If this sector is labor intensive then w will decline, and conversely.

The shift of resources affects p but not necessarily in the same direction as the direct effect. However, the net effect is in the same direction as that of the direct effect. This is seen by combining (A.4.5) and (A.4.18), noting that $T = T_1$,

$$(A.4.21) \quad \hat{p} \Big|_{k,eq.} = \left[\frac{I\Delta_1}{\Delta\sigma} (\sigma_D - 1) - 1 \right] T_1$$

and it can be shown that the bracketed term is negative. For that, use the definition of σ to obtain:

$$I\Delta_1 (\sigma_D - 1) - \Delta_1 (e_1 - ne_2 + \sigma_D I) = -\Delta_1 (I + e_1 - ne_2) < 0 . \quad \text{Hence, in equilibrium, the relative price of the active sector always declines. This}$$

is irrespective of the price elasticity or factor intensity.

The response of sectoral outputs is different for the active and passive sectors, reflecting the differences in the efficiency and cost effects on the supply side. For the case when sector 1 is the active sector, it gains resources when $\sigma_D > 1$. This results in an increase in output which is augmented by the efficiency effect. Sector 2, the passive sector, loses resources and its output must decline. When $\sigma_D < 1$, the output of the passive sector increases but this is also the case for the active sector where the efficiency effect is dominant. This can be seen by combining (A.4.16) with (A.4.18):

$$(A.4.22) \quad \hat{y}_1|_{k,eq.} = T_1 \left[\frac{e_1 \Delta_1 (\sigma_D - 1)}{\Delta_1} + 1 \right].$$

To show that the bracketed term is positive, it is sufficient to evaluate

$$\Delta_1 [e_1 (\sigma_D - 1) + (e_1 - ne_2 + \sigma_D I)] = \Delta_1 [\sigma_D (e_1 + I) - ne_2] > 0.$$

whereas for the passive sector we have

$$(A.4.23) \quad \text{sign } \hat{y}_2|_{k,eq.} = \text{sign } (e_2 \hat{\omega})|_{k,eq.} = \text{sign } e_2 \Delta_1 (\sigma_D - 1) \\ = \text{sign } (1 - \sigma_D).$$

FACTOR AUGMENTING TECHNICAL CHANGE

Resource Allocation

As indicated in the chapter, the production functions are written in terms of efficiency inputs. Furthermore, the factor intensities are determined by ω and the technology index A_{ik} . Specifically, a change in A_{ik} changes factor intensities at any given ω .

For the i -th sector, the relationships between factor intensities in efficiency and physical units are:

$$\hat{k}_1^e = \sigma_1 \hat{\omega}^e$$

$$\hat{k}_1^e = \hat{a}_{ik} + \hat{k}_1$$

$$\hat{\omega}^e = \hat{a}_{ik} + \hat{\omega}.$$

From which it is derived:

$$(A.4.24) \quad \hat{k}_1 = \sigma_1 \hat{\omega} + (\sigma_1 - 1) \hat{a}_{ik}.$$

The specific contribution of FATC, as compared to NTC, is in that $\hat{k}_1|_{\omega} \neq 0$, except for the special case of $\sigma_1 = 1$. Thus, to evaluate the effect of FATC on the equilibrium position of the economy, it is natural first to examine the effect on resource allocation for a given ω . In some sense, an improvement in the efficiency of resources should have a similar effect to that of increasing the quantity of resources. In what follows, this reasoning is observed.

The analysis of the effect of resources on resource allocation started with equations (A.2.3) - (A.2.4) which can now be organized as:

$$\Delta_1 \hat{\ell}_1 + \Delta_2 \hat{\ell}_2 = \rho_1 \hat{k}_1 + \rho_2 \hat{k}_2 - \hat{k} \quad (\text{A.4.25})$$

$$\Delta_1 \hat{\rho}_1 + \Delta_2 \hat{\rho}_2 = \ell_1 \hat{k}_1 + \ell_2 \hat{k}_2 - \hat{k}.$$

Recall $\Delta_1 = \ell_1 - \rho_1 = -\Delta_2$, $\ell_2 = -\ell_1 \hat{\ell}_1 / \ell_2$, $\hat{\rho} = -\rho_1 \hat{\rho}_1 / \rho_2$, and simplify to obtain

$$\hat{\ell}_1 = \frac{\ell_2}{\Delta_1} (\rho_1 \hat{k}_1 + \rho_2 \hat{k}_2 - \hat{k}) \quad (\text{A.4.26})$$

$$\hat{\rho}_1 = \frac{\rho_2}{\Delta_1} (\ell_1 \hat{k}_1 + \ell_2 \hat{k}_2 - \hat{k}).$$

Using (A.4.24), we can write $\hat{k}_1 = \hat{k}_1|_T + \hat{k}_1|_\omega$ which is the sum of the rates of change evaluated under constant technology ($\hat{k}_1|_T$) and under constant ω ($\hat{k}_1|_\omega$). To evaluate $\hat{\ell}_1$ and $\hat{\rho}_1$ for constant ω define

$$(\text{A.4.27}) \quad -\hat{a}_\rho = \rho_1 \hat{k}_1|_\omega + \rho_2 \hat{k}_2|_\omega$$

$$(\text{A.4.28}) \quad -\hat{a}_\ell = \ell_1 \hat{k}_1|_\omega + \ell_2 \hat{k}_2|_\omega$$

Then

$$(\text{A.4.29}) \quad \hat{\ell}_1|_\omega = -\frac{\ell_2}{\Delta_1} [\hat{a}_\rho + \hat{k}]$$

$$(A.4.30) \quad \hat{\rho}_1|_{k,m} = -\frac{\rho_2}{\Lambda_1} (\hat{a}_\ell + \hat{k})$$

The effect of FATC on resource allocation can then be viewed as a change of resources only if $\hat{a}_\rho$ and $\hat{a}_\ell$ have the same sign. That may or may not be the case. These terms can be evaluated explicitly by substituting (A.4.24) in (A.4.27-28) to obtain:

$$\begin{aligned} -\hat{a}_\rho &= \rho_1(\sigma_1 - 1) \hat{a}_{1k} + \rho_2(\sigma_2 - 1) \hat{a}_{2k} \\ (A.4.31) \end{aligned}$$

$$-\hat{a}_\ell = \ell_1(\sigma_1 - 1) \hat{a}_{1k} + \rho_2(\sigma_2 - 1) \hat{a}_{2k}$$

Thus the signs of these terms depend on the signs of $\hat{a}_{ik}(\sigma_i - 1)$ and on the weights ℓ_1 and ρ_1 .

In the cases where $\hat{a}_\rho \hat{a}_\ell > 0$ the improvement in efficiency acts like a change in resources. In that particular case the effect on resource allocation, at given m and k , is in the same direction for labor and capital:

$$(A.4.32) \quad (\hat{\rho}|_{k,m}) (\hat{\ell}_1|_{k,m}) > 0.$$

Specifically, with $\hat{a}_\rho$ and $\hat{a}_\ell$ positive, there is a capital saving technical change. That acts like an increase in the overall capital labor ratio and as a result, with m held constant, the share of the labor intensive sector in total resources declines.

In the case where the two measures of efficiency have different signs the share of a sector in labor employment will move in opposite direction to its share in the capital stock.

Note that when FATC occurs only in one sector, (A.4.32) is maintained.

The shift in the $l_1(\omega)$ and $\rho_1(\omega)$ caused by FATC causes a change in sectoral outputs produced under constant ω . This is the allocation effect of technical change. It can be evaluated by combining (A.4.2), (A.4.25) and (A.4.24). The result will differ from the expression in (A.4.4) derived under NTC. The difference is the allocation effect (AE). The term for sector 1 is:

$$(AE)_1 = S_{L1} \hat{l}_1|_{\omega} + (1 - S_{L1}) \hat{\rho}_1|_{\omega}$$

Using (A.4.26) - (A.4.31) and simplifying to obtain:

$$(AE)_1 = [\delta_{11}(\sigma_1 - 1)\hat{a}_{1k} + \delta_{12}(\sigma_2 - 1)\hat{a}_{2k}] / \Delta_1$$

where

$$\delta_{11} = S_{1L} l_2 \rho_1 + (1 - S_{1L}) l_1 \rho_2 > 0$$

$$\delta_{12} = l_2 \rho_2 > 0$$

From the foregoing discussion it is evident that a positive (negative) value for $(\sigma_i - 1)\hat{a}_{ik}$ implies labor (capital) saving TC in the i -th sector. Thus, a positive value for the denominator implies an overall labor saving technical change. With sector 1 being labor intensive ($\Delta_1 > 0$), $(AE)_1 > 0$. Note that if the FATC takes place in one sector only, then the allocation effect is uniquely signed independent of the position of the economy.

CHAPTER 5 - RATE OF CAPITAL ACCUMULATION

INTRODUCTION

Up to this point it was assumed that the amount of resources available to the economy was exogenously given. In this chapter this assumption is removed with respect to capital. At any point in time, individuals and firms generate savings which are invested by the firms. Such an investment affects the size of the capital stock of the firms and consequently the amount of capital in the economy. The response of the economy to changes was explored in Chapter 3. In this chapter we ask what can be said about the direction and the magnitude of the changes in the overall capital stock at any given state of the economy. Specifically the question is asked under what conditions does the capital labor ratio approach a limiting value. The answer of course depends, among other things, on the state of the technology, whether or not it is held constant, and on the nature of its change. The issues that result from technical change are different from the issues that exist under constant technology and therefore it is desirable to separate the discussions and to start with the discussion of accumulation under constant technology.

Basically, the changes that take place in the capital stock in the sectoral economy under investigation are affected by the same forces which act in a single sector economy. If the savings generated at any point are larger than the depreciated capital and the capital required to provide the new addition to the labor force then there is an increase in the capital labor ratio. The question is whether such an increase slows down and eventually stops. This is the case in a single sector neoclassical economy. The question is why should not this result be immediately applicable in the two sector case.

The answer is related to the nature of the demand and the supply of the capital goods as compared to the consumption goods.

This question does not come up in the one sector model where there is only one good: Thus, intuitively speaking, the question should also have a simple solution if in the case of more than one sector, each sector produces both capital and consumer goods. For instance, agriculture produces investment goods (livestocks, land improvements) in addition to food. Similarly, the industrial sector produces machines both for consumption and investment.

Under this line of reasoning, the extreme deviation from the one sector model is the case where one sector specializes in consumer goods whereas the other sector specializes in capital goods. It is therefore desirable to start the analysis with this extreme case. But even here, the analysis can be greatly motivated by that of the one sector model and that will be the theme of our presentation. For purpose of reference we give in the next section a short review of the one sector solution. This is then followed by the two-sector model with the specialization in the production of capital goods. This discussion is then followed by modifications which diversify the production of each sector to the two kinds of goods, consumption and capital.

As we shall see, some results may be sensitive to various assumptions, one of which is related to savings behavior. It turns out that this dependence is not as important as it is often thought. This will be shown by dealing with a rather general savings function.

