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for Grains**

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CROP INSURANCE AND DROUGHT COMPENSATIONS FOR GRAINS

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ABSTRACT

Farming in semi-arid areas is often impeded not only by low levels of yields due to the low levels of precipitation but also by the very high risks that the unstable weather conditions entail. Farmers must reckon both with large fluctuations in the prices for their products - a risk that farmers everywhere must face, and with the effects of the extremely volatile weather conditions on their production. In some years they are even confronted by catastrophies, when their entire yield is totally destroyed by droughts.

To protect themselves against these risks, farmers adopt various strategies, including conservative farm management practices which not only reduce their earning potential but, from the point of view of the national economy, result in an underutilization of the agriculture resources.

Crop insurance has long been recognized as a potentially important tool of reducing farmers' risks and thereby allowing a better utilization of the national resources. Nevertheless the experience with actual programs that have been implemented in a number of countries is mixed at best. In Australia, for example, it was concluded that crop and rainfall insurance do not play a major role in farmers' risk management. A World Bank study in a number of developing countries could not find a convincing evidence that agricultural insurance has a significant positive economic impact on agricultural production.

The purpose of this research was to evaluate the performance and determine the desirability of the drought compensation program that is administered by the Government of Israel in the Northern Negev. This evaluation is summarized in Chapter 1 which provides also a description of the operating principles of the program and evaluates its desirability in the various climatic sub-regions.

Clearly, the performance of the program and its potential contribution to agricultural production, depends to a very large degree on the extent to which farmers are averse to risk, and on the strategies that they are likely to adopt in order to protect themselves against these risks in the absence of an insurance program.

The performance of the program and its desirability to the national economy also depends on the "moral hazard" that it entails, i.e. on the risks that farmers may decide to take, knowing that they are protected by the program. These risky practices also leads to a wasteful utilization of resources. Chapters 2 and 3 examine several criteria for evaluating the economic and financial implications of farmers' response to risk, of the measures that farmers take in order to protect themselves against these risks, and of the resulting effects of the drought compensations program.

The last three chapters further extend the analysis of farmers' behavior under uncertainty and their response to risk. Chapter 4 develops a non-parametric test of the expected utility hypothesis. It determines the conditions under which the hypothesis holds, showing that it depends on whether there exists a feasible solution to a system of linear inequalities which is defined the farmers' choice set. When that solution exists, it can be "manipulated" to yield boundaries on the coefficient of absolute risk aversion. The test is applied to data on choices of land allocations that are modeled as choices over lottery sets. The empirical results are that the expected utility hypothesis cannot be rejected in most of the cases. These results are in contrast to results obtained in laboratory experiments. Chapter 5 develops an econometric procedure for estimating the Arrow-Pratt coefficients of risk aversion. Farmers' land allocation among different crops and time allocation between leisure and labor, provide the framework for testing Arrow's hypotheses of decreasing absolute risk aversion and of increasing relative risk aversion. The empirical analysis in the chapter supports these two hypotheses. Chapter 6 analyzes in more details farmer's own labor supply under uncertainty and the demand for hired labor. The

effects of increases in income, in the mean of the random variable, in risk, and in the degree of risk aversion are investigated for different "types" of risks. The analysis indicates that the results are mostly unambiguous in farms where the only labor is that of the owner. In farms that employ also hired labor, unambiguous results are assured under the additional assumption that the coefficient of absolute risk aversion is non-decreasing with labor.

OBJECTIVES OF THE RESEARCH PROPOSAL

The original research proposal had three main objectives:

(i) A theoretical evaluation of alternative policies and programs aimed at stabilizing agricultural incomes and assisting farmers in disaster years.

(ii) A theoretical study of farmers' attitude toward risk and an evaluation of the potential effects of crop insurance and other agricultural stabilization schemes on their production decisions.

(iii) An empirical analysis and an evaluation of the Drought Compensation Program which is implemented by the Government of Israel in the Northern Negev.

Chapter 1 summarizes the finding of this empirical analysis. (The complete report was written in Hebrew and is not included in the main report). Chapter 2 develops criteria for evaluating investment and production alternatives in agriculture on the basis of alternative assumptions on farmer's attitude toward risk and apply these criteria in order to analyze farmer's choices of crop rotations and the effects of the Drought Compensation Programs on their Choices. Chapter 3 develops another set of criteria that are often grouped in the literature under the title "Safety First", and apply these criteria to evaluate the effects of and the overall benefits from the Drought Compensation Program. Chapters 3 through 5 extend the theoretical analysis of farmers' attitude toward risk and develop methods for empirically estimating their aversion to the various risks in production - some of which are due to unstable weather conditions and others are due to market conditions and to policy shifts. Finally, chapter 6 examines the potential effects of uncertainty on farmers' labor supply.

Chapter 1:

Wheat Production in Semiarid Areas and The Role of Drought Compensation: The Israeli Experience in The Northern Negev

David Bigman, Nir Becker, and Hector Barak

WHEAT PRODUCTION IN SEMIARID AREAS AND THE ROLE OF DROUGHT COMPENSATION: THE ISRAELI EXPERIENCE IN THE NORTHERN NEGEV*

The objective of this paper is to summarize an extensive, two-year study on alternative methods of wheat production in the Northern Negev of Israel and on the desirability of a drought compensation program that the government administers in this area.

An important conclusion that came out of our study is that new technological innovations in the field of satellite and aerial photography in agriculture have the potential of providing an extremely efficient and inexpensive instrument to monitor the implementation of that program and, possibly also, other forms of crop insurance programs. In a number of field crops, these technologies may permit a drastic reduction of the moral hazard that all too often undermines crop insurance programs, thereby making them a much more attractive policy instrument.

We believe that the lesson from the Israeli experience can be instructive to other areas, and it is our intent to renew, through this report, the debate over the desirability of government involvement in crop insurance programs.

In recent years there seems to be a consensus among economists that governments should stay out, as much as possible, of the finance and implementation of crop insurance programs (e.g., the collection of studies in Hazell and Valdez, 1986); and international agencies such as the World Bank have thumbed down any support for these programs. With the new technologies of aerial photography in agriculture, this consensus should be reconsidered, and government involvement, that is often pivotal in semiarid areas, may be less objectionable.

I. Background

Since the full report is written only in Hebrew, we will spare our readers some agony by briefly reviewing the geographical and institutional setting. The Northern Negev is the largest agricultural land in Israel. It contains 2.7 million dunam of cultivated land: 1.2 million of which is used for grazing, 1.2 million is used for nonirrigated field crops (mostly wheat), and 0.3 million for irrigated crops--almost half of it for wheat (although this may vary from year to year). The climate in the area is typical semiarid, characterized by a low and very unstable level of precipitation-- both from year to year and within each year. There are also considerable differences in the climate between subregions in the area, as illustrated in Table 1.

Most of the nonirrigated lands are cultivated by the Kibbutzim (communal farms), and until the mid-1970s a large portion of their income came from agriculture--especially wheat. Over the last decade the share of industry has continuously increased and, with the expansion of the irrigated lands, they have also diversified their crops. Since 1964, the government implemented a drought compensation program that was centered on this region. Even before that year, however, the government supported the local settlers during years of severe drought, its major objective being to solidify the settlements in this region which at that time was very sparsely populated.

The program covers about 85 percent of the cultivated land in the Northern Negev (north of Beer Sheva, south of Kiryat Gat). The remaining 15 percent, situated south of what is known in local jargon as the "drought line," is not covered by the program but still cultivated by the local settlers. The cultivation practices in these latter lands will serve us as a reference to

Table 1. Distribution of Precipitation in the Three Subregions of the Northern Negev

	Dorot (north)	Lahav (central)	Gilat (South)
	millimeters		
Annual average	330	306	218
Maximum	608	562	569
Minimum	92	137	42
Standard deviation	147	115	89
Coefficient of variation	0.45	0.38	0.41

evaluate the practices in the lands covered by the program--albeit with some reservations that we will mention later on.

The program compensates the wheat growers up to 100 percent of their production costs whenever the yield falls below their break-even level. (Wheat growers north of Kiryat Gat, i.e., in the central and northern regions of the country, are compensated only up to 60 percent of their costs.) The break-even level (and, thus, the calculated production costs) is determined by a joint committee consisting of government representatives, representatives of the local settlers, and field experts.

Until the early 1980s, this committee was also in charge of determining the actual yields in a drought year in order to determine the actual amount of compensations. To assess the actual yields, members of the committee physically visited the affected fields in order to inspect their state first-hand and take various samples. This process was necessarily very tedious and clumsy and often resulted in arduous disputes. In the early 1980s this system was replaced by another method by which the evaluations were done on the basis of a biological model of wheat growing. The model estimates output on the basis of climatic, geographical, and managerial data. This is also the model that we have used in our analysis. The model was fed actual climatic data for the year that were collected in a number of observation points throughout the region and produced yield estimates. These estimates were compared with samples and, on the basis of these two sources, the final estimates were determined. Over the years the model proved to be accurate and highly efficient in minimizing haggling, disputes, and estimation costs. Furthermore, the model was continually revised, its parameters reestimated with the accumulated data, and from time to time new variables were added to take into account new technologies, etc.-

II. The Practices of Wheat Growing in the Northern Negev

In this study we have focused on the following managerial decisions that we have found to be pivotal in farming the dry lands in the Northern Negev.

From year to year: crop rotation which is primarily a decision on the percentage of the land that is laid idle.

Within the year: (i) sowing date and the decision whether or not to sow that year and (ii) harvest date and the decision whether or not to harvest that year.

The decision on crop rotation is especially crucial in dry farming. By laying idle part of their lands, farmers prevent escalation in losses due to pest activity and allow an accumulation of water in the lower layers of the soil. Furthermore, the practices of soil tillage during the years in which the land is fallowed may play an important role in allowing the accumulation of water. The model that we have used was not constructed, however, to detect the effects of soil tillage practices, and we have left them as a subject that requires further research.

The decision on the sowing date is designed so as to guarantee the seed with at least the minimum level of moisture necessary for successful budding. The period suitable for sowing in the Northern Negev is from early November to mid-December. An early sowing secures relatively warm temperatures that are ideal for budding and for the growth of the plant at the early stages. In mid-November, however, the amount of rainfall may not suffice to provide the minimum level of moisture which is necessary for budding.¹ The later the sowing date the higher the probability of having sufficient moisture but the lower the temperatures. Studies in the area have shown that for every single

day in which the sowing of wheat is postponed beyond mid-November, there is a loss in yield equal, on average, to 0.5 percent to 1.5 percent of the total yield, depending on the region.

In our visits to the area, we have found striking differences in the farming practices between the two sides of the drought line. North of the line, in the area covered by the drought compensation program, farmers invariably have sown their land in early November, irrespective of the amount of rainfall. South of the line, in the area not covered by the program, sowing was postponed as much as possible to assure the minimum level of moisture. The operating rule in the lands south of the line was the following: If at any date prior to mid-December there was more than the minimum level of moisture in the soil (the equivalent of 80 mm of rain), they would start sowing at that date. If there was insufficient moisture even by mid-December, they would not sow these lands at all in that year.²

Obviously, these differences in the practices north and south of the drought line are but one manifestation of the moral hazard which accompanies the compensation program.

The decision whether or not to harvest in a given year depends primarily on the yield (which includes not only the grains but also the hay). It may also depend on the possibility of using the fields for grazing.