ONE SECTOR MODEL

The model to be discussed in this section is due to Solow. The economy is assumed to consist of one good which is produced with two inputs, labor and capital. The production process is summarized by a production function $Y=F(K,L)$. Under the assumption of full employment, K and L represent the available resources. Labor is assumed to grow exogenously at a constant rate n . The growth of the capital stock is determined by that part of output which is not consumed. The demand by consumers, referred to as final demand, is represented by the consumption function. In its simplest form, the consumption function is given by

$$C = c Y$$

Where c is the consumption rate. Thus, the addition to the capital stock from the current output is $s Y$, where $s = 1-c$ is the saving rate. For simplicity, assume that capital does not depreciate, so that the rate of growth of the capital stock is

$$\hat{K} = SY/K$$

or expressed on a percapita basis:

$$\hat{k} = \hat{K} - \hat{L} = s Y/K - n$$

The production function, $F(K,L)$ is assumed to be linear homogenous, twice differentiable. To prevent some technical difficulties, it is also assumed that $f'(k) \rightarrow 0$ as $k \rightarrow \infty$. We can then write for $Y/K = \frac{Y/K}{K/L} = f(k)/k$, consequently with $\hat{k} = \dot{k}/k$

$$\dot{k} = sf(k) - nk \equiv \varphi(k)$$

with s and n given, the rate of growth of the capital labor ratio depends on k . The question is whether there is a value for k such that the capital labor ratio remains unchanged. This question can be rephrased by noting that nk represents the maintenance requirement, that is, the growth in the capital labor ratio computed for the existing L necessary to provide the additional labor with the same k . On the other hand, $sf(k)$ represents the per capita saving which constitutes the source for the capital accumulation. An equality of the two terms implies that the per capita saving is equal to the maintenance requirement. The existence of such a k is demonstrated in Figure 5.1. The function $sf(k)$ is drawn under the assumption of constant s . The maintenance requirement is representative by the straight line nk . Now, by the concavity of $f(k)$, if $f'(0) > n/s$, then the two functions intersect at k^* , so that $\varphi(k^*) = sf(k^*) - nk^* = 0$. Note that the requirement that $f'(0) > n/s$ can be replaced by another condition, which has an economic substance, by requiring that there is some k at which the capital labor ratio does not decline. This assumption and the concavity of $f(k)$ are sufficient for determining k^* . Furthermore, having said this, it is clear that the constancy of s can also be disposed of, as long as $sf(k)$ remains concave.

Figure 5.1

SPECIALIZATION IN CAPITAL GOODS UNDER CONSTANT SAVINGS RATE

In this initial model we repeat the assumptions made in the previous section with respect to population growth, depreciation and savings. In addition, it is assumed that industry specializes in the production of capital goods whereas agriculture produces only consumer goods. Consequently, the savings function is identical with the demand function. Under the constant saving rate, the demand function has a unit income elasticity and price elasticity:

$$(5.3.1) \quad x_2 = sy = s(py_1 + y_2)$$

Imposing the equilibrium conditions of $x_1 = y_1$:

$$(5.3.2) \quad x_1 = \frac{1-s}{s} \frac{x_2}{p}$$

The short run equilibrium position of the economy is shown by point B in Figure 6-2, where equilibrium production of y_2 is given by sy . The effect of this production on the (net) capital labor ratio is again given by

$$(5.3.3) \quad \dot{k} = sy - mk$$

Assume that $\dot{k} > 0$, in this case the production set of the economy expands and this can be shown by an expansion of the transformation curve. From Chapter 3 we know that such an expansion leads to an expansion of the equilibrium output of the capital intensive sector. Therefore, if the industrial sector is capital intensive, its output will increase implying an increase in the production of capital goods. The question is whether such an increase eventually slows down to the point where it just meets the capital requirement for replacement and new labor. Obviously, the answer depends on several factors:

- (1) To what extent new capital affects the (supply) price. This effect depends on the difference in capital intensities between the two sectors and the elasticities of substitution.
- (2) To what extent does demand respond to such a change in price as well as to the expansion effect. This is a somewhat general question because in this section the response is explicitly given by the savings equation.
- (3) How productive is the new capital in the industrial sector. If its marginal productivity diminishes rapidly, the process of accumulation will not go very far.

The choice to illustrate the case where the industrial sector is capital intensive is done on purpose, because it is in this case, mainly due to the story told above, that there may not be a unique steady state. Now, what emerges from the foregoing discussion is that in this case accumulation increases the relative importance of the industrial sector, and that such an expansion depends on the technology in the industrial sector. This observation is utilized to give a general proof on the existence of a stable steady state.

Proposition - Under specialization in the production of capital goods and constant savings rate there exists a stable steady state.

Proof - Referring to Figure (5.2) it is noted that the value of the equilibrium output (y) is larger than $\bar{y}_2(k) \equiv f_2(k)$, which is the value of output under specialization in the industrial product. It immediately follows that $sf_2(k) < sy(k) \equiv y_2(k)$ ^{1/}. At the same time, the industrial output cannot exceed its capacity, that is, $y_2(k) < f_2(k)$. Consequently we have

$$(5.3.4) \quad s\bar{y}_2(k) = sf_2(k) < y_2(k) < f_2(k) \equiv \bar{y}_2(k)$$

The lower and the upper bounds of the inequality are now expressed in terms of the industrial production function. By the concavity of that function, we know that there exist values of k , say $\underline{k}^*$ and $\bar{k}^*$ such that

$$(5.3.5) \quad sf_2(\underline{k}^*) - m\underline{k}^* = 0$$

$$(5.3.6.) \quad f_2(\bar{k}^*) - m\bar{k}^* = 0$$

$$\underline{k}^* < \bar{k}^*, \quad 0 < s < 1$$

Combining (5.3.4), (5.3.5), and (5.3.6), we have:

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^{1/} $y(k)$ implies the equilibrium value of y under the specified conditions of the model.

$$sy(\underline{k}^*) - m\underline{k}^* > sf_2(\underline{k}^*) - m\underline{k}^* = 0$$

$$sy(\bar{k}^*) - m\bar{k}^* < f_2(\bar{k}^*) - m\bar{k}^* = 0$$

The two points $sy(\underline{k}^*)$ and $sy(\bar{k}^*)$ are shown in Figure (5.3). Since the function $sy(k)$ is continuous in k , there is a value of k , $\underline{k}^* < k^* < \bar{k}^*$, such that $sy(k^*) - mk^* = 0$. Following the same reasoning, it is clear that there exists a steady state which is locally stable.

However, it should be noted that the steady state which was just established need not be unique.

SPECIALIZATION IN CAPITAL GOODS AND UNSPECIFIED SAVINGS FUNCTION

The above proof can be easily generalized to other forms of savings behavior. Note that the essence of the proof consisted of:

- (1) $\bar{k}^*$ determined by (5.3.6). This is independent of the savings assumption.
- (2) Continuity of $y(k)$ in k ; and
- (3) $\underline{k}^*$ as defined by (5.3.5). This last value depends on the saving behavior. However, it is to be noted that this value was used in the proof in a specific way that can be now made part of the assumption of the proposition. The essence of $\underline{k}^*$ was to establish that at this point there is a positive accumulation. Consequently, any savings behavior that results in a positive accumulation at some k and does not destroy the continuity of the function $y(k)$ leads to a stable steady state. It has just been argued that:

Proposition - Under specialization in the production of capital goods and a savings behavior that maintains the continuity of $y_2(k)$, a necessary and sufficient condition for the existence of a stable steady state is that there is a value of k at which there is a non negative change in the capital labor ratio.

A formal proof can be constructed along the lines discussed above: Let $\bar{k}^*$

be defined by (5.3.6), and recall that $y_2(k) < f_2(k)$, then

$$y_2(\bar{k}^*) - m\bar{k}^* < f_2(\bar{k}^*) - m\bar{k}^* = 0$$

By the hypothesis, there exists $k^0 < \bar{k}^*$

$$(5.3.7) \quad y_2(k^0) - mk^0 \geq 0.$$

Thus, the function $y_2(k)$ alternates signs within the interval $[k^0, \bar{k}^*]$.^{1/} By the continuity of $y_2(k)$ there exists a value, say k^* , such that

$$(5.3.8) \quad y_2(k^*) - mk^* = 0$$

This proves sufficiency. Necessity is self evident, since if $\dot{k} < 0$ for all k , then by definition there exists no steady state.

Local stability around k^* is obtained if in the neighborhood of k^* , for $k < k^*$, $\dot{k} > 0$ and $k > k^*$, $\dot{k} < 0$. When $y_2(k)$ is differentiable, this condition requires

$$(5.3.9) \quad \hat{y}_2(k^*)/\hat{k} < 1$$

The steady state is globally stable when (5.3.9) holds for all k .

The solution is illustrated in Figure 5.4. The right hand panel shows the function $f_2(k)$, which under specialization in industrial production gives $\bar{y}_2(k)$. This value serves as the y_2 intercept of the transformation curve drawn for that particular value of k in the left hand panel. To illustrate, for k_A , read $f_2(k_A)$ on the right hand panel, this point is marked as $\bar{y}_{2A}$. The transformation curve

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^{1/} When $y_2(k^0) = mk^0$, for all k^0 maintaining (5.3.7) there is no alternation of signs, but the rest of the statement follows.

which corresponds to k_A starts from this point. In a similar way it is also possible to generate $\bar{y}_{1A}$, but this is not necessary for our analysis.

The maintenance requirement for the capital labor ratio is given by the line mk in the right hand panel. It is now a simple matter to examine the sign of $\dot{k}$ which depends on the location of the short run equilibrium point. Thus, if the short run is represented by A^* , the production of capital goods is just sufficient to meet the capital maintenance requirement and there is no change in the capital labor ratio. If the short run equilibrium point is to the left of A^* , like at ^0A then $\dot{k}(^0A) < 0$ whereas to the right of A^* , like at A , $\dot{k}(A) > 0$. The essence of the proposition is that there exists a k for which a point like A (with $\dot{k} > 0$) is an equilibrium point.

If A^* is the short run equilibrium then k_A is the steady state capital labor ratio and A^* indicates the long run equilibrium output composition. Suppose that the short run equilibrium is given by A , rather than by A^* , then $\dot{k}(A) = y_{2A} - mk_A > 0$. Let $k_B = k_A + \dot{k}(A)$. The corresponding capital requirement for maintenance of k_B is mk_B . The increment of k to k_B induces a change in $\dot{k}$, the direction and magnitude of which depends on the location of the equilibrium point on the new transformation curve which in turn depends on the saving relation. However, we can divide the points on the transformation curve to subsets in accordance with the rate of change of $\dot{k}$ (sign of $\partial \dot{k} / \partial k$). To do so, pass through y_{2A} on the right panel a line with a slope of m , find its intersection with the vertical k_B . Denote this point by $\bar{B}$ and mark it on the transformation curve. For all the points to the left of $\bar{B}$, $\dot{k}$ declines, that is $\dot{k}$ is smaller than it was at k_A . Denote the set of such points by $S^-(\dot{k})$. For points to the right of B there is an increase of $\dot{k}$, the set of such points is denoted by $S^+(\dot{k})$. However, it is to be noted that such an increase in $\dot{k}$ cannot continue forever as the process is bounded. To see this, draw the transformation curve for $\bar{k}^*$, and let C be the equilibrium

point on it. Then $\dot{k}(C) < 0$ for any C such that $y_{2C} < \bar{y}_2(k_C)$. This is the essence of the proposition.

Global stability is achieved if initially $\dot{k} > 0$, the sequence of subsequent equilibrium points belong to a corresponding sequence of S^- and conversely for $\dot{k} < 0$ where the equilibrium points belong to a sequence of S^+ . We thus see that the nature of the dynamics of capital accumulation depends on the sequence of the short run equilibria which in turn depends on the supply and demand conditions.^{1/} It may be useful to take up some reference cases which indicate the nature of such dependence.

The first case is that of equal factor intensities in the two sectors for all k so that $Af_1(k) = f_2(k)$ for all k .^{2/} This case is illustrated in Figure 5.5 where k_A is the initial capital labor ratio, and A is the short run equilibrium point. The transformation curve is a straight line as was shown in Chapter 2. Let the demand curve be given by a straight line from the origin (unitary income elasticity). Then, as shown in the figure, $\dot{k}_A > 0$. Let, as before, $k_B = k_A + \dot{k}_A$, and B is the new equilibrium point. Now, note that $\dot{k}_B < \dot{k}_A$, the reason being that $f'_2(k)$ is declining with k . Since the transformation curves obtained for different values of k are parallel, that means that the differences in y_2 obtained for given values of y_1 , such as $\bar{y}_{2B} - \bar{y}_{2A}$, are diminishing with k . Consequently, in this case a sufficient condition for a steady state is that the income elasticity for the consumption good is non negative. This certainly covers

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^{1/} Solow (, p. 48) in discussing the Uzawa model stated, "It seems paradoxical to me that such an important characteristic of equilibrium path should depend on such a casual property of the technology." Apparently, it depends on supply and demand in which technology plays a role, but not an exclusive role.

^{2/} The condition of equal $w(k)$ for all k for the two functions is the same condition as that imposed for Hicks neutral technical change. Consequently we can write $f_1(k)/f_2(k) = \text{constant}$ for all k .

the fixed saving rate considered above and reproduced in Figure 5.5.

By examining the equal factor intensity case we have eliminated the price effect, and thereby isolated the income effect. Now, by eliminating the restriction of constant price there are two distinct possibilities to be considered, depending on the factor intensities. When the consumption good is capital intensive, its relative price is declining with k and consequently, the substitution effect on the demand side contributes to the convergence to steady state. As k increases, there is a drift away from capital goods to consumption goods and thus slowing down the rate accumulation. The substitution effect behaves in an opposite direction when the capital good sector is capital intensive. It is in this case that the substitution effect can pull the consumption good down and thus diverting increasingly more resources to the production of capital goods. A closer look at this case indicates that there are two more factors to be considered. First, the strength of the substitution effect, as reflected by the magnitude of the demand elasticity. When this elasticity is small the price effect is unimportant. Second, the rate at which capital accumulation effects prices depends not only on the difference in factor intensity but also on the elasticities of substitution in the two sectors. When the elasticities of substitution are large the affect of accumulation on factor prices is relatively small. The discussion can then be concluded as follows: for the rates of accumulation ($\dot{k}$) to increase locally with the capital labor ratio it is necessary that the capital producing sector be capital intensive and that the demand elasticity and the elasticities of substitution be "sufficiently large." More explicit statements are given in the Appendix. It is only in a quantitative framework that the terms "sufficiently large" can be given a more precise meaning.

FIGURE 5.2

FIGURE 5.3

CHAPTER 6 - DISTORTION IN THE FACTOR MARKETS¹

This chapter is the first deviation from the standard model in the direction of reality. An important assumption in the standard model is that factor prices are the same in both sectors. This assumption is a poor description of reality when agriculture is one of the sectors considered. Except in very mature economies, the agricultural wage rate is below that of the rest of the economy. The question is what effect does a modification of this assumption have on the analysis.

The model to be examined consists of equations (2.1) - (2.4) and:

$$p_1(\partial F_1/\partial K_1) = r_1, \quad p_1(\partial F_1/\partial L_1) = w_1. \quad (6.1)$$

The four equations in (6.1) together with the four equations (2.1) - (2.4) specify the supply side of the model. The demand specification is unchanged.

The analysis in this chapter is conducted in terms of the intersectoral price ratios, namely:

$$\delta_r = r_1/r_2, \quad \delta_w = w_1/w_2 \quad \text{and} \quad \delta = \delta_w/\delta_r = \omega_1/\omega_2$$

where $\omega_1 = w_1/r_1$. At this point, these ratios are given exogenously. They are endogenized in subsequent discussion.

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¹This chapter is based on Mundlak (1970) and Mundlak and Tropp (1980).

2. Factor Intensity

The technological conditions are the same as in the standard model so that, in the absence of distortion, $k_2(\omega) > k_1(\omega)$. Under distortion, $\omega_1 \neq \omega_2$, so that resource allocation is affected. Also, it is possible to have a situation where factor intensity is reversed.

Utilizing the factor intensity assumption, write

$$\underline{\omega}(k) = \omega_2(k), \bar{\omega}(k) = \omega_1(k), \bar{\omega}(k) > \underline{\omega}(k). \quad (6.1)$$

Define

$$\Gamma(k) \equiv \bar{\omega}(k)/\underline{\omega}(k), \delta \equiv \omega_1/\omega_2. \quad (6.2)$$

The functions $\omega_1(k_1)$ are drawn in Figure 6.1. For $\omega_1 = \omega_2 = \omega_C, (\delta = 1)$, the factor intensities are k_{1C} and k_{2C} . Next consider $\delta < 1$. For instance, ω_{1H} corresponding to point H. That determines uniquely $k_1(\omega_{1H}) \equiv k_{1H}$. Thus the pair (ω_1, ω_2) determines uniquely the pair (k_1, k_2) .

As will become clear below, it is desirable to differentiate between changes in ω_1 that maintain δ constant and those that do not. The pair (ω_1, ω_2) can be characterized by (ω_2, δ) . Holding δ constant, variations in ω_1 are generated by variations in ω_2 . So we can write $k_1(\omega_1) = k_1(\delta\omega_2) = k_1(\omega_2|\delta)$. Holding δ constant, the function is monotonic and can be inverted to give $\omega_2(k_1|\delta)$. In terms of Figure 6.1, the function goes through points such as H'. The location of $\omega_2(k_1|\delta)$ with respect to $\omega_1(k_1)$ depends on δ . There are four distinct cases of interest, $\delta < 1$, $1 < \delta < \Gamma(k)$, $\delta = \Gamma(k)$ and $\delta > \Gamma(k)$. These possibilities are illustrated in Figure 6.2, except for the case of $\delta = \Gamma(k)$ in which case $\omega_2(k_1)$ becomes equal to $\omega_2(k_2)$. We thus have for the feasible sets of ω_2 :

Figure 6.1

Figure 6.2

For $\delta \in (0, 1]$, $\omega_2 \in [A, B'(\delta)]$

$\delta \in [1, \Gamma(k)]$, $\omega_2 \in [A, B''(\delta)]$

$\delta \in [\Gamma(k), \infty)$, $\omega_2 \in [B'''(\delta), A]$.

For each case it is possible to select a feasible value for ω_2 and select the corresponding values for k_1 and k_2 . It can thus be shown that it is in the last case where $\delta > \Gamma(k)$ that the factor intensity reverses so that $k_1(\omega_2) > k_2(\omega_2)$. To maintain the full employment condition, for any feasible ω_2 , given $\delta > \Gamma(k)$ the result is $k_1(\omega_2) > k > k_2(\omega_2)$.

Proposition (1): Given the conditions of the model,

$$k_1(\omega_1) \gtrless k_2(\omega_2) \iff \delta \gtrless \Gamma(k).$$

Before continuing we reflect on the implications of this proposition to agriculture. In general agricultural wages are lower than non-agricultural wages. However, that does not imply a particular relationship with respect to the rate of return in the two sectors. Thus, $r_1/r_2 \equiv \delta_r$ can be smaller, equal or larger than $w_1/w_2 \equiv \delta_w$. The case of equality, $\delta_r = \delta_w$ is referred to as equal distortion and therefore $\delta = \delta_w/\delta_r = 1$ and $\omega_2(k_1|\delta=1) = \omega_1(k_1)$. With $\delta_r > \delta_w$, $\delta < 1$ and agriculture, if it were labor intensive for $\delta = 1$, is forced to still lower capital labor ratios. This is simply to say that low labor cost relative to capital charges leads to a low capital-labor ratio.

3. Resource Allocation and the Transformation Curve

For any pair of feasible values (ω_1, ω_2) there is a corresponding pair $(k_1(\omega_1), k_2(\omega_2))$. Under the full employment conditions, the resource

allocation is uniquely determined:

$$l(\omega_1, \omega_2, k) = \frac{k - k_2(\omega_2)}{k_1(\omega_1) - k_2(\omega_2)}.$$

Holding k constant, it is clear that:

$$\text{sign } \frac{\partial l}{\partial \omega_1} = \text{sign } [k_2(\omega_2) - k]$$

$$\text{sign } \frac{\partial l}{\partial \omega_2} = \text{sign } [k - k_1(\omega_1)].$$

This leads to the next proposition:

Proposition 6.2: The share of the labor intensive sector in the labor force is monotonically increasing with either ω_1 or ω_2 .

An increase in ω_1 increases $k_1(\omega)$ and L_1 , hence K_1 must increase as well and therefore K_2 must decline. An increase in ω_2 increases $k_2(\omega_2)$, but by proposition 6.2 L_2 declines and L_1 increases, also $k_1(\omega_1)$ remains constant, therefore K_1 must increase and K_2 decline. That can be summarized by:

Corollary: The share of the labor intensive sector in the total capital stock is also monotonic in ω_i , $i = 1, 2$.

It then immediately follows:

Proposition 6.3. The output of the labor (capital) intensive sector increases (decreases) with an increase of either ω_1 or ω_2 .

The question is how to evaluate the feasible production set of the economy in the short run. One possibility is to generate outputs by changing the wage-rental ratio in one sector while keeping the other constant. That would imply a change in δ . Another possibility is to change both ω_i at the same rate and thereby keep δ constant. This latter possibility

is actually the generalization of the case of $\delta = 1$ which leads to the transformation curve in the competitive system. It results in a transformation curve which depends on k and δ , denoted by $T(k, \delta)$. The important property of the transformation curve is that it is the efficiency frontier. That means that for any $y = (y_1, y_2) \in T(k, \delta)$, an increase in output of one product can only take place at a cost of a reduction of output of the other product. Let $y_1(y_2)$ represent the value of y_1 that corresponds to y_2 on the transformation curve $T(k, \delta)$, then if $y_1(y_2)$ is differentiable, $\hat{y}_1/\hat{y}_2 \Big|_{\delta, k} < 0$.