To analyze the crop rotation decisions, we inquired first about the actual farming practices in the various subregions. We found that many Kibbutzim included fodder in their crop rotations for two reasons: First, they needed fodder to feed their animals, and they found it cheaper to grow the fodder themselves. Second, the inclusion of fodder in the crop rotation prevented the spread of pests that are typical to wheat and added important nutrients to

the soil. On the other hand, though, fodder extracts water from the deeper layers of the soil and, as a result, the yield of wheat in that field the following year is smaller than the yield in a field that was fallowed in the previous year.

As a conclusion of our inquiry, we have focused on the following alternative crop rotations:

- I. Wheat and Fallow, i.e., every year one-half of the land is fallowed.
- II. Wheat and Wheat and Fallow, i.e., every year one-third of the land is fallowed.
- III. Wheat and Fodder and Wheat and Fallow, i.e., every year one-fourth of the land is fallowed.
- IV. Wheat and Wheat and Fodder and Wheat and Fallow, i.e., every year one-fifth of the land is fallowed.

In all of these alternatives, sowing takes place in early November. We have also considered, as a fifth alternative, the practice south of the drought line, whereby the sowing decision is reexamined every day from early November through mid-December on the basis of the moisture level at the three top layers of soil. We denote this alternative "dynamic."

Finally, we examined the possibility of adding supplementary irrigation in mid-November to secure optimal moisture level during budding. This possibility applies only to 20 percent of the lands that have the appropriate infrastructure for irrigation. Until recent years, farmers used to irrigate the wheat in these fields twice during the season: in mid-November and in mid-February. With these irrigations they have obviously eliminated the risk

of a drought. In the last three years, following a succession of droughts countrywide, there has been a considerable shortage in water for irrigation. This has forced farmers to choose either one of the following two alternatives: Either to irrigate all of their irrigated fields only once a year (i.e., in mid-November) or irrigate only part of their irrigated fields twice a year, as before. Having only one irrigation exposed them to the risk of drought. It allowed, however, a more efficient use of the water. In this study, we have considered the alternative of supplementary irrigation in mid-November for the above four crop rotations, I through IV, and the effect of the drought compensation program on these methods.³ All together we have, therefore, examined nine methods: four dry farming rotations, four with supplementary irrigation, and one with a dynamic decision process with respect to the sowing date.

III. The Methodology

To evaluate the effects of the drought compensation program, we have calculated the yield and the net profit over a period of 20 years for each of the above methods with and without the compensation program. Toward that end, we have collected daily climatic data of temperatures and precipitation in each of the three subregions that characterize the Northern Negev. These are: Dorot, the northern subregion; Lahav, the central subregion, and Gilat, the southern subregion. The yield was calculated by the biological model mentioned earlier.⁴ The model estimates the daily growth of wheat on the basis of the daily climatic data on temperature, precipitation, and evaporation as well as the agronomical characteristics such as soil composition at different layers (up to the depth of 210 cm), soil salinity, etc., and the managerial decisions such as the sowing date, crop rotation, fertilization,

pesticidation, etc. The revenue over the 20-year period was calculated for the nine production methods from the yield under the assumption that the price of wheat is \$180 per ton. This was the average price for the imported wheat in Israel (including freight, insurance, etc.) over the last three years (not including premia for quality). The revenue estimate includes also the revenue from hay and fodder. To estimate the cost we divided the total costs into eight categories and estimated each of the categories in the three sub-regions. The cost estimates were based on the calculations of the drought compensation committee (which calculates the cost for the central region only), estimates of the farmers themselves, and estimates of field experts in the regional extension services.

We have thus obtained nine distributions of net profit for the nine methods of farming over a period of 20 years. From these distributions we could determine the efficient methods and eliminate the inefficient ones. Our main efficiency criterion was the second stochastic dominance (SSD), but we have examined also a number of other criteria. By comparing the efficiency frontier without the drought compensation program to that with the program, we could determine the effects of the program on the choice of the farming method. We have also estimated the net benefit (or loss) to the overall economy and the value of the dollar saved under each of the farming methods in order to determine the overall desirability of the program.

IV. The Choice of a Farming Method Without Compensation

Table 2 summarizes the average yield and profit data for the four crop rotations when there is no irrigation and when sowing is in mid-November. At the price of \$180 per ton, wheat production would be desirable in the Dorot (northern) subregion only. In the Lahav (central) and even more so in the

Table 2. Average Output and Profit per Dunam Under Alternative Crop Rotations Without Irrigation

	Crop rotation			
	I	II	III	IV
<u>Dorot</u>				
Output (kg/dunam)	133	166	162	179
Revenue (dollars/dunam) ^a	23.86	29.80	29.13	32.17
Cost (dollars/dunam)	21.77	26.83	27.53	29.39
Profit (dollars/dunam) ^a	+2.09	+2.96	+1.60	+2.78
Break-even price (dollars/ton) ^b	164.00	161.00	170.00	164.00
<u>Lahav</u>				
Output (kg/dunam)	86	105	104	112
Revenue (dollars/dunam) ^a	15.50	18.97	18.70	20.13
Cost (dollars/dunam)	15.88	20.00	20.45	22.11
Profit (dollars/dunam) ^a	-0.38	-1.04	-1.76	-1.98
Break-even price (dollars/ton) ^b	183.00	190.00	196.00	197.00
<u>Gilat</u>				
Output (kg/dunam)	39	49	47	51
Revenue (dollars/dunam) ^a	7.06	8.84	8.46	9.25
Cost (dollars/dunam)	13.65	17.14	18.04	18.72
Profit (dollars/dunam) ^a	-6.59	-8.30	-9.58	-9.47
Break-even price (dollars/ton) ^b	350.00	348.00	383.00	367.00

^aWhen the average price is \$180 per ton.

^bThe price at which revenue equals cost.

Gilat (southern) subregions, farmers would incur substantial losses in the long run.⁵

It should be noted, though, that the expected profit in the Dorot subregion does by no means ascertain that this land would indeed be farmed because of the high risks involved. The profit does indicate, however, that wheat production in that subregion is desirable from the point of view of the economy as a whole in that, on average, it offers a cheaper substitute to the imported wheat.

Furthermore, the loss incurred in the central and southern subregions does not necessarily imply that the farming of these lands is not desirable from the point of view of the economy as a whole. As noted earlier, there are a number of important national objectives other than the production of import substitutes that can only be achieved by assuring that the land is farmed. Among them are preventing the desertification of the Northern Negev, enabling Israel to sustain a higher degree of self-sufficiency in wheat supply, reasons related to national security, etc.

In all three subregions the question facing the policymaker is, therefore, what would be the most efficient way to make certain that all the national objectives are achieved, the main policy instrument being the drought compensation program.

Table 3 summarizes the profit characteristics under the different crop rotations when there are no compensations. In this table we have included also the discretionary dynamic method of sowing and the four crop rotations under supplementary irrigation. In addition to the mean and the standard deviation, the table also calculates a Baumol-type utility function given by

(1)

$$U = \mu - k\sigma$$

Table 3. Profit Characteristics Under Different Crop Rotations: No Compensation

	Crop rotation										No crop
	No irrigation				Supplementary irrigation						
	I	II	III	IV	Dynamic	I	II	III	IV		
dollar per dunam											
<u>Dorot</u>											
Average	2.09	2.96	1.60	2.78	3.36	2.39	3.32	2.30	3.70	0	
Standard deviation	16.27	20.63	19.95	22.16	23.01	15.61	19.13	19.17	20.42	0	
Coefficient of variation	7.79	6.96	12.47	7.97	6.65	6.53	5.76	8.33	5.52	0	
$\mu - 0.1 \cdot \sigma$	0.46	0.90	-0.39	0.56	1.06	0.83	1.41	0.38	1.66	0	
Second stochastic dominance criterion	Na	N	N	N	E ^b	E	E	N	E	E	
<u>Lahav</u>											
Average	-0.38	-1.04	-1.76	-1.98	+3.06	+1.88	-0.59	-0.73	-1.97	0	
Standard deviation	14.96	19.02	19.20	20.05	18.82	16.02	19.98	20.02	20.53	0	
Coefficient of variation	-39.38	-18.27	-10.31	-10.13	6.15	8.52	-33.68	-27.42	-10.42	0	
$\mu - 0.1 \cdot \sigma$	-11.11	-14.11	-14.82	-16.43	+1.18	+0.28	-2.59	-2.73	-4.02	0	
Second stochastic dominance criterion	N	N	N	N	E	E	N	N	N	E	
<u>Gilat</u>											
Average	-6.59	-8.30	-9.45	-9.47	-2.78	-4.90	-6.35	-7.09	-7.17	0	
Standard deviation	10.05	12.79	11.96	13.22	10.59	10.75	13.35	13.90	16.36	0	
Coefficient of variation	-1.53	-1.54	-1.25	-1.40	-3.94	-2.18	-2.10	-1.96	-2.28	0	
$\mu - 0.1 \cdot \sigma$	-7.59	-9.58	-10.65	-10.79	-3.84	-5.97	-7.68	-8.48	-8.80	0	
Second stochastic dominance criterion	N	N	N	N	E	N	N	N	N	E	

^aNa = nonefficient on the basis of the second stochastic dominance criterion.

^bE = efficient on the basis of the second stochastic dominance criterion.

where μ is the mean profit and σ its standard deviation. In this illustrative calculation we assumed the risk coefficient k to be 0.1.

In land where infrastructure for irrigation exists, our calculations clearly show that supplementary irrigation would be an efficient method. Other calculations, not reported here, indicate that, in the last three years as water for irrigation became much scarcer, farmers could have gained by switching to supplementary irrigation in all the lands that could have been irrigated rather than continuing their present practice of irrigating part of these lands twice during the season while growing wheat without irrigating in the remaining part. The main reason why farmers did not change their practice was that supplementary irrigation exposed them to considerable risks--as Table 3 clearly illustrates--while the drought compensation program covered only the nonirrigated land. As a result of our findings, the program was extended in 1987 to cover also land receiving supplementary irrigation.

Most of the land in the Northern Negev does not have, however, the necessary infrastructure for irrigation. Leaving aside in this report the question of whether investment in this infrastructure is profitable for the farmers and desirable from the general economy point of view, we concentrate in the remainder of the paper on the nonirrigated lands.

In these lands, the general conclusion that came out of our calculations is that the more extensive the crop rotation the lower the net profit (in Dorot) or the higher the loss (in Lahav and Gilat). The dynamic method, which is the least extensive, would yield the highest profit in Dorot and Lahav and the lowest loss in Gilat. Although we would caution farmers in the Northern Negev from adopting the ad-hoc rule for sowing under this method, it seems to us that further research on the optimal sowing date is likely to reap high rewards.

Finally, we should emphasize once more that, although wheat growing is likely to be profitable in Dorot and, under the dynamic method, also in Lahav, it is still possible that the local settlers would not have farmed this land had it not been for the drought compensation program, lacking the means and the access to the capital market to bear the risks involved. This is indicated by the fact that, in all subregions, the option of having no crops is always on the efficiency frontier.

V. The Choice of a Farming Method with Compensation

Table 4 summarizes the profit characteristics under the different crop rotations in nonirrigated land when the drought compensation is in effect. A clear pattern that comes out of our calculations: The more extensive the crop rotation the higher the profit. The reason is obvious. By compensating farmers for all their losses, the program eliminates all their risks. The more extensive crop rotations would, therefore, simply increase the farmers' entitlement for compensation without exposing them to higher risks. A farmer who would choose to fallow half his land would thus lose the entitlement for compensation on the laid-off land whereas the expected extra yield due to the fallow cannot compensate him for this loss. In short, the switch to more extensive crop rotations was the most important manifestation of the moral hazard that we have found in dry land farming.