To generate $T(k, \delta)$, it is noted that $\hat{\delta} = 0$ implies $\hat{\omega}_1 = \hat{\omega}_2$. By application of proposition 6.3, it is clear that under equal rates of change of ω_1 , the outputs of the two sectors move in opposite directions. Thus, maintaining the competitive conditions within each sector so that ω_1 is obtained from $\omega_1(k_1)$ and the full employment conditions defines the feasible set for (ω_1, ω_2) . Variations in ω_1 that leave δ constant generate the transformation curve $T(k, \delta)$. There are two common points to the family of $T(k, \delta)$ generated by variations of δ , those are the two end points $(\bar{y}_1(k), 0)$ and $(0, \bar{y}_2(k))$ as these points are independent of δ . This can be summarized by the following proposition:

Proposition 6.4: Under the assumptions of the model, the transformation curve is determined by k and δ . The end points obtained under specialization are common to all δ .

4. Supply Price

Under distortion, the slope of the transformation curve, $dy_2/dy_1 \Big|_{\delta}$, is not equal to the supply price. The evaluation of the slope, under the

constraint of constant δ , leads to

$$\frac{dy_2}{dy_1} = -h(\delta_r, \delta_w, w_2)p \quad (6.3)$$

where h is a positive function and becomes unity when $\delta_r = \delta_w = 1$.¹ The investigation of the concavity of the transformation curve can proceed by investigating the behavior of (6.3). The outcome of such an investigation is that distortion may destroy the concavity.² The derivation is long and is not presented here. Furthermore, it is only of secondary interest because what matters in studying the behavior of the economy is the relationships between the outputs and the supply price.

In studying the supply price, we follow the approach taken in Chapter 4 and view the supply price as the ratio of the average cost functions:

$$p^s = \frac{p_1(w_1, r_1)}{p_2(w_2, r_2)} \quad (6.4)$$

Upon differentiation, using Shepard's lemma:

$$\hat{p}^s = (S_{1L}\hat{w}_1 + S_{1K}\hat{r}_1) - (S_{2L}\hat{w}_2 + S_{2K}\hat{r}_2) \quad (6.5)$$

which after rearrangement can be written as:

$$\hat{p}^s - I\hat{w}_2 = S_{1L}\hat{\delta}_w + S_{1K}\hat{\delta}_r \quad (6.6)$$

where $I = S_{1L} - S_{2L}$.

The analysis is conducted in stages, the first one assumes constant distortion rates in the two factor markets, $\hat{\delta}_r = \hat{\delta}_w = 0$. This condition

¹Mundlak (1970).

²Cf. Bhagwati and Srinivasan (1971).

is referred to as the strong constant distortion (SCD). This condition leads of course to $\hat{\delta} = 0$. SCD is sufficient but not necessary for having a constant δ . It is possible to have $\hat{\delta}_w = \hat{\delta}_r \neq 0$, which also results in constant δ . This is referred to as weak constant distortion (WCD).

Under SCD we have

$$\hat{p}^s = I \hat{\omega}_2 = I \hat{\omega}_1. \quad (6.7)$$

Combining (6.7) and proposition 6.3 makes it possible to relate the directional changes of y_1/y_2 and p^s . Thus, $(y_1/y_2)/\hat{\omega}_2|_{\delta} > 0$ and $\text{sign } \hat{p}^s/\hat{\omega}_2|_{\delta} = \text{sign } I$. The question then is what is sign I? In the standard model ($\delta = 1$), $\text{sign } I = \text{sign } \Delta_1$. That is $\ell_1 > \rho_1 \iff S_{1L} > S_{2L}$. This however is no more true in the present case. Distortion may lead to factors receiving different shares of output than those called for in the standard model. Consequently the simple relationships between factor shares and factor ratios disappear. For instance, the labor intensive sector may pay a large share of output to capital. Then, an increase in the wage rental ratio increases the relative cost of production in the sector with the larger labor share which happens to be in this example the capital intensive sector. This is the deviation from the standard model. Under such a case, an increase in ω_2 increases y_1/y_2 but decreases p^s . That means negative relationship between the supply of y_1 and its price.

Formally the source of this difference is related to the properties of p^s and resource allocations as functions of factor prices. $p_1(w_1, r_1)$ is homogeneous of degree one in w_1, r_1 and consequently, p^s is homogeneous of degree zero in all four factor prices. On the other hand, ω_1 is

homogeneous of degree zero in the two factor prices w_1, r_1 alone. Consequently, doubling w_1, r_1 does not change ω_1 but it does change p^S . This problem does not exist in the standard model because there are only two, rather than four, factor prices.

There are several situations which leads to the reversibility of sign I . Those will not be fully described.¹ It is however useful to illustrate the nature of the analysis. Define

$$\theta_1(\omega_1) = \omega_1/k_1 = S_{1L}(\omega_1)/(1 - S_{1L}(\omega_1)).$$

Hence, $S_{1L}(\omega_1) = 1/[(1/\theta_1(\omega_1)) + 1] \equiv A(\theta_1), A'(\theta_1) > 0$. We also know that $d \ln \theta_1 / d \ln \omega_1 = 1 - \sigma_1$. We are interested in

$$I(\omega_2, \delta) = \{[S_{1L}(\delta \omega_2) - S_{2L}(\omega_2)] | I(\omega_2, 1) \geq 0\}. \quad (6.8)$$

It thus follows that

$$\theta(\omega_2, \delta) \equiv \frac{\theta_1(\delta \omega_2)}{\theta_2(\omega_2)} \begin{matrix} > \\ < \end{matrix} 1 \iff I(\omega_2, \delta) \begin{matrix} > \\ < \end{matrix} 0. \quad (6.9)$$

The simplest case is that of equal factor intensities for all ω_2 and $\delta = 1$. This is illustrated in Figure 6.3. There are two distinct cases, $\sigma < 1$ and $\sigma > 1$ shown in two different panels. For $\delta < 1$, $\omega_A = \omega_1$ and $\omega_B = \omega_2$. Now $I > 0$ if $\theta_A > \theta_B$, and this condition is met with $\sigma > 1$. The other cases can be illustrated in a similar way. Thus for this simple case we have the following possibilities:

- (1) $I > 0$ iff $\delta < 1$ and $\sigma > 1$ or $\delta > 1$ and $\sigma < 1$
- (2) $I < 0$ iff $\delta < 1$ and $\sigma < 1$ or $\delta > 1$ and $\sigma > 1$
- (3) $I = 0$ iff $\sigma = 1$ regardless δ or $\delta = 1$ regardless of σ .

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¹See Mundlak (1970) for a complete discussion of this subject.

Figure 6.3

5. Equilibrium Under the Strong Constant Distortion

The production set of the economy is bounded from above by $T(k, \delta)$. δ is uniquely determined by the pair δ_w and δ_r . Under the SCD the supply price can be written as $p^s(y_1 | \delta_w, \delta_r)$, and it is a continuous function of y_1 for all $y_1 \in T(k, \delta)$. As indicated in Section 2 above, for any (k, δ) , ω_1 is bounded, and therefore $k_1(\omega_1)$ are bounded and thus p^s is bounded. Let p and $\bar{p}$ be the extreme values of $p^s(y_1 | \delta_w, \delta_r)$ for $y_1 \in T(k, \delta)$. Thus, if an equilibrium price exists it must belong to the closed interval $[p, \bar{p}]$.

The demand functions $x_1 = D(p, y_2)$ corresponding to p and $\bar{p}$, are drawn in Figure 6.4. The intersection of these functions with $T(k, \delta)$ have the coordinates y^* and y^{**} . The demand conditions imply that any feasible $y = (y_1, y_2)$ with y_1 coordinates in $(0, y_1^{**})$ would be purchased with $p^d > \bar{p} \geq p^s$ and hence $p^d > p^s$. Similarly for $y_1 \in (y_1^*, \bar{y}_1)$ $p^d < p^s$. Both p^d and p^s are continuous in y_1 and therefore there exists $y \in T(k, \delta)$ at which $p^d = p^s$. Furthermore, the equilibrium output is characterized by $y_1 \in [y_1^{**}, y_1^*]$. The left hand panel of Figure 6.4 illustrates the

Fig. 6.4.--Short Run Equilibrium Under Strong Constant Distortion

determination of the equilibrium using the supply and demand prices.

The demand function drawn in the left panel expresses the demand price that corresponds to outputs read from the transformation curve.

Thus, for any $y \in T(k, \delta)$, we have $dy_2/dy_1|_{\delta} < 0$, and consequently,
 $p^d[x_1, x_2 | x_1^d = y_1, x_2^d = y_2] = p^d(y_1 | \delta)$ and $(\partial p^d / \partial y_1)|_{\delta} < 0$.

As indicated above, the supply price may decline with y_1 . In this case there may be multiple equilibria. Finally, p^s need not be monotonic in y_1 .

The foregoing discussion indicates existence of equilibria. A convergence rule can be established by assuming that when the cost of production, as represented here by p^s , is below the price that consumers are willing to pay, production will increase. Formally:

$$(p^d - p^s)dy_1 > 0. \quad (6.10)$$

Said differently, production increases when profits exist and declines when losses are realized.

Having established the existence of a short run equilibrium, it is now possible to examine the behavior of the economy in the long run in the face of capital accumulation and under technical change. However, to carry such exercise under the SCD seems somewhat artificial. It is expected that as the economy develops, the factor price differentials will change as well. For that reason we turn now to examine the effect of change of distortion on the equilibrium position of the economy, and delegate the additional comments on the behavior of the economy under capital accumulation to the appendix.

6. The Effect of Changes in Distortion

The inefficiency caused by distortion is best shown with the aid of the well known Edgeworth box diagram, as shown in Figure 6.5. Points A and B represent feasible allocations with $\delta < 1$ and $\delta > 1$ respectively. A movement from A to C represents higher production of both sectors. The same is achieved by a movement from B to D. What is to be noted is that the improvement is associated with a decline in $|\delta - 1|$. Note that the loss of efficiency is not indexed by $|\delta - 1|$, because the sign of $\delta - 1$ matters. However a decline in $|\delta - 1|$ from any existing point leads to improvement. We can then state:

Proposition 6.5. As $|\delta - 1|$ decreases, the efficiency of the economy improves in the following sense: Let $|\delta - 1| > |\delta^* - 1|$, and $(\delta - 1)(\delta^* - 1) > 0$, then for every $y \in T(k, \delta)$ there exists $y^* \in T(k, \delta^*)$ such that either $y^* > y$ or $y^* = y$. Further, the equality holds only for the extreme points which are obtained under specialization.

Thus referring to Figure 6.6, the figure is drawn subject to $|1 - \delta'| > |1 - \delta| > |1 - \delta''|$. If we define $\bar{S}(y^0) \equiv \{y: y \geq y^0\}$, $\underline{S}(y^0) \equiv \{y: y \leq y^0\}$, then a movement from y^0 to $y \in T(k, \delta') \cap \underline{S}(y^0)$ is caused by $\hat{\delta}(1 - \delta) < 0$ whereas a movement from y^0 to $y \in \bar{S}(y^0) \cap T(k, \delta'')$ is caused by $\hat{\delta}(1 - \delta) > 0$.

A change in δ has an effect on resource allocation which is different from what has been observed thus far in the standard model. This can be seen in Figure 6.5 where a movement from A to C implies an increase in k_1 and a decline in k_2 . That is, the sectoral factor intensities move in opposite directions. Writing the logarithmic differential of the two production functions (see Chap. 2):

Figure 6.5

Figure 6.6

$$\hat{y}_1 = \hat{l}_1 + (1 - s_{1L})\hat{k}_1$$

$$\hat{y}_2 = \hat{l}_2 + (1 - s_{2L})\hat{k}_2.$$

Then if $\hat{k}_2 < 0$ and $\hat{y}_2 > 0$, $\hat{l}_2$ must be positive which implies negative $\hat{l}_1$ and therefore positive $\hat{k}_1$. This kind of reasoning leads to the following proposition, where we use $S(\underline{y}^0) = \bar{S}(\underline{y}^0) \cup \underline{S}(\underline{y}^0)$.

Proposition 6.6. Let $\underline{y}^0 \in \text{interior } T(k, \delta^0)$ and $\underline{y} \in T(k, \delta) \cap S(\underline{y}^0)$, where $\delta = \delta^0(1 + \hat{\delta})$. For a sufficiently small $|\hat{\delta}|$, the directional changes of the variables uniquely determined by $\hat{y}$ and δ are:

$$\hat{\omega}_1/\hat{\delta} > 0 \quad \hat{k}_1/\hat{\delta} > 0 \quad \hat{l}_2/\hat{\delta} > 0 \quad \hat{p}_1/\hat{\delta} > 0$$

$$\hat{\omega}_2/\hat{\delta} < 0 \quad \hat{k}_2/\hat{\delta} < 0 \quad \hat{l}_1/\hat{\delta} < 0 \quad \hat{p}_2/\hat{\delta} < 0.$$

In what follows, the expansion of the feasible production set due to a decline in $|1 - \delta|$ is referred to as the expansion effect. A positive expansion effect means an expansion of the output of the two products. This however is not the only effect.

A change in δ requires a change in δ_w and δ_r . Moreover, under the WCD, we have $\hat{\delta}_w = \hat{\delta}_r \neq 0$ but $\hat{\delta} = 0$. Thus, in this case the price changes at any feasible production, even though the production set remains constant. The effect of a change in distortion on price depends only on the direction of the change in distortion and not on the relationships of the δ 's to 1. Thus an increase in δ_w and δ_r implies a relative increase in the cost of production in sector 1 and therefore an increase in p^S .

In the general case, where $\hat{\delta}_r \neq \hat{\delta}_w$, the directional change in $p^S(\omega_1 | \delta_r, \delta_w)$ depends on the signs of $\hat{\delta}_r$ and $\hat{\delta}_w$. If they have the same sign, then the result is obvious. If not, the sign of the corresponding

change in p^s will depend on the dominating term. For instance, if only one δ changes, then its sign determines the directional change in p^s .

It is now possible to summarize the discussion on the effect of change in distortion on the short run equilibrium.

Proposition 6.7. The economy is in short run equilibrium, $y \in T(k, \delta)$.

Factor intensity and factor shares are different in the two sectors ($\Delta_1 I \neq 0$). The distortion in the factor market is reduced, $\hat{\delta}(1 - \delta) > 0$. The expansion effect is positive for the two products whereas the price effect acts in opposite direction on the other two sectors.¹

Thus, the output of the sector whose cost of production is reduced by the change in distortion will expand whereas the direction of the change in the other output depends on the relative strength of the income and substitution effect.

7. Implications for the Long Run Analysis

There is a relationship between the stability of a short run equilibrium and the behavior of the economy in the process of capital accumulation. If we define σ to be the overall elasticity of substitution under SCD: $\hat{\omega}/\hat{k}|_{\delta} = 1/\sigma$, then it is shown in the appendix that σ is positive if and only if the short run equilibrium at which it is evaluated is stable. Thus, in this case, capital accumulation increases the wage rental ratio.

Now suppose σ is negative so that the economy is not in a stable short run equilibrium. We can now show that this can only be locally so and that in the large the wage rental ratio must increase with capital

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¹There is also a possibility of no price effect.

accumulation. This result follows from the monotonicity of $\omega_1(k_1)$. Referring to Figure 6.7,

Figure 6.7

the economy is in an initial resource endowment k_A and wage-rental ratio ω_A . Capital accumulation increases the capital-labor ratio to k_B . The equilibrium point now results in $\omega_B < \omega_A$. This is the meaning of $\sigma < 0$. Note however that this process is bounded from below. Let $k_2(\omega_A) \equiv \underline{k}(\omega_A)$, then under CD for all $k > \underline{k}(\omega_A)$, $\omega(k) > \omega_A$. In words, there is a capital labor ratio, beyond which ω_A is nonadmissible.

This result, which assures global stability, is obtained under the SCD. It is of course possible to change the distortion constantly to offset capital accumulation so as to keep the wage rental ratio in one of the sectors declining. This would require accumulating the capital in one sector and the labor in the other sector. This situation may occur under a very capital intensive industrialization program which extracts capital from agriculture and prevent labor migration. Such a program is

of course inconsistent with a flow of resources in response to differential remuneration. It can thus be a result of government policies.

As indicated earlier in this chapter, it is more reasonable to expect that in the long run capital accumulation will eventually result in a decline of distortion. Whether or not this is the case it depends on how the distortions are formed. This is discussed in the next chapter.

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CHAPTER 7 - THE DYNAMICS OF RESOURCE ALLOCATION1. INTRODUCTION

The discussion in Chapter 6 analyzed the behavior of the economy under the assumption of constant distortion rates. The discussion extends the results of the unconstrained competitive model. However, it is still not general enough for empirical analysis in that it assumes implicitly that the rate of distortion is given exogenously and as such it is not determined within the economic system itself. This constitutes a serious limitation for long run analysis. Furthermore, it assumes implicitly that at any point in time, intersectoral resource allocation is performed so as to maintain the exogenously given distortion rate. If this assumption has empirical validity, what is the mechanism that stops the resource flow at the point of constant distortion rate and thereby prevents resources to move until the price differentials disappear? It is perhaps fair to say that the analysis of constant distortion is basically a short run analysis and as such the formentioned limitation is not of serious consequences.

To overcome those limitations, the factor price differentials are now endogenized. This is done by recognizing that the process of resource allocation requires real time. The generation of intersectoral factor price differentials leads to a flow of resources from low to high returns occupations. However, such an adjustment is not sufficiently fast to close the gap in one period. In fact, it is not sufficiently fast to affect the resource allocation within a given time interval. Consequently, the resource allocation at any given time can be considered to be

predetermined. In such a model the rate of distortion is endogenous and it is determined by the changes in factors' supply and demand that occur over time. The main features of this approach are introduced in a very simple model in the following section. Subsequent sections introduce details necessary to confront the model with the data.

2.CONSTANT RESOURCES AND TECHNOLOGY

The basic assumption is that at any time t the technology and resource allocation between the two sectors are predetermined. Consequently, outputs are also predetermined. The economic problem of a closed economy in the short run is to determine the relative product price, p . The solution is shown in Figure 7.1. Point A in panel I indicates the fixed supply of the two products. The price is determined from the demand curve that goes through A. In other words, p is determined by writing the demand function as $p(y_1, y_2)$. The solution is also illustrated in panel II where the demand function $x_1 = D(p, y_2)$, drawn for $y_2 = y_{2A}$, intersects with the fixed supply y_1 .

The fundamental assumption that resource allocation and technology are given determines the marginal productivities of labor and capital in the two sectors. Once the product price p is determined we also have the value marginal productivities. It is assumed that competition prevails within each sector and consequently, factor prices are forced to equal the value marginal productivities. Rewriting equation (6.1):

$$p_1(\partial F_1 / \partial K_1) = r_1$$

(1)

$$p_1(\partial F_1 / \partial L_1) = w_1$$

If $w_1 = w_2$ and $r_1 = r_2$ the economy is in equilibrium. If not, it is assumed that the resources will move according to:

$$(2) \quad (w_2 - w_1)\dot{\ell} < 0$$

$$(3) \quad (r_2 - r_1)\dot{\rho} < 0$$

where $\ell = L_1/L$. $\rho = K_1/K$

Under constant resources $\dot{\ell} = \dot{L}_1$ and $\dot{\rho} = \dot{K}_1$. With resources held constant.

(2) and (3) determine the resource allocation in (t+1) :

$$L_1(t+1) = L_1(t) + \dot{L}_1(t)$$

(4)

$$L_2(t+1) = L_2(t) - \dot{L}_1(t)$$

$$K_1(t+1) = K_1(t) + \dot{K}_1(t)$$

(5)

$$K_2(t+1) = K_2(t) - \dot{K}_1(t)$$

Under the present assumptions of constant resources and technology, this process converges to an equilibrium point at which $w_2 = w_1$, $r_2 = r_1$, and consequently, $\dot{L}_1 = \dot{K}_1 = 0$ and hence $\dot{y}_1 = \dot{y}_2 = \dot{p} = 0$. Furthermore,

this equilibrium is stable. To see this, assume first that $(w_2 - w_1)$ $(r_2 - r_1) > 0$. Assume, that each of the terms is positive, so that there is an outflow of resources from agriculture. Consequently, agricultural output will decline and that of non-agriculture will increase. As a result p will increase and that will cause the factor price differentials to decline. The process is illustrated in Figure 7.1. The output in $t+1$ is shown by point B. The limit of the process is given by point C, where factor prices are equal for both sectors and thus the point is located on the competitive transformation curve. Note that as the economy moves to C, p increases. That is the agricultural supply declines and the price increases. This relationship is in opposite direction to that obtained for movements along the transformation curve.

The foregoing discussion basically asserts the existence of a stable competitive equilibrium, a subject discussed in Chapter 2. If this is true, why then was the system in disequilibrium initially? The answer is given by the discussion of Chapters 3 and 4. Changes in resources and technology lead to changes in the equilibrium position. In the present discussion resources and technology were held constant and therefore there is no reason for the economy not to converge to equilibrium. The effect of such changes is taken up below.