In Lahav (except for the dynamic method) and in Gilat, the average compensation is much higher than the average profit. Perhaps somewhat crudely stated, this reflects the fact that farmers choose to cultivate each year the land, and lose money in the process, for the sole purpose of acquiring entitlement to receive compensation. The more land they cultivate, the higher their average compensation, which more than offsets their production losses.

Table 4. Profit Characteristics Under Different Crop Rotations with Drought Compensation

	Crop rotations				
	I	II	III	IV	Dynamic
	dollar per dunam				
<u>Dorot</u>					
Average	5.75	7.95	7.45	9.03	4.90
Standard deviation	11.60	14.52	12.90	14.64	21.50
Coefficient of variation	2.02	1.83	1.73	1.62	4.39
$\mu - 0.1 \cdot \sigma$	4.59	6.50	6.16	7.57	2.75
Average compensation	3.65	4.98	5.85	6.24	1.61
Compensation as percent of profit	64.00	63.00	79.00	69.00	33.00
<u>Lahav</u>					
Average	5.71	7.67	7.94	8.57	5.09
Standard deviation	9.97	12.12	10.71	11.82	18.20
Coefficient of variation	1.75	1.58	1.35	1.38	3.58
$\mu - 0.1 \cdot \sigma$	4.71	6.46	6.87	7.39	3.27
Average compensation	6.09	8.73	9.71	10.65	2.01
Compensation as percent of profit	107.00	114.00	122.00	124.00	40.00
<u>Gilat</u>					
Average	2.98	4.46	4.83	5.78	0.53
Standard deviation	4.66	5.64	4.37	5.14	9.37
Coefficient of variation	1.56	1.26	0.90	0.89	17.58
$\mu - 0.1 \cdot \sigma$	2.51	3.90	4.39	5.27	-0.41
Average compensation	9.57	12.78	14.42	15.26	3.31
Compensation as percent of profit	321.00	286.00	298.00	264.00	625.00

Table 5 summarizes the economic gains or losses from the program. It calculates the net gain for the national economy due to the transition from any one of the methods on the efficiency frontier without compensation to the efficient method with compensation. Without having precise knowledge of the farmers' utility function, we could not determine, of course, which of the efficient methods they would have chosen had they not received the drought compensation. We thus had to examine all the possible methods on the efficiency frontier. Their choice of crop rotation with the program was easier to determine both because our calculations have indicated that the more extensive rotations clearly dominate the less extensive ones and because we have substantiated our results with an extensive field inquiry among the farmers in the various subregions.

As Table 5 clearly indicates, if farmers in Dorot had chosen "method II" without the program, then the program would have yielded a loss as a result of the switch to the more extensive method IV. If, however, the (more risk averse) farmers in that subregion had chosen not to cultivate the land at all without compensation, then the program would have yielded a substantial gain of \$2.78 per dunam. In Lahav and Gilat the program yields only losses by inducing farmers to abandon the more cautious and less extensive dynamic method or by changing their decision not to cultivate the land at all without compensation in favor of the most extensive and, hence, most risky crop rotation when the program is in effect.

VI. Constrained Compensation Program

The moral hazard induced by the drought compensation program is a common phenomenon in practically all insurance programs. In our case the moral hazard was manifested in two ways. First, farmers were induced to switch to

Table 5. Economic Gain or Loss from Drought Compensation: Present Program

Without compensation		With compensation		Net gain from compensation
Method	Net gain	Method	Net gain	
dollar per dunam				
<u>Dorot</u>				
I	2.09			+0.69
II	2.96			-0.16
IV	2.78	IV	2.78	0.00
Dynamic	3.36			-0.58
Ø	0.00			+2.78
<u>Lahav</u>				
Dynamic	3.06			-5.04
		IV	-1.98	
Ø	0.00			-1.98
<u>Gilat</u>				
0.0	0.0	IV	-9.47	-9.47

more extensive crop rotations; second, the program eliminated the incentive to take precautionary measures before sowing so as to secure a minimum level of moisture in the soil. In our study we were able to assess the financial losses that can be attributed to these effects of the moral hazard as we shall later see. These effects of the program raised considerable criticism in Israel over the years and repeated calls, both within the government and among field experts, to abolish the program in its entirety or replace it by direct income transfers. The intention of the latter proposal was to provide the financial support to the local settlers in a form of lump-sum payments that would not have any effect on their farming practices. In our report we objected to that proposal on the grounds that, by abolishing the drought compensation program, the government would eliminate not only the moral hazard induced by the program but also its positive effects. Our main concern was that, without some form of insurance, farmers will not be able or willing to bear the risks; and they may choose either not to cultivate these lands at all or to select farming methods that are much less extensive than the desirable ones for the national economy.

Recent technological developments in the area of aerial photography made possible an alternative approach that we have recommended in our report. Under the proposed scheme, participation in the drought compensation program will be conditioned on an agreement to follow by a predetermined crop rotation that will set clear limits on the farming practices. Field experts and government officials will determine the desirable crop rotation that can best meet national goals--broadly defined--for each geographical subregion. Aerial photography will be used to monitor farmers to see that they do indeed comply with these constraints. It will allow experts to determine not only what crop

has been grown in each field or whether the land has been laid idle, but also the tillage practices of the soil, particularly in years when the field is fallowed.

Table 6 illustrates the overall economic gains that can be expected from the constrained compensation program. Under the constrained program, farmers in the Dorot subregion would have to observe the crop rotation designated as method II, and farmers in the Lahav subregion would have to follow the dynamic sowing method. We have not included the Gilat subregion in these calculations because, at the price of \$180 per ton, wheat growing in that subregion would not be desirable for the national economy unless objectives other than substituting the imported wheat are taken into account.

Perhaps more meaningful than the latter figures is the calculation the net gain that can be expected from constraining the present compensation program. These gains are detailed in Table 7. In this table we examined constraints on the program which are slightly different from those in Table 6. In the Dorot subregion farmers must observe the method II crop rotation. This would result in net economic gains of \$0.16 per dunam and a savings of \$1.26 per dunam in the average compensation, i.e., a reduction of 20 percent in the budget for compensation. In Lahav we examined three different constraints. If farmers are required to observe the method II crop rotation, the net economic gain would total \$0.94 per dunam, and the budgetary savings would be \$1.92 per dunam or 18 percent. The savings will be much higher if more severe restrictions are enforced.

Most dramatic are the gains in the Gilat subregion. Indeed, as a result of our recommendations, the Ministry of Finance, which is in charge of the program, has already decided to constrain the program as of 1988 to the method II crop rotation (i.e., to fallow each year one-third of these lands

Table 6. Economic Gain or Loss from Constrained Drought Compensation

Without compensation		With constrained compensation		Net gain from constrained compensation
Method	Net gain	Method	Net gain	
dollar per dunam				
<u>Dorot</u>				
I	2.09			+0.87
II	2.96	II	2.96	-0.00
IV	2.78			+0.16
Dynamic	3.36			-0.40
Ø	0.00			+2.96
<u>Lahav</u>				
Dynamic	3.06	D	3.06	0.00
Ø	0.00			+3.06

Table 7. Economic and Fiscal Gain from the Constraints on Drought Compensation

	Present program		Method	Constrained program		Change in compensation	Change in net gain
	Method	Compensation		Net gain	Net gain		
	dollar per dunam						
<u>Dorot</u>							
IV	6.24	2.78	II	4.98	2.96	-1.26	+0.16
<u>Lahav</u>							
IV	10.65	-1.98	II	8.73	-1.04	-1.92	+0.94
			I	6.09	-0.38	-4.56	+1.60
			Dynamic	2.01	+3.06	-8.64	+5.04
<u>Gilat</u>							
IV	15.26	-9.47	II	12.78	-8.30	-2.48	+1.17
			I	9.57	-6.59	-5.69	+2.88
			Dynamic	3.31	-2.78	-11.95	+6.69

instead of one-fifth before that). Our estimates indicate that, even with this mild restriction, the Ministry is likely to save \$2.48 per dunam or some 16 percent of the total compensation budget to that subregion. These savings would allow the Ministry to expand the program at no extra costs to the 150,000 dunams of the lands south of the "drought line" which, at the present, are not covered.

VII. Conclusions

Moral hazard, which is a well known and mostly inevitable by-product of all insurance programs, creates special difficulties in crop insurance programs.⁶ In our study on the drought compensation programs in the Northern Negev in Israel, the moral hazard was reflected mostly by the transition from the least extensive crop rotation (which fallows some 50 percent of the land under cultivation) to the most extensive crop rotation (which fallows only 20 percent of the land).

These effects of the program caused a considerable waste in the budget and losses to the national economy. Nevertheless, drastic measures to abolish the program in its entirety--as has sometimes been suggested--may be even less desirable and more wasteful because the high risks involved in dry land farming may deter farmers from cultivating these lands at all.

This dilemma, which is well known in many crop insurance programs, may be resolved, at least for selected field crops, with the use of aerial photography to monitor the crop rotation and soil tillage practices. Our study indicates that, by imposing appropriate restrictions on participating farmers with respect to their crop rotation, a large portion of the moral hazard can be eliminated and the fiscal budget which is required for the program can be drastically reduced.

Footnotes

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¹To lower that risk, the seeds are sown 8 cm into the soil.

²In 1985 the early rains came only in December 20. In one Kibbutz they decided to sow all their land; in a neighboring Kibbutz they decided not to sow at all.

³Until 1987 fields under irrigation--even once a year--were not included in the compensation program. As a result of our recommendation, this restriction has been canceled.

⁴The model has been constructed and estimated by H. Tzaban. It had been later developed by Tahal Co. on behalf of the government ministry in charge of the program.

⁵Farming south of the drought line was profitable only because it allowed farmers to make use of their excess capacity in machinery and labor. Hence, the relevant costs in these lands were the marginal costs.

⁶Since our study examines a drought compensation program, the other problem that often undermines crop-insurance programs (viz., adverse selection) creates, of course, no difficulty here.

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Chapter 2:

**Cardinal Criteria for Ranking Investment Alternatives and
for Evaluating Portfolio Choices by Means of The mean and
The Variance: A Theoretical Analysis and an Empirical Analysis
of Farmers' Choices under Uncertainty in the Northern Negev**

David Bigman

CARDINAL CRITERIA FOR RANKING INVESTMENT ALTERNATIVES AND FOR EVALUATING
PORTFOLIO CHOICES BY MEANS OF THE MEAN AND THE VARIANCE

Despite well-known limitations, the decision rule for evaluating investments that has been proposed by Harry Markowitz (1952) on the basis of their expected return and variance, is still highly popular. This rule, which is often referred to in the literature as the E-V rule, is known to be consistent with the expected utility (EU) criterion for ranking alternative investment portfolios only if some restrictions are placed on either the agents' preferences or on the set of random variables comprising the choice set. Two familiar examples of such restrictions require that the agent's utility function be quadratic or that the random alternatives be normally distributed. Recently, Jack Meyer (1987) has demonstrated that a sufficient condition that ensures the consistency of the E-V rule with the EU criterion is that the choice set be composed of random variables which differ from one another only by location and scale parameters.

This paper examines the risk premium as a criterion for evaluating alternative investment portfolios and develops an expression for approximating the risk premium in terms of the mean, the variance and the coefficient of relative risk aversion. It shows that if this approximation is sufficiently close then the E-V will be consistent with the EU criterion under fairly general conditions. The paper then proceeds to examine additional criteria for ranking alternative portfolios within the efficient set when some knowledge is available on the degree to which agents are risk averse.

I. The Criterion for Evaluation and the Approximation Method.

Let $Y = (y_1, \dots, y_n) \in \Omega^n$ be a set of possible outcomes of a given portfolio. To simplify the notations - and without limiting the generality - I will assume that all outcomes are equally probable and that they are all non-negative⁽¹⁾. Let μ denote the expected outcome of (or the expected return from) that portfolio, and let y_E denote the 'certainty equivalent' (CE) outcome. Obviously, the ranking of portfolios by their EU is consistent with their ranking by their corresponding CE outcomes. The advantage of the latter as a criterion for evaluation is that they indicate not only whether a given portfolio would be preferred over another one, but also provide a clear cardinal measure which shows by how much would the investor be better off in selecting the one over the other, and express in money value his likely gains.

To illustrate this criterion and describe the method of approximation, let us turn to Figure 1. Consider the set of (two) possible outcomes (y_1, y_2) indicated in the Figure by point A. The mean value of these two (equally likely) outcomes is determined in the Figure at the intersection of the (negative) 45° line that crosses through A with the (positive) 45° ray from the origin at E. The agent's preferences are assumed to be represented by a continuously differentiable and (strictly) concave utility function that has the expected utility property. In the Figure, these preferences are shown by the indifference curve that crosses through A. The intersection of that indifference curve with the (positive) 45° ray from the origin at B determines the CE outcome y_E .

A comparison of the portfolio given by the outcomes at A with another portfolio given by the outcomes at A^0 shows that the former has a higher mean ($\mu > \mu^0$) but also a higher spread. In the two dimensional illustration, the higher

FIGURE 1.

spread is indicated by the steeper slope of the ray OA, which shows the ratio between the two outcomes, relative to that of OA^0 . Furthermore, the outcomes at A do not dominate the outcomes at A^0 in the sense of the first criterion of stochastic dominance (i.e. Pareto - dominance) since $y_1^0 > y_1$. The choice between these two portfolios clearly depends on the degree to which agents are risk-averse. If agents are risk-neutral, they select the portfolio on the basis of its expected outcome and hence prefer A over A^0 . The difference $(\mu - \mu^0)$ expresses the potential gains from this choice in money terms. If agents are extreme risk-averse, and make the selection on the basis of the Max-Min criterion, then they prefer A^0 over A. The difference $(y_1^0 - y_1)$ then expresses their potential gains from this choice.

In general, these two portfolios are included in the efficient set from which the final selection of the most desirable alternative would be made. To identify that selection we must have more knowledge on the agents preference ordering and, in particular, on the degree to which they are risk averse. Without that knowledge, we cannot determine whether the portfolio at A would be preferred over the one at A^0 , no matter how larger y_2 is than y_2^0 and how small is the difference between y_1^0 and y_1 . In other words, the order of investment portfolios only on the basis of their expected outcome and their spread - however measured - is (notoriously) incomplete.

These, of course, are familiar problems in portfolio selection which every textbook in financial analysis must encounter early on. What, however, if we have some knowledge on the agents' preferences; what if we know that they are neither risk-neutral nor the most extreme (Maximin) risk-averse; what, in other words, if we know the range within which their true degree of relative

risk-aversion is likely to be? Can we use that knowledge to determine a more complete order of investment portfolios and thus a smaller efficient set? Clearly, the E-V rule does not offer any guidance in that direction even when the mean and the variance do provide all of the relevant information about the distribution of possible outcomes, nor are the other criteria that establish the second- degree stochastic dominance relationship (e.g. Mean-Gini) helpful.

To identify such a criterion, consider agents having a 'rational' preference ordering (i.e., complete, transitive and reflexive) that can be represented by an appropriate utility function having the expected utility property. To represent 'ordinary' risk-averse agents, Kenneth Arrow (1964) added the following requirements:

$$(i) \quad U'(y) > 0$$

$$(ii) \quad U''(y) < 0$$

$$(iii) \quad \frac{d}{dy} \left[- \frac{U''(y)}{U'(y)} \right] \leq 0$$

$$(iv) \quad \frac{d}{dy} \left[- \frac{U''(y) \cdot y}{U'(y)} \right] \geq 0$$

Ordinary agents are thus assumed to be strictly risk-averse, having non-increasing absolute risk-aversion and non-decreasing relative risk aversion. By referring back to the Figure, notice that outcomes along the (negative) 45° line that cross through A (strictly) inside the segment $\bar{A}A$ represent mean-preserving (strict) reductions in spread. If the initial spread of the outcomes at A is not "too large", we can apply a Taylor-series approximation of A around the expected outcome at E, yielding:

$$(1) \quad U(y_1^A) = U(\mu) + (y_1^A - \mu) \cdot U'(\mu) + 0.5 \cdot (y_1^A - \mu)^2 \cdot U''(\mu) + \dots$$

Initially, I will assume that this series converge rapidly enough and disregard the remainder beyond the second order approximation, thereby treating the expected utility as a function of the first two central moments of the distribution:

$$(2) \quad EU(Y^A) \approx U(\mu) + 0.5 \cdot U''(\mu) \cdot V(Y)$$

where $V(Y)$ is the variance of the outcomes at A, and $\approx$ indicates an approximation. I will comment, however, later on the third order approximation which depends on the third central moment of the distribution, and on what S.C. Tsiang (1972) has termed "the preference for skewness". The (certain) outcomes at B can be expressed as a fraction of the mean, i.e., $y_E = (\alpha \cdot \mu)$: $\alpha \leq 1$. Obviously, the smaller that fraction the more risk-averse the agents. $\alpha = 1$ indicates that the agents are risk neutral, whereas $\alpha = y_1/\mu$ represents agents that select portfolios on the basis of the maximax rule. The fraction $(1-\alpha)$ represents the loss in utility, expressed in terms of money, on account of the spread - and the risk - of the outcomes at A, and is often referred to as the Atkinson's measure of inequality (or spread - see Antony Atkinson, 1970). Applying a Talor-series approximation of B around E, we get;

$$(3) \quad U(y_E) = U(\mu) + (\alpha - 1) \cdot \mu \cdot U'(\mu) + 0.5 \cdot (1-\alpha)^2 \cdot \mu^2 \cdot U''(\mu) + \dots$$

Hence, if we can assume that the remainder beyond the second order of the series can be neglected, we get:

$$(4) \quad EU(Y^B) \approx U(\mu) + (\alpha - 1) \cdot \mu \cdot U'(\mu) + 0.5 (1-\alpha)^2 \cdot \mu^2 \cdot U''(\mu)$$

equating $EU(Y^B) = EU(Y^A)$ and inserting the results in (2) and (4) we therefore get:

$$(5) \quad R(\mu) \cdot CV^2(Y) \approx 2 \cdot (1-\alpha) + (1-\alpha)^2 \cdot R(\mu)$$

where: $R(\mu) = - \frac{U''(\mu) \cdot \mu}{U'(\mu)}$

is the coefficient of relative risk aversion at μ , and $CV^2(Y) = V(Y)/\mu^2$ is the square of coefficient of variation of the outcomes at A. By disregarding elements of the third order in (4) and defining the index I_B as that value which (exactly) equates

$$EU(Y^B) = U(\mu) - I_B \cdot \mu \cdot U'(\mu) + 0.5 \cdot I_B^2 \cdot \mu^2 \cdot U''(\mu)$$

we obtain, from (5), the following second-order approximation:

$$(6) \quad I_B \approx \frac{\sqrt{1 + R^2(\mu) \cdot CV^2(Y)} - 1}{R(\mu)}$$

When $(1-\alpha)^2$ is "sufficiently small" (i.e., if the spread of the outcomes at A is not "too large" and agents are not too risk-averse), we can use the first-order approximation. By defining the index I_A as that value which equates:

$$EU(Y^B) = \left\{ U(\mu) - I_A \cdot \mu \cdot U'(\mu) \right\},$$

we obtain from (2) and (5) the following first-order approximation:

$$(7) \quad I_A \approx 0.5 \cdot R(\mu) \cdot CV^2(Y)$$

Notice that by the mean-value theorem, (granted that the utility function is continuously differentiable) the fraction $(1-\alpha)$ is exactly given by:

$$(1-\alpha) = \frac{U(\mu) - EU(Y^A)}{\mu \cdot U'(\mu)} : y_E \leq y \leq \mu$$

and this fraction is therefore invariant with respect to linear transformations of the utility function. It is therefore a well defined cardinal measure of risk which expresses the risk as the percentage loss in the expected return on account of the spread. Furthermore, if the utility function is (strictly) concave, then the ratio: $\{U(\mu) - EU(Y^A)\} / \{\mu \cdot U'(\mu)\}$, and thus also the approximation I_A , establishes an upper bound on the true value of $(1-\alpha)$. The second-order approximation of $(1-\alpha)$ by the index I_B establishes an upper bound on the true value of that fraction, i.e. $I_B \geq (1-\alpha)$ if $U''' \geq 0$; it establishes a lower bound, i.e. $I_B < (1-\alpha)$, if $U''' < 0$. As is well known, for agents having decreasing or constant absolute risk-aversion, U''' must be positive. Tsiang termed this relation "skewness preference", which characterises "normal" risk-averse agents. For normal agents, therefore, the first-order approximation by the index I_A establishes an upper bound on the second-order approximation I_B and the latter, in turn, establishes an upper bound on $(1-\alpha)^{(2)}$, i.e.,

$$I_A > I_B \geq (1-\alpha).$$

My objective is to examine the potential use of these two approximations for a consistent ranking of investment portfolios. Notice that the index I_A itself (and not merely the index $(1-\alpha)$ it approximates) is, by definition, also invariant with respect to linear transformations of the utility function, and, in that sense, a well defined cardinal measure. This can also be shown for the index I_B . From the expressions for these two indices in (6) and (7) we can also observe that the two are scale independent if and only if the coefficient of risk aversion is constant. In other words, both indices would be independent of the units of measurement (and thus, possibly, represent agents that have no

money illusion) if and only if the corresponding utility is a log-linear constant elasticity function of income.

From (6) we can observe that I_B strictly rises with the variance, and strictly falls with the mean, provided that $R'(\mu) > 0$ ⁽³⁾. In the appendix it is also shown that the trade-off between the mean and the variance along the curve which leaves the value of the index constant, is strictly positive, i.e.,

$$\left. \frac{d\mu}{dV} \right|_{dI_B = 0} > 0$$

Likewise, it can be shown that I_A rises with the variance, falls with mean (provided that $R'(\mu) > 0$), and along the "indifference curve" for which $dI_A = 0$, the slope $d\mu/dV$ is (strictly) positive. It should also be noted that a rise by one percent in the coefficient of relative risk aversion $R(\mu)$ would raise also the index I_B , but at a rate lower than one percent. ⁽⁴⁾ In contrast, the index I_A rises at the same rate as the rise in the coefficient $R(\mu)$ (i.e., the corresponding elasticity is equal to 1). This difference between the two indices should make no difference for the mere ranking of investment portfolios but it may be important when we want to quantify the resulting monetary gains or losses.