3. RESOURCE ACCUMULATION

The model is now extended to allow for changes in resources, beginning with capital accumulation. At first, it is assumed that there is only one

capital good, produced in non agriculture. The investment can be allocated between the sectors. However, once capital has been allocated to a given sector, the cost of changing the allocation ("cost of transfer") is prohibitive so that existing capital is committed.

There are now two demand components in non agriculture, x_2^d demand by consumers and x_2^i demand by investors. The resource utilization identities are:

$$y_1 \equiv x_1^d$$

$$y_2 \equiv x_2^d + x_2^i$$

As before, $y_1(t)$ is predetermined in t , and the demand by consumers is given by equation (2.17). The saving equation generates the demand for new capital goods. Let

$$(6) \quad x_2^i = s(\cdot)y$$

$$y = py_1 + y_2.$$

The aggregate demand for the non agricultural product can be written by using (2.17) and (6) to obtain:

$$(7) \quad x_2 = H(p, y_1) + s(py_1 + y_2) = \phi(p)$$

$$\partial H / \partial p > 0, \quad \frac{\partial sy}{\partial p} = sy_1 + y \frac{\partial s}{\partial p}$$

Under the constant saving rate, $\partial Dx_2 / \partial p > 0$. If s is allowed to vary with p , we assume that the aggregate demand $\phi(p)$ is increasing in p (that is declining in p_2). Thus, the equilibrium p is uniquely determined. The solution is illustrated in Figure 7.2. Under the assumption of $\phi'(p) > 0$, we find p_A such that $x_1^d = (p_A, y_2)$ passes through the initial point A. Once p_A is determined, we can draw the consumer's demand $x_1^d = D(p_A, x_2^d)$ and its intersection with the supply y_1 , at point B, determines x_2^d . Thus, x_2^1 is determined as the difference $y_2 - x_2^d$.

The investment is now allocated to the two sectors according to (3). Note however that now $\dot{K} \neq 0$, and therefore $\dot{\rho} = (\hat{K}_1 - \hat{K})\rho$. Therefore, a negative value for $\dot{\rho}$ does not imply a decline in the agricultural capital stock. We turn to a discussion of the determinants of ρ in a subsequent section. At the present it is important to note that the decline in the relative importance of agriculture in factor employment is consistent with an increase in agricultural output even when the technology is held constant.

With capital accumulation, the transformation curve of the economy expands constantly and consequently there is no a priori reason to assume that the economy will converge to the competitive equilibrium as was the case with constant resources. A possible dynamics is illustrated in Figure 7.3, where the initial point is A_0 and the equilibrium solution for the economy with initial resources is given by C_0 . In the next period the

economy moves to A_1 , whereas the competitive equilibrium moves to C_1 . Since capital accumulation is assumed to have caused the disequilibrium in the first place, it is not surprising that it may also increase the degree of distortion. Whether or not that actually happens depends on the rates of factor adjustment a subject which we take up subsequently. But the process also depends on the rate of capital accumulation. Note that thus far it was assumed that the labor force, or population, is held constant. In such a case, the economy may accumulate capital forever. The purpose for holding population constant thus far was to emphasize the role of resource changes in generating disequilibrium. Having done this, we can now assume that there is a population growth at rate n . Chapter 5 is still relevant for the present model. We thus know that with $n \neq 0$ there is a steady state solution. That is at some point, the growth in the capital labor ratio will slow down. At this point, the rate of factor mobility may be large enough to yield a convergence to the competitive solution. Looking at the process from the point of view of factor price differentials, there may be a long time period at which factor price differentials will not decrease. or even increase. while at the same time factor mobility will be in line with equations (3) and (4). The dynamics of the economy can be examined with reference to a Liapunov function of the state variables $\delta_w = w_1/w_2$ and $\delta_r = r_1/r_2$. Define

$$\phi(\delta_w, \delta_r) = (\delta_w - 1)L_1 + (\delta_r - 1)K_1.$$

Under constant resources and technology, $\phi \geq 0$. It reaches its minimum at $\delta_w = \delta_r = 1$. At any other point we have:

$$(8) \quad \dot{\phi} = \dot{\delta}_w L_1 + \dot{\delta}_r K_1$$

If we assume

$$(8) \quad \dot{\delta}_w(\delta_w - 1) < 0 \quad \dot{\delta}_r(\delta_r - 1) < 0$$

then $\dot{\phi} < 0$ and under this assumption the system converges uniformly to the competitive equilibrium. The essence of the discussion in this section has been that under resource change there is no reason to expect (8) to exist everywhere, and consequently uniform convergence cannot be expected. Yet, the situation may change once the rate of accumulation declines and the economy is approaching a steady state solution. At that point, the intersectoral resource flow results in approaching an almost stable transformation curve (or production frontier) and the convergence to equilibrium becomes just a question of time.

Figure 7.1

Figure 7.2

Figure 7.3

CHAPTER 8 - THE EVOLUTIONARY APPLICATION OF TECHNIQUES AND CAPITAL ACCUMULATION¹

The Production Structure in Agriculture

The agricultural technology consists now of two techniques, "traditional" and "modern," denoted as 1 and 2 respectively. They are represented by production functions homogenous of degree 1, twice differentiable and concave. Let L_j and K_j be the labor and capital assigned to technique j , $j = 1, 2$, and L_A , K_A be the total agricultural use of labor and capital. Define $\ell = L_1/L_A$ and $k_j = K_j/L_j$, then the agricultural technology and full employment conditions are:

$$\ell k_1 + (1 - \ell)k_2 = k_A \quad (8.2.1)$$

$$k_A = K_A/L_A$$

$$Y_1/L_A = \ell f_1(k_1) \quad (8.2.2)$$

$$Y_2/L_A = (1 - \ell)f_2(k_2) \quad (8.2.3)$$

For simplification, it is assumed that for any admissible ω , $k_1(\omega) < k_2(\omega)$ and that $\omega_j(k_j) \rightarrow \infty$ as $k_j \rightarrow \infty$ and $\sigma_j < \infty$. The unit isoquants of the two functions are drawn in Figure 8.1. The choice of the technique to be implemented under cost minimization depends on ω . Specifically, it is noted that there exists a line tangent to the two isoquants with slope $\bar{\omega}$. For $\omega < \bar{\omega}$, technique 1 is more efficient in that its cost for producing a unit output is smaller than that of technique 2. The situation is reversed for $\omega > \bar{\omega}$.

The conditions for the selection of techniques can also be expressed

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¹This section is based on Danin Yigal and Yair Mundlak, "Introduction of Techniques and Capital Accumulation." Rehovot: The Center for Agricultural Economics Research, Working Paper 7909, 1979.

FIGURE 8.1

FIGURE 8.2

FIGURE 8.3

in terms of the capital labor ratios. Let $\tilde{k}_j = k_j(\tilde{\omega})$. Then technique 1 is used alone when $k_A \leq \tilde{k}_1$, technique 2 is used alone when $k_A \geq \tilde{k}_2$. When $k_A \in (\tilde{k}_1, \tilde{k}_2)$ the two techniques will be used simultaneously with

$$l = (\tilde{k}_2 - k_A) / (\tilde{k}_2 - \tilde{k}_1) \quad (8.2.4)$$

as follows from the full employment conditions. Note that at $\tilde{\omega}, l$ is a function of k_A alone and $dl/dk_A < 0$. Thus, when the two techniques are used simultaneously, the relative importance of the traditional technique is declining with capital accumulation. This results reflects two conditions, cost minimization and full employment. As such it reflects only the supply side. However the amount of k_A itself is determined by the equilibrium solution.

In turning to the equilibrium analysis it is noted that it is possible to aggregate the two techniques into an aggregate production function. This is shown in Figures 8.1-8.3. Figure 8.1 shows the unit isoquant for agriculture, $F_A = 1$, marked by the solid line. It is identical to F_1 for $k_A \leq \tilde{k}_1$, it is obtained as a combination of $F_1 = 1$ and $F_2 = 1$ for $k_A \in (\tilde{k}_1, \tilde{k}_2)$ and is identical to $F_2 = 1$ for $k_A \geq \tilde{k}_2$. The solid line in Figure 8.2 shows the wage rental ratio in agriculture, ω_A , as a function of k_A . Note however that the inverse function $k_A(\omega_A)$ does not exist for $k_A \in (\tilde{k}_1, \tilde{k}_2)$. The solid line in Figure 8.3 shows the aggregate average labor productivity in agriculture.

These results can be summarized analytically. The competitive conditions for agriculture are

$$pf'_j(k_j) \leq r \quad (8.2.5)$$

$$p[f_j(k_j) - f'_j(k_j)k_j] \leq w. \quad (8.2.6)$$

Conditions (8.2.5) and (8.2.6) imply:

$$pf_j(k_j) \leq w + rk_j. \quad (8.2.7)$$

The conditions are written as inequalities. A technique is not used if it generates value marginal productivity below factor price and it is used when equality exists. Thus a technique is not used when it generates loss as the inequality in (8.2.7) implies. It is used under equality. Thus for the two techniques to be used simultaneously (8.2.5) and (8.2.6) should indicate equalities. The solution of these equalities results in $\tilde{k}_j$. That is

$$f'_1(\tilde{k}_1) = f'_2(\tilde{k}_2)$$

$$f_1(\tilde{k}_1) - \tilde{k}_1 f'_1(\tilde{k}_1) = f_2(\tilde{k}_2) - \tilde{k}_2 f'_2(\tilde{k}_2)$$

The solution $(\tilde{k}_1, \tilde{k}_2)$ substituted in (8.2.5) and (8.2.6) yields the real factor prices $\tilde{r} \equiv f'_j(\tilde{k}_j)$, $\tilde{w} \equiv f_j(\tilde{k}_j) - \tilde{k}_j f'_j(\tilde{k}_j)$.

Agricultural output Y_A , is the sum of outputs produced by the individual techniques: $Y_A = Y_1 + Y_2$. The average labor productivity in agriculture is given by:

$$Y_A/L_A \equiv f_A(k_A) = \lambda f_1(k_1) + (1 - \lambda) f_2(k_2). \quad (8.2.8)$$

When the two techniques are used, the Euler condition implies:

$$f_j(\tilde{k}_j) = \tilde{r}\tilde{k}_j + \tilde{w}. \quad (8.2.9)$$

substituting (8.2.9) in (8.2.8) and utilizing (8.2.1) yields the aggregate average labor productivity for agriculture for $k_A \in [\tilde{k}_1, \tilde{k}_2]$:

$$f_A(k_A) = \tilde{r}k_A + \tilde{w}. \quad (8.2.10)$$

This function is linear, with $\tilde{r}$ and $\tilde{w}$ representing the marginal productivities of capital and labor, respectively, and those are, of course, fixed.