Let us now consider the outcomes at A^0 , having a smaller mean and a smaller spread ⁽⁵⁾. The CE income y_E^0 can be expressed as a fraction α_0 of the corresponding mean income μ_0 at E^0 , and the latter can be expressed as a fraction β of the mean income μ at E . Hence, $y_E^0 = \alpha_0 \cdot \beta \cdot \mu$. By comparing the CE outcomes, we can therefore conclude;

$$(8) \quad EU(Y) > EU(Y^0) \quad \text{if and only if} \quad (1 - \alpha) < (1 - \alpha_0 \cdot \beta)$$

By using the same procedure as above we can also obtain the first and second order approximations I_A^0 or I_B^0 , e.g.,

$$(9) \quad I_A^0 \approx 0.5 \cdot R(\mu_0) \cdot CV^2(Y^0)$$

By inserting these approximations to (8) we can therefore obtain a criterion for ranking the two portfolios that is specified in terms of these approximations, as suggested in the following proposition:

PROPOSITION 1: If the utility function is continuously differentiable and strictly concave, the coefficient of absolute risk aversion does not rise with income and the coefficient of relative risk aversion does not fall with income, then a necessary and sufficient condition for $EU(Y) > EU(Y^0)$ is:

$$(10) \quad 0.5 \cdot \left| R(\mu) \cdot \frac{V(Y)}{\mu} - R(\mu_0) \cdot \frac{V(Y^0)}{\mu_0} \right| < (\mu - \mu_0)$$

The proposition thus states that although the index I_A is only a first-order approximation, the ranking of investment portfolios on the basis of this index is, nevertheless, consistent with their ranking on the basis of their CE outcomes.

The proof of the proposition follows from the definition of I_A as that value which equates

$$U(y_E) - U(\mu) - I_A \cdot \mu \cdot U'(\mu),$$

and the definition of I_A^0 that value which equates

$$U(y_E^0) - U(\mu_0) - I_A^0 \cdot \mu_0 \cdot U'(\mu_0).$$

To prove that the condition is sufficient, use Taylor's formula to expand $U(\mu)$ and $U'(\mu)$ about μ_0 yielding:

$$U(\mu) \geq U(\mu_0) + (\mu - \mu_0) U'(\mu_0) - U(\mu_0) + (1-\beta)/\beta \cdot \mu_0 \cdot \mu_0 \cdot U'(\mu_0)$$

and $U'(\mu) \leq U'(\mu_0)$. Hence,

$$U(y_E) \geq U(\mu_0) + ((1-\beta)/\beta) \cdot \mu_0 \cdot U'(\mu_0) - (I_A/\beta) \cdot \mu_0 \cdot U'(\mu_0)$$

From the latter inequality together with the earlier definition of I_A^0 we obtain, after some algebra, the following sufficient condition:

$$U(y_E) \geq U(y_E^0) \Leftarrow I_A < \left\{ 1 - (1 - I_A^0) \cdot \beta \right\}$$

By inserting the values of the approximations in (7) and (9) into the latter condition we obtain the criterion in (10) as a sufficient condition.

To prove that the condition is necessary, use Taylor's formula to expand $U(\mu_0)$ and $U'(\mu_0)$ about μ yielding:

$$U(\mu_0) \geq U(\mu) - (1-\beta) \cdot \mu \cdot U'(\mu), \text{ and } U'(\mu_0) \geq U'(\mu).$$

Hence,

$$U(y_E^0) \geq U(\mu) - [1 - \beta(1 - I_A^0)] \cdot \mu \cdot U'(\mu)$$

And the latter inequality determines the following necessary condition:

$$U(y_E) > U(y_E^0) \Rightarrow I_A < \left\{ 1 - (1 - I_A^0) \cdot \beta \right\}$$

This condition, together with the definitions of I_A and I_A^0 establishes the criterion in (10) as a necessary condition. Q.E.D.

Earlier it was noted that I_A establishes an upper bound on the true value of $(1-\alpha)$. To establish a lower bound on $(1-\alpha)$ in terms of the first order approximation, notice that, by definition:

$$I_A = \frac{U'(y)}{U'(\mu)} \cdot (1 - \alpha) < \frac{U'(y_E)}{U'(\mu)} \cdot (1 - \alpha) : \quad y_E < y < \mu$$

Expanding $U'(y_E)$ about μ , we therefore get, for 'normal' agents:

$$I_A = (1 - \alpha) \cdot \left[1 + \left(\frac{\mu - y}{\mu} \right) \cdot R(\mu) + \frac{1}{2} \left(\frac{\mu - y}{\mu} \right)^2 \cdot \frac{U''(\mu) \cdot \mu^2}{U'} \right]$$

$$< (1 - \alpha) + (1 - \alpha)^2 R(\mu)$$

From the latter we can determine a lower bound on the true value of $(1 - \alpha)$, given by:

$$(11) \quad (1 - \alpha) \geq \frac{\sqrt{1 + 4 \cdot R(\mu) \cdot I_A} - 1}{2 \cdot R(\mu)}$$

An appropriate selection of the upper and lower bounds determines the following two working criteria for comparing two investment alternatives:

COROLLARY 1: A sufficient condition for $\alpha > \alpha_0 \beta$

is:

$$(12) \quad (1 - I) > \beta \left\{ \frac{1 + 2R(\mu_0) - \sqrt{1 + 4 \cdot R(\mu_0) \cdot I^0}}{2R(\mu_0)} \right\}$$

where I and I^0 represent either the first-order approximations I_A and I_A^0 , or the second-order approximations I_B and I_B^0 .

It can also be verified that (12) is a necessary condition for

$$I > 1 - (1 - I^0) \beta.$$

COROLLARY 2: A necessary condition for $\alpha > \alpha_0 \beta$ is:

$$(13) \quad \left\{ \frac{\sqrt{1 + 4 \cdot R(\mu) \cdot I} - 1}{2 \cdot R(\mu)} \right\} \cdot \left\{ 1 - (1 - I^0) \cdot \beta \right\}$$

Likewise, it can also be verified that (13) is a sufficient condition for $I > 1 - (1 - I^0) \cdot \beta$.

Notice also that by combining the conclusions of COROLLARIES 1 and 2 we obtain, in fact, an alternative proof of PROPOSITION 1 with respect to the

first-order approximations I_A and I_A^0 . Furthermore, since, for 'normal' agents, $I_A \geq I_B$, the lower bound in (11) can also be determined in terms of the second-order approximation, and COROLLARIES 1 and 2 can also be specified in terms of the second-order approximations I_B and I_B^0 . This, in turn, establishes the following:

COROLLARY 3: If the utility function is continuously differentiable and strictly concave, the coefficient of absolute risk-aversion not rising with income, and the coefficient of relative risk-aversion not falling with income, then a necessary and sufficient condition for $EU(Y) > EU(Y^0)$ is:

$$I_B < 1 - (1 - I_B^0) \cdot \beta$$

Several additional corollaries of PROPOSITION 1 and the above three COROLLARIES are obvious and appealing:

- (i) If agents are risk-neutral i.e., if $R(\mu) = R(\mu_0) = 0$ the choice depends only on whether or not $\mu > \mu^0$.
- (ii) If $\mu = \mu_0$, the choice depends only on whether or not $V(Y) < V(Y^0)$.
- (iii) The left-hand-side of (10) measures the reduction in risk, expressed, in terms of money, on account of the reduction in spread; the right-hand-side measures the reduction in the expected outcome. Portfolio A would therefore be preferred over A^0 if the rise in risk that it entails, measured in these money terms, does not exceed the rise in the average outcome that it provides.

- (iv) If both $\mu > \mu^0$ and $V(Y) < V(Y^0)$ then $EU(Y) > EU(Y^0)$ for all positive $R(\mu)$. In that case we can conclude that Y dominates Y^0 , in the sense that it would be selected by all risk-averse agents for which the coefficient of relative risk aversion does not fall with income. Eq. (10) determines in fact a stronger sufficient condition for dominance which requires

$$\mu > \mu_0 \text{ and } \frac{V(Y)}{\mu} < \frac{V(Y^0)}{\mu_0}.$$

- (v) The famous "paradox" of Karl Borch (1969), warns us that this criterion must be applied with caution. Thus, if $\mu < \mu_0$ but $V(Y) < V(Y^0)$, we must first examine whether it is not the case that Y^0 dominates Y according to the first criterion of stochastic dominance (FSD), i.e., that all the outcomes in Y^0 are larger than the outcomes in Y, before we apply the above criterion.⁽⁶⁾

- (vi) With a constant relative risk aversion, the criterion can also be written as: $EU(Y) > EU(Y^0)$ if and only if:

$$(14) \quad \mu - k \cdot \frac{V(Y)}{\mu} > \mu_0 - k \cdot \frac{V(Y^0)}{\mu^0}$$

where $k = 0.5 \cdot R(\mu)$. The latter closely resembles but is different from the criterion suggested by Baumol for ranking investment portfolios, which is based on an ad-hoc expected utility function of the form $EU(Y) = \mu - k^* \sigma$; where σ is the standard deviation

- (vii) If $EU(Y) = EU(Y^0)$ according to the criterion of the proposition and $\mu > \mu_0$, then $V(Y) > V(Y^0)$. In other words, the indifference curve of the E - V rule is indeed positively sloped.

To see this formally, write the first order approximation of the CE outcome as

$$y_E = \mu - 0.5 \cdot R(\mu) \cdot \frac{V}{\mu}$$

Taking the total differential of the latter equation and setting $dy_E = 0$ yields

$$(15) \quad \left. \frac{dV}{d\mu} \right|_{dy_E=0} = \frac{\mu}{R(\mu)} + \left(\frac{V}{\mu} \right) \left(\frac{R(\mu) - \mu \cdot R'(\mu)}{R(\mu)} \right) > 0$$

since

$$R(\mu) - \mu R'(\mu) = \mu^2 \cdot \frac{d}{d\mu} \left(\frac{U''}{U'} \right) \geq 0$$

where the latter inequality follows from our earlier assumption that the marginal absolute risk aversion does not rise with income. In the appendix this inequality is shown to hold also for the second-order approximation of the CE outcome.

From the latter equations it is easy to derive the slope of the indifference curve between the mean and the standard deviation, given by:

$$\left. \frac{dS}{d\mu} \right|_{dy_E=0} = \frac{1}{R(\mu)} \cdot \frac{\mu}{S} + \frac{1}{2} \cdot \frac{S}{\mu} \cdot \left(\frac{R(\mu) - \mu \cdot R'(\mu)}{R(\mu)} \right)$$

Thus, with a constant absolute risk aversion,

$$\left. \frac{dS}{d\mu} \right|_{dy_E=0} = \frac{1}{R(\mu)} \cdot \frac{\mu}{S} > \frac{\mu}{S} \iff R(\mu) < 1$$

and,

$$\left. \frac{d^2 S}{d\mu^2} \right|_{dy_E=0} = \frac{R(\mu) - \mu \cdot R'(\mu)}{R^2(\mu) \cdot S}$$

In this case, therefore, the indifference curves are straight lines whereas in the general case they rise upward as illustrated in Figure 2. The Figure

FIGURE 2.

illustrates also the case of constant marginal relative risk aversion (i.e., $R'(\mu) = 0$), in which case:

$$\left. \frac{dS}{d\mu} \right|_{dy_E=0} = \frac{\mu}{R(\mu) \cdot S} + \frac{1}{2} \frac{S}{\mu}$$

The latter expression can be written, in elasticity terms, as:

$$\left. \frac{dS/S}{d\mu/\mu} \right|_{dy_E=0} = \frac{2 + R(\mu) \cdot CV^2(Y)}{2 \cdot R(\mu) \cdot CV^2(Y)}$$

Hence,

$$\left. \frac{dS/S}{d\mu/\mu} \right|_{dy_E=0} > 1 \iff R(\mu) < \frac{2}{CV^2(Y)}$$

Furthermore, in this case,

$$\left. \frac{d^2 S}{d\mu^2} \right|_{dy_E=0} = \frac{2 \cdot \mu}{R(\mu) \cdot S^2} > 0$$

II. Investment Choice under Partial Information on the Agents Preferences.

The main application of these criteria is to establish an order among investment portfolios when we do have some knowledge on the degree to which agents are risk averse. To illustrate this use, consider the following numerical example: Let $Y = (5; 95)$ and $Y^0 = (10, 10)$. Obviously, both Y and Y^0 are in the efficient set under the first criterion of stochastic dominance. Furthermore, Y would be preferred by the risk neutral whereas Y_0 would be preferred by the extreme risk-averse agent who chooses according to the Maximin criterion. By inserting the corresponding values to (10) we can determine the critical value of $R(\mu)$ for which the two portfolios are equally worthwhile in terms of their expected utility, given by $R^*(\mu) = 1.975$. We can therefore conclude that Y would rank higher than Y^0 for all agents having coefficients of relative risk aversion

smaller than that critical value. I will refer to this condition as the condition for R^* - stochastic dominance.

To consider the application of this criterion to determine a more complete order of investment portfolios, let us turn to Figure 3. Portfolio A has a mean of μ_1 and a variance of V_1 . Under the above conditions, portfolios in the north-west region I dominate A, whereas portfolios in the south-east region III are dominated by A (granted though that FSD cannot be established). Without further knowledge on the agents preferences we cannot rank the portfolio A vis-a-vis portfolios in the other two orthants II and IV. From (15) we know, however, that along the indifference curve:

$$\left. \frac{dV}{d\mu} \right|_{dy_E = 0} \geq \frac{2\mu}{R(\mu)}$$

For a given value $\bar{R}(\mu)$ we can divide these two orthants into four sub-regions, marked in the Figure as II_1, II_2, IV_1 and IV_2 respectively, by drawing the line that crosses through A and has the slope $dV/d\mu = 2\mu/\bar{R}(\mu)$. For agents having a coefficient of relative risk aversion smaller than or equal to $\bar{R}(\mu)$, it can now be concluded that portfolios in the regions II_2 and IV_2 are inferior to A. We still cannot rank, however, the portfolio A vis-a-vis portfolios in the region II_1 and IV_1 , without further knowledge on the agents preference ordering.

With a constant relative risk aversion and (say) under the first-order approximation, the critical level of the coefficient of relative risk-aversion can be approximated by:

FIGURE 3.

$$(16) \quad R^*(\mu) = \frac{2(\mu - \mu_0)}{\frac{V(Y)}{\mu} - \frac{V(Y^0)}{\mu_0}}$$

For an empirical illustration of these criteria, I will evaluate the choice of crop rotation and irrigation practices in wheat production in the Northern Negev in Israel. The climate in that region is typical semi-arid, characterized by low and very unstable levels of precipitation - both from year to year and within each year. The government implement in that region an extensive drought compensation program, and the data presented here were collected in comprehensive study that evaluated the program ⁽⁷⁾. Table 1 summarizes the main profit characteristics of nine alternative cultivation practices over a period of 20 years, when the compensation program is implemented ⁽⁸⁾. An important tenth alternative is to refrain from growing wheat altogether. None of these alternative "dominates" any of the others on the basis of the first criterion of stochastic dominance (FSD). The choice of the most desirable cultivation practice thus depends not only on its profit characteristics (the mean and the variance), but also on the extent to which farmers are risk-avert.

The Table examines 6 different criterias for the selection: (i) Maximize μ ; (ii) Minimize V ; (iii) Minimize the coefficient of variation, and (iv)-(vi): Maximize the estimated CE outcome for three different values of marginal relative risk aversion: (iv) $R(\mu) = 0.5$; (v) $R(\mu) = 1.0$ and (vi) $R(\mu) = 2.0$. The seventh criterion selects the crop rotation so as to maximize the coefficient of relative risk aversion under which production is still desirable, (given, via (16), by $R(\mu^*) = 2 \cdot \mu^2/V$).

Table 1: Profit Characteristics under Different Crop Rotations

Performance criterion	C R O P R O T A T I O N									
	1	2	3	4	5	6	7	8	9	10
Mean (μ)	4.09	4.26	4.62**	4.78**	3.66	3.99	4.22	4.40	4.60	0
Variance (V)	16.87	18.63	19.15	21.16	12.01**	5.01	16.26	17.01	18.65	0*
Coefficient of Variation	1.004	1.013	0.951	0.962	0.947	0.971	0.955	0.937**	0.939**	0*
$\mu - 0.25 \cdot \frac{V}{\mu}$	3.06	3.17	3.56	3.67*	2.84	3.05	3.26	3.43	3.59**	0
$\mu - 0.5 \cdot \frac{V}{\mu}$	2.03	2.08	2.52	2.57**	2.02	2.11	2.30	2.46	2.58*	0
$\mu - 1 \cdot \frac{V}{\mu}$	-0.04	-0.11	+0.44	+0.35	+0.38	0.23	+0.37	+0.53**	+0.55*	0
MAX R(μ) under which production is profitable	1.98	1.95	2.21	2.16	2.23	2.12	2.19	2.28	2.27	..

* First choice under the criterion

** Second choice under the criterion

Another route for evaluation is a pair-wise comparison of all the alternatives. To simplify the presentation, I will examine now only the first five alternatives, and calculate, for each two alternative the smallest value of the coefficient of relative risk aversion for which one alternative would be preferred over the other. The results of these calculations are presented in Table 2.

Table 2. Pair-Wise Comparison of Crop Rotations

	Row rotation is preferred over the noted column rotation by all agents having $R(\mu)$ equal to or larger than:			
	(1)	(2)	(3)	(4)
(1)	..	1.37	26.60	4.57
(2)	—	..	3.23	19.42
(3)	—	—	..	1.36
(4)	—	—	—	..
(5)	1.02	1.10	2.14	1.96

The results indicate that among these 5 alternatives, (5) would be preferred over all the others by all agents having a coefficient of relative risk aversion larger than 2.14.

A third method of ranking the rotation alternatives is by means of the first and second order approximations of the CE outcome. These comparisons are given in Table 3. They show that for small values of the coefficient of relative risk aversion, the ranking of the rotation alternatives by means of the first-order approximations is consistent with the ranking by means of the second-order approximations, and the difference between the CE outcomes under the two methods

of approximations is of the order of magnitude of less than 2 percent. For high values of the coefficient, the first-order approximations are biased downward significantly and the rankings under the two methods are no longer consistent with each other. A comparison of the two rankings obtained by the second-order approximations with the two values of the coefficient $R(\mu)$, shows, however, that in the case under consideration, the two are still consistent.

TABLE 3

Ranking Rotation Alternatives by Means of the First and Second Order Approximations of the CE Outcome

(a) $R(\mu) = 0.5$

Rotation	1-st Order Approximation			2-nd Order Approximation		
	I A	$\hat{Y}$ E	Rank	I B	$\hat{Y}$ E	Rank
(1)	0.252	3.06	4	0.238	3.12	4
(2)	0.257	3.17	3	0.242	3.23	3
(3)	0.226	3.56	2	0.215	3.61	2
(4)	0.232	3.67	1	0.220	3.73	1
(5)	0.224	2.84	5	0.213	2.88	5

(b) $R(\mu) = 2.0$

(1)	1.01	-0.04	4	0.622	1.57	4
(2)	1.03	-0.11	5	0.630	1.58	3
(3)	0.91	+0.44	1	0.575	1.96	2
(4)	0.93	+0.35	3	0.581	1.99	1
(5)	0.90	+0.38	2	0.570	1.56	5

APPENDIX

(a) To prove that:

$$\left. \frac{d\mu}{dV} \right|_{dI_B=0} > 0$$

take the total differential of (6) and equate $dI_B=0$. After some algebra, this yields:

$$\left. \frac{d\mu}{dV} \right|_{dI_B=0} = - \frac{R/\mu^2}{\left[\frac{V}{\mu^3} \cdot (R' \cdot \mu - 2R) - I_B^2 \cdot R' \right]} > 0$$

The denominator is negative because $R' > 0$ and $R' \cdot \mu < R$ under assumptions (iii) and (iv) on the utility function.

Likewise, it can be shown that:

$$\left. \frac{d\mu}{dV} \right|_{dI_A=0} = - \frac{R^2}{(\mu R' - 2R) \cdot V} > 0$$

(b) Proof of corollary (vi) for the second-order approximation of the CE outcome:

Under the second-order approximation, the CE outcome can be written as

$$y_E = \mu - \frac{\sqrt{\mu^2 + R^2(\mu)} \cdot V}{R(\mu)} - \mu$$

Taking the total differential of the latter equation and setting $dy_E=0$, yields, after some algebra, the following differential equation

$$d\mu \left[1 + \frac{(R(\mu) - \mu \cdot R'(\mu))}{R^2(\mu)} \cdot \left\{ 1 - \frac{\mu}{\sqrt{\mu^2 + R^2(\mu)} \cdot V} \right\} \right] - \\ - dV \cdot \left[\frac{R^3(\mu)}{2 \sqrt{\mu^2 + R^2(\mu)} \cdot V} \right]$$

and the expressions in the two square brackets are strictly positive under the assumption of non-increasing absolute risk aversion which implies that $R(\mu) - \mu \cdot R'(\mu) \geq 0$. If the marginal absolute risk aversion is constant,

$R(\mu) = \mu \cdot R'(\mu)$, and

$$\left. \frac{d\mu}{dV} \right|_{dy_E=0} = \frac{R^3(\mu)}{2 \cdot \sqrt{\mu^2 + R^2(\mu)} \cdot V}$$

From the latter equality we can derive the conditions on $R(\mu)$, V and μ under which this slope is larger or smaller than 1.

FIGURE A-1.

Footnotes

- (1) In other words, the choice of the investment portfolio does not depend on the arbitrary choice of the "zero" point.

- (2) Clearly, $(1-\alpha) = \frac{U'(y)}{U'(\mu)}$. $I_A < I_B$, since $U'(y) > U'(\mu)$ for $y < \mu$.

Equally obvious: $I_A = I_B - 0.5 I_B^2 \cdot \mu^2 \cdot U''(\mu) > I_B$. Finally, if $U''' \geq 0$:

$$(1-\alpha) \cdot \mu \cdot U'(\mu) - 0.5 (1-\alpha)^2 \mu^2 \cdot U''(y^*) \leq I_B \cdot \mu \cdot U'(\mu) - 0.5 \cdot I_B \cdot$$

$\mu^2 \cdot U''(\mu)$ for $y^* < \mu$. Hence, in that case:

$$I_A > I_B \geq (1 - \alpha).$$

- (3) The derivatives are given by:

$$\frac{dI_B}{dV} = \frac{1}{2} \cdot \frac{R}{\mu^2 (1 + I_B \cdot R)} > 0, \text{ and}$$

$$\frac{dI_B}{d\mu} = \frac{\frac{V}{\mu^3} (\mu \cdot R' - 2R) - I_B^2 \cdot R'}{2 (1 + I_B \cdot R)} < 0$$

Since, with non-decreasing absolute risk aversion: $R < \mu \cdot R'$.

- (4) That elasticity is given by:

$$\frac{\partial I_B}{\partial R(\mu)} \cdot \frac{R(\mu)}{I_B} = \frac{R(\mu) \cdot CV^2(\mu)}{I_B \cdot (1 + R(\mu) \cdot I_B)} = 1$$

and the ratio on the right-hand-side is strictly larger than 1; It is also strictly smaller than 2 if and only if both I_B and $R(\mu)$ are strictly positive.