For $k_A \notin [\tilde{k}_1, \tilde{k}_2]$ only one technique is used as was already stated above and the agricultural production function is identical with that of the activated technique. We can then summarize the discussion by giving a complete description of the agricultural production function as follows:

TABLE 8.1: Forms of the Aggregate Agricultural Function and Some Derivatives for Different Values of k_A

Function	$k_A \in (0, \tilde{k}_1]$	$k_A \in [\tilde{k}_1, \tilde{k}_2]$	$k_A \in [\tilde{k}_2, \infty)$
$f_A(k_A)$	$f_1(k_A)$	$\tilde{w} + \tilde{r}k_A$	$f_2(k_A)$
$f'_A(k_A)$	$f'_1(k_A)$	$\tilde{r}$	$f'_2(k_A)$
$\omega_A(k_A)$	$\frac{f_1(k_A)}{f'_1(k_A)} - k_A$	$\tilde{w}$	$\frac{f_2(k_A)}{f'_2(k_A)} - k_A$

As indicated above, for $k_A \in [\tilde{k}_1, \tilde{k}_2]$, $\omega_A(k_A)$ is constant and therefore the inverse function does not exist; instead we have a correspondence of $k_A(\tilde{\omega})$. This is well illustrated in Figure 8.2. However, $f_A(k_A(\tilde{\omega}))$, and the marginal productivity of capital, $f'_A(k_A(\tilde{\omega}))$ are uniquely defined for any k_A , as the above table tells us.

The Transformation Curve

Having established the production structure in agriculture, we turn to examine the effect of the introduction of the modern technique on the economy. This is done under the assumption that the technology in non-

agriculture remains as before. To introduce the notation it is written here as:

$$y_N = l_N f_N(k_N) \quad (8.2.11)$$

$$\text{where } l_N = L_N/L, \quad l_A = L_A/L$$

$$L = L_N + L_A$$

$$k_N = K_N/L_N$$

$$K = K_A + K_N.$$

Hence, by the full employment conditions:

$$k = l_A k_A + l_N k_N \quad (8.2.12)$$

$$l = l_A + l_N. \quad (8.2.13)$$

The competitive conditions consist of (8.2.5), (8.2.6) and:

$$f'_N(k_N) = r \quad (8.2.14)$$

$$f_N(k_N) - k_N f'_N(k_N) = w \quad (8.2.15)$$

It is also assumed that for all admissible ω , $k_N(\omega) > k_A(\omega)$. The analysis can be conducted for any other factor intensity assumption.

The transformation curve is obtained by solving for y_A and y_N from equations (8.2.2), (8.2.3) and (8.2.11) subject to equations (8.2.12) - (8.2.15). However, when the two techniques are in use, the system is not uniquely determined in terms of ω , and therefore it is better to express the system in terms of k_A . As it is shown in the last section, $\omega(k_A)$ is uniquely determined and therefore so is $k_N(\omega)$. We now demonstrate the

derivation in Figure 8.4, which demonstrates that the transformation curve consists of segments of constant p and varying p .

For $k_A \leq \bar{k}_1$ only technique 1 is in operation and the corresponding segment of the transformation curve displays increasing p with y_A as usual. Similarly for $k_A \geq \bar{k}_2$. In both cases, $\omega(k_A)$ is monotonically increasing. For $k_A \in [\bar{k}_1, \bar{k}_2]$, ω is constant and so is $k_N(\omega)$. That fixes the marginal productivities in the N-sector. Similarly, as shown in Table 8.1, the marginal productivities in agriculture are also constant in this interval. Consequently, p is constant and the corresponding segment of the transformation curve is a straight line.

The foregoing discussion in fact establishes a continuous monotonic relationship between p and ω . The discussion is summarized in Figure 8.4. Panel I presents the $\omega_A(k_A)$ and $\omega_N(k_N)$ functions. The numbers in parentheses on the $\omega_A(k_A)$ identify the utilized techniques. Panel II presents the corresponding $p(\omega)$ function. Note specifically that $\bar{p} = p(\bar{\omega})$ and the function is continuous but not differentiable at that point. The transformation curve is given in panel III. The numbers in parentheses indicate the techniques utilized. Thus, with high agricultural production, only technique 2 is used, along with the N-technique, as it is indicated by (2, N). The middle segment indicates that it is obtained by the use of the two techniques, whereas when the share of agriculture declines, only technique 1 is used in agriculture. Panel IV of the figure portrays the corresponding agricultural supply relationships. It should be noted that when the two techniques are used the agricultural supply is perfectly elastic. This of course follows from the foregoing discussion. We subsequently return to the significance of this result.

$$y < \tilde{y} < \hat{y}$$

$$k_A < k_N$$

FIGURE 8.4

$$y < \tilde{y} < \hat{y}$$

$$k_A > k_N$$

FIGURE 8.5

The whole discussion was conducted under the assumption that $k_A < k_N$. The question is to what extent do the findings depend on this assumption. This question is analyzed in Figure 8.5 which is drawn under the assumption of $k_2 > k_1 > k_N$ for every admissible ω . We now see that the traditional technique is associated with a large agricultural production and the use of the modern technique with a small agricultural production. The reversal of the relationship between the use of the technique and the relative importance of agricultural production is also reflected in the reversal of the relationship between the use of the technique and p . However, what has not changed is the relationship between the choice of a technique and the wage rental ratio or the capital labor ratio in agriculture. This is basically the essence of the process. As the wage rental ratio increases, the labor intensive techniques become unprofitable and producers have to resort to more capital intensive techniques. What is also shown is that the process takes place gradually, there need not be a jump from one technique to another. The reader may find it useful to tell the story of gradual change in terms of the market forces.

Before concluding this section it is desirable to mention some special cases that the reader may find useful to work out for himself:

- (1) $\tilde{\omega} = \underline{\omega}(k)$. In this case one technique will not be used individually: the traditional technique when agriculture is labor intensive ($k_A < k_N$) and the modern technique when agriculture is capital intensive. Consequently, the (1, N) segment is eliminated from Figure 8.4 and the (2, N) segment is eliminated from Figure 8.5.

- (2) $\tilde{\omega} = \bar{\omega}(k)$: This is the opposite of the previous case, and the results follow accordingly.
- (3) $\tilde{\omega} = \underline{\omega}(k) = \bar{\omega}(k)$: In this case, $\omega_N(k_N)$ intersects $\omega_A(k_A)$ at the point of $k_N = k$. The modern and the traditional techniques are both used together.
- (4) $\tilde{\omega} \notin [\underline{\omega}(k), \bar{\omega}(k)]$. Only one technique is used. The choice of that technique depends on which sector is more capital intensive and on whether $\tilde{\omega} < \underline{\omega}(k)$ or $\tilde{\omega} > \bar{\omega}(k)$.

The Effect of Capital Accumulation

The addition of demand functions facilitates the determination of the short run equilibrium. From the point of view of the present discussion it is desirable to distinguish between two cases: $p = \tilde{p} \equiv p(\tilde{\omega})$ and $p \neq \tilde{p}$. The latter case is identical to that discussed in Chapter 3. Thus, we turn to the case where the two techniques are used simultaneously and prices are constant. In this case, capital accumulation generates only income effect which is represented graphically by a shift of the transformation curve. However note that in this case the Rybczynski proposition cannot be applied because $y(\tilde{p})$ is not a function but a set.

Still it is possible to sign the income effect of capital accumulation on output composition. It follows directly from the full employment conditions in (8.2.12) that for any constant values of k_A and k_N an increase in k increases the proportion of the capital intensive sector in total resources. Thus, under the assumption of $k_N > k_A$, $(\partial \ell_A / \partial k)|_{k_A, k_N} < 0$. Since $y_i = \ell_i f_i(k_i)$, $i = A, N$, any output combination obtained under given k_i is shifted by capital accumulation so that the output of the capital intensive sector increases and that of the labor intensive sector declines.

This is illustrated in Figure 8.6 where k_1 at point B are the same as at point B^* .

FIGURE 8.6

Bringing the changes in demand and supply together, and maintaining $p = \bar{p}$, the equilibrium quantities of the two sectors increase, and the rates of increase are proportional to the income elasticities.

Turning to factor employment, it is to be noted that the only way to increase y_N is to increase its proportion in employment since $k_N(\bar{\omega})$ is constant. An increase in l_N is compensated by a decrease in l_A . Since y_A also goes up, this is accomplished by increasing k_A , that is by substituting the traditional technique by the modern technique. The discussion can be concluded with the following proposition.

Proposition: When a new technique, more capital intensive than the old one, is introduced in the production of one of the products, the economy is undergoing the following changes. For relatively low values of the overall capital labor ratio only the old technique is used. For relatively high values of the overall capital labor ratio only the new technique is used. In both of these cases, the response of the economy to changes in the overall capital labor ratio is covered by the results discussed in Chapter 3. For intermediate values of the capital labor ratio the

techniques are used simultaneously. Changes in the overall capital labor ratio which do not change the state of coexistence of the two techniques do not generate a change in price and therefore there is no substitution effect on demand. Consequently, an increase in k generates a positive income effect on the two products, the relative importance of the new technique increases and resources are shifted to the capital intensive sector.

Discussion

The main point of the foregoing discussion was that an introduction of new capital intensive techniques is subject to capital constraint and the rate of adoption of the technique depends on the rate of capital accumulation. It should be noted that this finding is obtained under the assumption that the capital goods are homogeneous so that horses can be converted to tractors and vice versa. Still if the economy does not have enough capital it may find it useful to use horses beside tractors. Introducing the assumption that the existing capital goods are heterogeneous only reinforces the results.

Looking at it from the other side, if the capital intensive technique is available, the degree of its application will depend on two factors: First, the threshold capital-labor ratio required to activate it, $\tilde{k}_2$, relative to that of the traditional technique, $\tilde{k}_1$. Second, it depends on the overall capital labor-ratio, k , and on its rate of growth. If initially $\tilde{k}_2 \leq k$, the new technique will take over at a rate determined by gross investment. If, on the other hand, $\tilde{k}_2 > k$, the rate of introduction will depend on net investment. From this it can be inferred that the accumulation of capital generates demand for capital intensive techniques. Thus, firms which specialize in the production of techniques (R and D) will aim at the production of

capital intensive techniques. However, to be able to sell such techniques, the threshold factor intensity of the new technique should not be too far off from that of existing techniques.

The story can be told by looking at the firm level. In the absence of a new capital intensive techniques, capital accumulation increases the capital-labor ratios, thus increasing real wages and decreasing the real rental rate on capital. Thus, the owners of capital will be interested to invest their capital in techniques which prevent the rate of return from falling. This generates the demand for the capital intensive techniques.

Technical Change

The foregoing analysis generalizes the formulation of technical change that was analyzed in Chapter 4. Referring to equation (4.2), the techniques can be identified with the values of A_L or A_K . Such a parameterization of the sequence of new techniques is convenient analytically and can be exploited when possible. However, it should be noted that when more than one technique is used in production, the summary of the technology by (4.2) gives a distorted view, as will be explained below when the empirical implications are discussed.

To accommodate the formulation within the present framework, we start with the NTC where $A_K = A_L$. Thus, let $F_2(K,L) = AF_1(K,L)$, $A > 1$. In this case the new technique dominates completely the old technique and it is disadvantageous to employ the two techniques simultaneously. If the two techniques are used simultaneously, it can be explained in terms of additional considerations, which have been neglected so far.