- (5) While this may be self evident and follows directly from the earlier derivations, a heuristic explanation is the following: Consider the two portfolios indicated in Figure 1 by A and D. They both have the same spread and thus also the same (Atkinson's) measure $(\mu - y_E)/\mu$, indicated in the Figure by $(1-\alpha)$. D thus represents a spread-preserving reduction in mean from μ to μ_0 . The portfolio A^0 represents a mean-preserving (strict) reduction in spread relative to D. Hence, the measure of spread at A^0 , given by $(\mu_0 - y_E^0)/\mu_0$ is (strictly) smaller than $(1-\alpha)$. Thus

$$1 - \alpha_0 = \frac{1}{\mu_0} (\mu_0 - y_E^0) < (\mu - y_E) \cdot \frac{1}{\mu} = 1 - \alpha$$

- (6) Borch claimed that for every point on the E-V indifference map we can find another point having a lower mean and a lower variance, and another point having a higher mean and a higher variance. In the case of two variables, the "paradox" is illustrated in Figure A-1, where A denotes a portfolio having a higher mean and a higher variance and yet $A \text{ FSD } B$. Clearly, B must be on the E-V indifference map for some finite value of relative risk aversion. The same holds for all portfolios in the shaded domain AOC. Likewise, points in the shaded domain above A, such as E, represent a higher mean and a higher variance, and yet clearly $E \text{ FSD } A$.

- (7) See David Bigman, Hector Barak and Nir Becker (1988).

(8) These alternatives are determined by:

(i) Irrigation Regime:

No irrigation: 1,2,3,4,5.

Supplementary irrigation at the beginning of the season: 6,7,8,9.

(ii) Crop rotation

Fallow once every 2 years: 1,6

Fallow once every 3 years: 2,7

Fallow once every 4 years: 3,8

Fallow once every 5 years: 4,9

In alternative 5 the decision whether or not to grow in any given year is determined every year independently on the basis of the level of moisture in the soil by mid-December.

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Chapter 3:

Safety First Criteria and The Measurement of Risk

David Bigman

SAFETY FIRST CRITERIA AND THE MEASUREMENT OF RISK

Safety-first criteria for the choice among alternatives involving uncertain prospects are first and foremost concerned with the risk of failing to achieve certain minimum goals. These criteria are commonly expressed by a probability statement of the form

$$(1) \quad \Pr(X \leq z) \leq \alpha$$

where X is a random variable (say, profits), z is the minimum (profit) goal--often referred to also as the "disaster level," $\Pr$ is a probability statement, and α is an acceptable limit on the probability of failing to reach that goal.

A. D. Roy's (1952) safety-first criterion seeks to minimize the probability α that a disaster will occur, i.e., that the agent will fail to reach even its minimum goal. Lester Tesler (1956) approaches the choice as a two-step procedure whereby the agent first eliminates from consideration alternatives that fail to meet the safety requirements specified by equation (1) for a given level of α and then selects among the remaining alternatives the one (or ones) that maximizes the expected utility. Shinji Kataoka (1963) and Willam Baumol (1963) select the alternative that, for a given α , maximizes the level of the (minimum) goal level, i.e., maximizes the α -fractile of the profit distribution. (See also David Pyle and Stephen Turnovsky, 1970; Enrique Arzac, 1976.)

It is well known that these criteria are incompatible with the standard utility theory. Tesler's and Kataoka's criteria need not satisfy the continuity and independence axioms nor do they correspond to any real or vector-valued (lexicographic) utility. Roy's criterion can be derived from a utility function, but this function will not, in general, be strictly monotonic increasing.¹

The main concern of this paper is with the probability statement in equation (1) as an expression of risk. The principal deficiency of this measure is that it does not reflect the "depth" of the disaster because it provides the same measure of risk regardless of whether the fall below z was very large or very small. Consider the following illustration: Let $(x_1, \dots, x_n)$ be the values that X can obtain--with equal probabilities--and let x_1 be one of the x_i 's that falls strictly below z . An increase in x_1 by Δ , such that $(x_1 + \Delta)$ is still smaller than z , will not change the probability measure of the risk. This increase is bound, however, to have an effect on the consequences of failing to reach the goal and, hence, on the perception of risk. To take this example to the extreme, consider an increase in all the x_i 's that are strictly smaller than z by an amount equal to $\xi_i = (z - x_i - \epsilon)$. This increase will still leave unchanged the probability measure of risk, irrespective of how small (though positive) ϵ is.

The objective of this paper is to define a measure of risk that reflects both the probability of failing to reach the goal and the depth of the disaster. The proposed measure is based on a definition of risk as the expected loss in utility as a result of failing to reach the minimum goal, and it, therefore, reflects the properties of the Von Neumann-Morgenstern utility function. The common properties of this function will allow us to determine the exact functional form of the expression for risk.

I. The Measure of Risk

Let X be a discrete (continuous) random variable, taking the values $(x_1, \dots, x_n)$ [or $x \in (A, B)$]--assumed, for simplicity, to be nonnegative--with a given cumulative probability distribution $F(x)$. In the discrete case I

will adopt the convention that the payoff prospects are in nondecreasing order so that $x_i \leq x_j$ whenever $i < j$. The preferences between alternative prospects are assumed to be represented by an increasing, strictly concave, and twice continuously differentiable utility function $U(x)$.

The concern with risk is manifested first and foremost by focusing on the failures to reach the minimum goal (or the "target"). Payoffs that exceed that level would have no effect on the measure of risk. To manifest this concern, the measure of risk would, therefore, focus on the truncated vector of payoffs

$$(x_1^t, \dots, x_n^t)$$

where

$$x_i^t = \min\{x_i, z\}, \quad i = 1, \dots, n$$

(i.e., on the interval $(A, z]$ in the continuous case). The risk measure is defined as the (proportional) expected loss in utility due to the failure to reach the target z . In the discrete case this measure is given by

$$(2) \quad R(F, z) = \frac{\sum_{i=1}^n p_i U(z) - \sum_{i=1}^n p_i U(x_i^t)}{\sum_{i=1}^n p_i U(z)}$$

where $(p_1, \dots, p_n)$ are the respective probabilities (densities). The risk measure is, thus, a function of the probability distribution of the payoffs and the target--or the disaster--level z . Given our definition of the truncated vector of payoffs, this measure can also be written as

$$(3) \quad R = \sum_{i=1}^h p_i \left[\frac{U(z) - U(x_i)}{U(z)} \right]$$

where h is the highest value such that $x_h < z$. For a given z , this measure will coincide with the probability measure in equation (1) only for a very degenerate utility function which obtains the value of 0 for all $x_i < z$ and the value of 1 for all $x_i \geq z$. Another degenerate case is when the utility function has the form $U(x) = x$. In this case R will measure the expected size of the shortfall below z , relative to that target, i.e.,

$$R[U(x) = x] = \sum_{i=1}^h p_i \left(\frac{z - x_i}{z} \right).$$

I will later refer to the latter measure as the expected (unconditional) "relative gap". Sometimes the relative gap is taken as a measure of risk. Our definition in equation (2) indicates, however, that this measure does not correspond to a strictly concave utility function and, thus, does not represent risk-averse agents.

The definition of risk in equation (2) establishes a one-to-one correspondence between that measure and the individual (subjective) preference ordering. To determine the mathematical structure of R , we must therefore specify the functional form of the utility function. This, however, may raise a problem because the mathematical structure of this expression cannot be uniquely determined--the reason being that $U(x)$ is not a unique representation of a given set of preferences. Indeed, it is well known that any positive linear transformation of $U(x)$ will also represent the same preferences. To identify more specifically the group of agents for whom the measure of risk is being

defined, one must therefore place restrictions on $U(x)$. A common practice in the literature on risk bearing² is to place restrictions on $r(x)$, defined as

$$r(x) = \frac{-U''(x)}{U'(x)}.$$

Indeed, John Pratt (1964) has shown that $r(x)$ provides a unique representation of the agent's preferences. In this paper I take a somewhat different approach by placing the restriction directly on the risk measure itself. Later on we shall see that the two approaches are, indeed, closely related. Specifically, I will make the following assumption:

ASSUMPTION 1: The risk measure is invariable with respect to proportional shifts.

In other words, R is homogeneous of degree zero in x and z so that doubling all the payoffs and the target level z does not change the measure of risk. One interpretation of this assumption is that agents have no money illusion. This restriction, together with the common properties of the utility function, is sufficient to determine the specific functional form of the risk measure as we see in the following theorem.

THEOREM 1: If the risk measure defined in equation (2) is invariant with respect to proportional shifts in x and z and the utility function is monotonic (strictly) increasing, then R has the following form:

$$(4) \quad R = F(z) \cdot [1 - (1 - G)^{1-\xi} \cdot (1 - S)]$$

where

$$G = \sum_{i=1}^h p_i \cdot \frac{(z - x_i)}{z}$$

is the expected (conditional) relative gap (when a failure occurs), p_i' are the normalized probabilities such that $p_i' = p_i/F(z)$, and

$$S = \sum_{i=1}^h p_i' \cdot \left[1 - \left(\frac{x_i}{\bar{x}_h} \right)^{1-\xi} \right]$$

is a measure of the spread of the payoffs that fall below z , ξ being the coefficient of relative risk aversion.

PROOF: The starting point for the proof is the definition of the "certainty equivalent" payoff, namely, that payoff which, if received with certainty, will provide the same utility as the original (random) payoff.³ In the present analysis where attention is focused on the prospects of failing to reach the target, I will define the certainty equivalent of these prospects only, i.e., of $x_1, \dots, x_h$. This is defined as that payoff, denoted by x_h^* , such that

$$(5) \quad \sum_{i=1}^h p_i' U(x_h^*) = \sum_{i=1}^h p_i' U(x_i).$$

Hence,

$$(6) \quad U(x_h^*) = \sum_{i=1}^h p_i' U(x_i)$$

where $p_i' = p_i/F(z)$ are the normalized (conditional) probabilities, $F(z)$ being the probability that a failure will occur so that

$$\sum_{i=1}^h p_i' = 1.$$

By inserting this definition into equation (3), together with the definition of the normalized probabilities, we get

$$(7) \quad R = F(z) \cdot \left[1 - \frac{U(x_h^*)}{U(z)} \right].$$

Since R is invariant with respect to proportional shifts in x and z , we can multiply all the payoffs and the target by $[1/z]$ (thereby expressing the payoffs in terms of their proportional deviations from z) without changing the measure of risk. Hence,

$$(8) \quad R = F(z) \cdot \left[1 - \frac{U(x_h^*/z)}{U(1)} \right].$$

By normalizing $U(1)$ to be 1, we can therefore conclude that U is similar to Cauchy's equation: $f(x \cdot y) = f(x) \cdot f(y)$. For positive x and y and when $f(x)$ is increasing such that $f(1) = 1$, its only solution is $f(x) = x^\delta$ for $\delta > 0$; and $f(x) = \log_e(x)$ for $\delta = 0$. Hence, the utility function for which the risk measure is invariant with respect to proportional shifts must be of the log-linear, constant elasticity form (see Pratt, 1964; Kenneth Arrow, 1971; and Anthony Atkinson, 1970). In other words, $U(x)$ can be written as

$$(9) \quad U(x) = x^{1-\xi}, \quad \xi \neq 1$$

and

$$U(x) = \log_e(x), \quad \xi = 1.$$

The normalization $U(1) = 1$ together with the requirements of strict monotonicity (i.e., $U' > 0$) and strict concavity (i.e., $U'' < 0$) imply that $0 < \xi < 1$.⁴ The parameter ξ is the coefficient of relative risk aversion defined as

$$\xi = - \frac{U''(x)}{U'(x)} \cdot x = r(x) \cdot x.$$

Hence, the restriction placed on the risk measure R in A-1 to be invariant with respect to proportional shifts is equivalent to placing a restriction on $r(x)$ by requiring the product $r(x) \cdot x$ to be constant, and both restrictions provide a unique representation of the agent's preferences.