The first is committed capital. So far it has been assumed that capital is homogeneous. If the new technique is embodied in a machine, then it is impossible to apply it with the old machine. This is the embodied technical change that was developed by Solow. In this case, the rate of introduction of the new technique will depend on the rate of gross investment rather than net investment. An old technique will disappear when its quasi rent becomes negative in line with the discussion following (8.2.7).

The second consideration is that of uncertainty. The new technique may be superior but firms do not know it and may require time to sample it. During this period of information collection, the various techniques will coexist.

Turning now to the FATC where $A_K \neq A_L$. This case, by definition affect factor intensities, except for the special case of $\sigma = 1$. The special case of $\sigma = 1$ can be treated for the present purpose like a NTC. Thus, FATC generates a situation such as those considered in Figures 8.1 - 8.3. Yet, the discussion in Chapter 4, like the general discussion in the literature, avoided the problem coexistence of techniques. It was also implicitly assumed that capital is not committed, and that there is no uncertainty. Under these assumptions, there is an immediate shift to the new more efficient technique. In deriving this result we ignored the difference in factor intensities of the two techniques. This can only be justified when the threshold capital-labor ratio does not exceed k , that is in terms of the previous notation, $\tilde{k}_2 \leq k$. This may be the case when the disparity in the threshold values, $\tilde{k}_2/\tilde{k}_1$, is

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close to 1. When this is not the case, then the analysis of the effect of FATC on the economy should be modified to allow for co-existence of the two techniques.

For purpose of simplification, we have dealt with two techniques, traditional and modern. The appearance of additional techniques can be handled in a very similar fashion. One case, however, is worthy of examination for its own interest and for future reference, the case of NTC in the modern technique. We select the modern technique to be the subject of the NTC for a purpose. As it has been argued that the process of capital accumulation causes a shift in the direction of capital intensive techniques, then other things being equal, the demand will call for development of the NTC to be implemented on the modern techniques. In a more detailed framework, the cost of producing and changing techniques, as well as the required research time, should be introduced. If the required time is significant, by the time the research is completed, the traditional technique may not be of any importance. Therefore, effort will be directed at increasing the productivity of the modern techniques. This consideration has a dynamic aspect. With time, the modern techniques become traditional and, therefore they have already been worked on so that the easy gains might have already been made and additional gains may be subject to increasing cost. Thus, both from the demand side and the supply side, it is likely that the effort of improving an existing technique will be aimed at the modern techniques.

An improvement in the productivity of a technique should increase the degree of its utilization. This can be established analytically by utilizing (8.2.4), writing the ratio of the labor employed in the modern to that of traditional technique:

$$\frac{1-\ell}{\ell} = \frac{k_A - \tilde{k}_1}{\tilde{k}_2 - k_A}.$$

This ratio is increased where both threshold values decline as should be expected. In fact, for a given k_A , this is the only way that $1-\ell$ can increase. This stems from the fact that $\tilde{\omega} = \omega_1(k_1) = \omega_2(\tilde{k}_2)$, and $\omega'_1(k_1) > 0$. Hence,

$$\frac{\partial(1-\ell)/\ell}{\partial \tilde{\omega}} = \left(\frac{1}{\tilde{k}_2 - k_A} \right) \left[-k'_1(\tilde{\omega})(\tilde{k}_2 - k_A) - k'_2(\tilde{\omega})(k_A - \tilde{k}_1) \right] < 0.$$

This is illustrated in Fig. 8.7, where $k_j(0)$ decline to $k_j(t)$. The net effect of this change is labor saving and thus can be expressed as a decline in the labor share at any level of k .

The foregoing discussion does not preclude technical change in the traditional technique. In this case, following the foregoing analysis, $\tilde{\omega}$ will increase, a change that will further increase the demand for capital intensive techniques.

Aggregation Over Techniques

The concept of a production function is identified here with a technique, however, in empirical application it is applied to aggregate data. The aggregation is done over products, firms, and techniques within firms. In terms of the present analysis, products, plants and firms can all be represented by techniques. So basically, the aggregates are generated by aggregation over techniques.

The foregoing analysis indicated that the aggregate production function can actually be viewed as the envelope of the available technology and as such it can be shown to behave like a production function. The question is whether the observations actually represent points on the envelope. The previous discussion already mentioned some reasons for observations falling off the frontier. In evaluating this subject it is first necessary to identify the envelope since the envelope itself is changing with the technology. For instance, referring to Fig. 8.7, it is possible to describe the following sequence of appearance of techniques: for time interval $0 \leq t \leq \tau_1$ only $F_1(0)$ was available. In this case the envelope is identical to $F_1(0)$. In time interval $\tau_1 < t \leq \tau_2$ technique 2 appears and the envelope now changes. NTC changes $F_2(0)$ to $F_2(t)$ and this new technique becomes available in $t > \tau_2$, again leading to a new frontier. Thus, technological change which takes the form of a sequence of new techniques leads to a sequence of envelopes of the available technology. An observation generated in period τ_1 may not be on the envelope of later periods because that was not feasible on τ_1 . For instance, point C on

$F_1(0)$ in Fig. 8.7 is on the envelope of τ_1 , but it is dominated by $F_2(0)$ and thus will be replaced by another point in period τ_2 . Thus, a sample consisting of observations A_1, C and A_2 can easily lead to an empirical isoquant which will be concave rather than convex.

The essence of this discussion is to point out the importance of identifying the relevant envelope. The identification should consist of variables explaining the appearance of new techniques (supply) and variables explaining the implementation of new techniques once they become available (demand). In the foregoing discussion, the demand side was determined by k . As long as $k \leq \bar{k}_1$, there is no implementation of $F_2(0)$ even if it is available. The identification of the supply is somewhat more difficult. It is generally agreed that the generation of techniques requires expenditures on research and development. The outcome depends on the scientific and technological level of the society in question which in turn depends on the quantity and quality of education. There are now many studies which attempt at qualifying the importance of these variables.

The above discussion has dealt with the movement of the frontier. But as indicated earlier, the movement to the new frontier is not necessarily immediate. That requires adding to the determinant of the demand side the constraints that determine the pace of adjustment. When capital is committed, the pace will be largely dependent on the rate of gross investment. If there is uncertainty about the quality of the new technique, then this will also have to be taken into consideration. We return to uncertainty considerations below.

The foregoing analysis can accommodate product aggregation. In this case, the output of the different techniques is now measured in value terms. Turning back to Fig. 8.7, we can now interpret $F_2(\tau)$ and $F_1(0)$ as the \$1 isoquant of the two techniques. The envelope at the initial prices will go through A_2 , A , and B_1 . An increase in the price of the capital intensive technique will move the unit isoquant closer to the origin and the new envelope will go through points B_2 and B_1 , so that points A_2 and A_1 now become inefficient. That makes it clear that the frontier depends on the price of the product associated with the various techniques. Since prices do not remain constant, it is clear that using this approach the frontier becomes variable, and perhaps stochastic. It is reasonable to assume that not all price variations will have the same effect on the frontier. Price variations that are perceived to be of longer duration (permanent) should have a stronger effect than transitory changes. The degree of such a differential effect depends on the case of adjustment as well as on the perception of the durability of such price changes.

The comments about the effects of prices apply also to expected prices which are also subject to variations.

Micro Analysis

The choice of techniques is done at the firm level and it is, therefore, desirable to examine more closely the decision at the firm level. To generalize, it is assumed that the available technology is given by $T = \{F_j(v_j, b_j) | F_j\}$, the production function of the j th

technique}, where v_j and b_j are vectors of inputs. The firm wants to maximize income subject to some constraints on b_j . Thus, the problem is to maximize:

$$L = \sum_j p_j F_j(v_j, b_j) - \sum_j w_j v_j + \lambda (b - \sum_j b_j)$$

where p_j is the price of the product of technique j , w_j is the vector of prices of the vector of inputs v_j and b is the constraint on $\sum b_j$.

The Kuhn Tucker necessary conditions for a solution are:

$$L_{v_j} = p_j F_{v_j} - w_j \leq 0$$

$$L_{b_j} = p_j F_{b_j} - \lambda \leq 0$$

$$\sum_j (L_{v_j} v_j + L_{b_j} b_j) = 0$$

$$v_j \geq 0$$

$$L_\lambda = b - \sum_j b_j = 0$$

$$\lambda L_\lambda = 0$$

The solution gives:

$$v_j^*(S), b_j^*(S), \lambda^*(S)$$

where S represents the state variables of this problem:

$$S = (b, p, w, T)$$

So the solution depends on the available technology T , on the constraint b and on the products and variable inputs prices. The solution determines

which techniques are used and the level of their use as determined by the allocation of fixed inputs b_j^* and variable inputs, v_j^* . Thus, we can determine the implemented technology:

$$IT(b, p, w, T) = \{F_j(v_j, h_j) \mid F_j(v_j^*, b_j^*) \neq 0, F_j \in T\}$$

Note that IT is a subset of T . Using duality theory, if T properly behaves, its frontier will behave like a production function. If T properly behaves, so is IT and its frontier behaves like a production function as well, but it is likely to differ from the frontier of T . Thus, the implemented production function is not necessarily the same as the production function dual to the available technology.

Figure 8.7

C. Evaluation of Research Achievements

In retrospect it is clear that the original research plan was too ambitious. The empirical analysis which was considered to be straight forward, has turned out to be more difficult than anticipated, and has not converged to a stage that can be reported here with the same level of accomplishment as that of the theoretical analysis. One phase of the empirical analysis, dealing with the demand for fruits in Israel was completed in a satisfactory form and will be written out for publication in the near future.

On the other hand, the work on the formulation of the process of growth has come out well and has extended beyond the original anticipation. Particular progress has been achieved in the analysis of technical change and resource allocation which are the main determinants of the growth process.

D. Description of Results of Cooperation

The principal investigator visited the cooperating investigator. However, the cooperative investigator was unable to visit the principal investigator. Nevertheless various aspects of the analysis were discussed by the two investigators.

E. Thesis

The study of agricultural growth (as well as sectoral growth in general) requires a formulation which takes into account explicitly the dynamics of resource allocation as well as the determinants of resource productivities.

F. Benefits to Agriculture

Extending the implication of this approach it is evident that agricultural growth is favorably affected by incentives and discouraged by taxing agriculture. This has very important policy implications.

G. Publications

Yair Mundlak "Various Aspects of Profitability of Milk Production" (Hebrew) Rehovot: The Center for Agricultural Economic Research, working paper # 8113 (1981)

David Guy "The Demand for Fruit in Israel" (Hebrew) Rehovot: Faculty of Agriculture M.S. dissertation. 1982

Javier M. Ekboir "Stability of Growth Models with Endogenous Population" (Hebrew) Rehovot: Faculty of Agriculture M.S. dissertation. 1983