By inserting equation (9) into (8), we get

$$(10) \quad R = F(z) \cdot \left[1 - \left(\frac{x_h^*}{z} \right)^{1-\xi} \right].$$

This can be written as

$$(11) \quad R = F(z) \cdot \left[1 - \left(\frac{\bar{x}_h}{z} \right)^{1-\xi} \cdot \left(\frac{x_h^*}{\bar{x}_h} \right)^{1-\xi} \right]$$

where

$$(12) \quad \bar{x}_h = \sum_{i=1}^h p_i x_i$$

is the average (conditional) shortfall below target. Hence, by definition,

$$(13) \quad G = (z - \bar{x}_h)/z.$$

By combining equations (6), (9), and (12), the ratio $(x_h^*/\bar{x}_h)$ can be written as

$$(14) \quad \left(\frac{x_h^*}{\bar{x}_h} \right)^{1-\xi} = \sum_{i=1}^h p_i \left(\frac{x_i}{\bar{x}_h} \right)^{1-\xi}.$$

Define S as

$$S = 1 - \left(\frac{x_h^*}{\bar{x}_h} \right)^{1-\xi}$$

$$= \sum_{i=1}^h p_i \left[1 - \left(\frac{x_i}{\bar{x}_h} \right)^{1-\xi} \right].$$

The measure S is a member of what Frank Cowell and Kiyoshi Kuga (1981) defined as a general class of entropy measures. It tends toward Theil's entropy measure as $\xi \rightarrow 1$. It is also related to Atkinson's (1970) class of inequality measures which can be written as

$$(15) \quad I = 1 - \left[\sum_{i=1}^h p_i \left(\frac{x_i}{\bar{x}_h} \right)^{1-\xi} \right]^{1/(1-\xi)}.$$

The basic difference between the two measures of spread, I and S^5 , is that I defines dispersion as

$$I = \frac{\bar{x}_h - x_h^*}{\bar{x}_h}$$

whereas S defines dispersion as

$$S = \frac{U(\bar{x}_h) - U(x_h^*)}{U(\bar{x}_h)}.$$

Thus, S measures the loss in utility on account of the spread in the payoffs; and it is, therefore, in line with Hugh Dalton's (1920) definition of inequality.⁶ By inserting equations (12), (13), and (14) into equation (11), we get equation (4).

Q.E.D.

The measure of risk R reflects the combined effect of several factors: (i) the probability of failing to reach the target, (ii) the average shortfall below target when a failure occurs, (iii) the spread of the payoffs below target, and (iv) the degree of risk aversion. It emphasizes that there is no way--and no rationale--of defining risk in an "objective" way without regard to the preferences of the corresponding agent whose perception of risk is reflected in R . The following conclusions are direct corollaries of the theorem.

COROLLARY 1:

- (i) R is nonnegative.
- (ii) R would be equal to zero if and only if $x_i \geq z$ for all i .
- (iii) R is strictly increasing with any increase in z .
- (iv) R is strictly increasing with any decrease in $x_i \leq z$.
- (v) R is strictly increasing with any decrease in $x_i > z$ to a level below z .
- (vi) R is strictly increasing with any increase in the spread of the payoffs below z .

It is easy to observe that the probability measure of risk in equation (1), i.e., $F(z)$, will satisfy neither (iv) nor (vi) although the changes noted in both are likely to be perceived as an increase in risk. In certain probability distributions, it may even be the case that an increase in z will not be recorded as an increase in risk by the probability measure $F(z)$ although it will always be recorded as an increase in risk by R .⁷

II. A Dual Measure of Risk

Another way of assessing the risk is by measuring the agent's "disutility" from failing to reach the target rather than the utility he derives from the given payoffs. The size of a failure is measured by

$$(16) \quad f_i = \max\{z - x_i; 0\} \quad i = 1, \dots, n.$$

Assume that the agent's disutility can be represented by a "disutility function" $d(f_i)$, which is monotonic (strictly) increasing, and (strictly) convex in f_i , i.e., $d' > 0$ and $d'' > 0$. Strict convexity of d is thus merely a reflection of strict concavity of the corresponding utility function. By normalizing $d(0)$ to be 0, we can express the agent's disutility as a function of the ratio (f_i/z) . A natural analog to the definition of risk in equation (2) in terms of the disutility function would be

$$(17) \quad R^D(F, z) = \sum_{i=1}^h p_i \cdot d\left(\frac{f_i}{z}\right).$$

In words, the measure of risk is the expected "gain" in disutility as a result of the failures to reach the target. On the basis of this definition and the earlier results in section I, we can derive the exact functional form of R^D , as indicated in the following theorem:

THEOREM 2: If the dual risk measure R^D is invariant with respect to proportional shifts in f and z and if the disutility (strictly) increases with the size of the failure, then this risk measure has the following form:

$$(18) \quad R^D = F(z) \cdot G^{1+\delta} \cdot (S^D + 1)$$

where G , the expected relative gap, is defined as before, and S^D is an entropy measure of the spread of the failures defined as

$$(19) \quad S^D = \sum_{i=1}^h p_i \cdot \left[\left(\frac{f_i}{f} \right)^{1+\delta} - 1 \right],$$

δ being the coefficient of relative "failure aversion" and $\delta > 0$ for concavity.

The proof of the theorem is essentially a mirror image of the proof of Theorem 1. It is also straightforward to observe that all the results of Corollary 1 apply also to the measure R^D . From the proof, we can also conclude that the two measures will be identical if and only if the two aversion elasticities ξ and δ are such that the utility function is the Bernoullian (logarithmic) function, in which case the spread measure becomes Theil's entropy measure.

Our interest in the measure R^D stems from two questions. First, do the two dual measures provide different characterization of risk? Second, how are the "ordinary" measures of risk (and, for that matter, how are the ordinary measures of spread such as the coefficient variation) related to these two?

To answer the first question, we have to consider the values of R and R^D at different values of the risk aversion parameters ξ and δ . When the agent is risk neutral, i.e., when $\xi = \delta = 0$, then

$$(20) \quad R = R^D = F(z) \cdot G.$$

In this case the two measures will be identical and risk will be measured by the (unconditional) average size of the shortfall below target. The reason is obvious. As noted earlier, the utility function in this case is piecewise linear of the form $U(x) = \min\{x, z\}$ and the disutility function is $d(x) = \max\{z - x, 0\}$ so that $d(x) = [z - U(x)]$.

When $0 < \xi < 1$ and $\delta > 0$, we are, in fact, comparing two power series of the form

$$(21) \quad R = \sum_{i=1}^h p_i \left[1 - \left(\frac{x_i}{z} \right)^{1-\xi} \right]$$

and

$$(22) \quad R^D = \sum_{i=1}^h p_i \left(\frac{z - x_i}{z} \right)^{1+\delta}.$$

Obviously, by the continuity of the risk measures thus defined, it will always be the case that, for every probability distribution of the payoffs and any value of ξ , we can find a corresponding value of δ that will equate the two measures. This value of δ will depend, however, not only on ξ but also on the distribution of the payoffs. Hence, we cannot establish a universal correspondence between the two risk aversion parameters which is independent of the specific probability distribution of the payoffs.

To examine how the two measures are related to each other, we turn to Figure 1. The payoffs in the figure are assumed to be uniformly distributed along the diagonal OC such that the normalized payoff (x/z) obtains values ranging from 0 to A. N denotes the target levels at which $x = z$. The probability measure of risk is thus given by the ratio ON/OA. The total size of all the gaps is measured by the triangle OMB, and the average size of the shortfalls below target, by the distance FG. The risk measure is the shaded area bounded by the solid curve OTB. The higher the degree of risk aversion (i.e., the larger the δ) the steeper the curvature. Hence, the broken line in the figure represents a more risk-averse agent. The most risk-averse agent is the one that is concerned only with maximizing the minimum payoff or minimizing the maximum failure. For this agent $\delta, = \infty$.

Consider, as an illustration, a given payoff denoted in the Figure by point R. The ratio (x/z) is thus given by RS, and the ratio (f/z) by SU. For $0 < \xi < 1$, the ratio $(x/z)^{1-\xi}$ is given by RT and $[1 - (x/z)^{1-\xi}]$ is thus given by TU. For an appropriate choice of $\delta > 0$, the ratio $(f/z)^{1+\delta}$ would also be given by TU. Hence, under an appropriate choice of the risk aversion parameters, the two measures will be identical.

To answer the second question, namely, how are the two dual measures related to the ordinary measures mentioned in the literature, I will examine here only the measures of spread. Interestingly, the most common measure of spread, namely, the coefficient of variation, is related to the dual measure of spread, S^D rather than to the "primal" measure S. In fact, simple algebra would show that, for $\delta = 2$, the two are related in the following way:

$$(23) \quad CV = \frac{(z - \bar{x})^2}{\bar{x}^2} \cdot S^D$$

where CV denotes the coefficient of variation. This relationship indicates, in turn, that the appropriate representation of the preference ordering which leads to the coefficient of variation as a measure of spread is, in fact, in terms of the disutility function $d(f)$.

Figure 2 illustrates the two measures of spread--taken across the payoffs that fall below target only. These are, therefore, the spread measures that are included in the risk measures illustrated in Figure 1. Under risk neutrality, the spread will be measured by the sum of the areas of the two triangles ODE and EFB. Risk aversion is manifested by the curvature of the curves OE and EB. The broken line thus represents a higher degree of risk aversion. In the limit, the agent will be concerned only with the distance $[\max - \min]$. The measure of spread itself is the shaded areas in the Figure.

III. Risk and Stochastic Dominance

I will conclude this paper with a few observations on the relationship between the risk measures that were defined in the preceding sections and the familiar stochastic dominance rules for ordering uncertain prospects. For notational convenience, I will examine only the dual risk measure R^D although an extension to the "primal" measures R is straightforward. To make explicit the choice of parameters that determine this measure of risk, I will denote it as $R^D(\delta, F, z)$ where δ , F , and z are defined as above. With this notation, it is easy to observe that

$$(24) \quad R^D(-1, F, z) = F(z)$$

$$(25) \quad R^D(0, F, z) = F(z) \cdot G.$$

The following theorem summarizes the relationships between these measures of risk and the stochastic dominance rules for agents having constant relative risk-aversion elasticity.

THEOREM 3: Let F and H denote two (cumulative) probability distribution functions of a random variable, x , taking values in the (closed) interval, $[A, B]$.

1. A necessary and sufficient condition for first-degree stochastic dominance of F over H is

$$R^D(-1, F, z) \leq R^D(-1, H, z), \quad \text{for all } z \in [A, B]$$

with inequality holding for at least one z in that interval.

2. A necessary and sufficient condition for second-degree stochastic dominance of F over H is

$$R^D(0, F, z) \leq R^D(0, H, z), \quad \text{for all } z \in [A, B]$$

for all $z \in I$, with inequality holding for at least one z in that interval.

The proof of the theorem follows directly from the definition of the risk measure in (24) and (25) and is omitted for brevity. The theorem cannot be extended to third-degree stochastic dominance because the risk measure is defined only for utility or (disutility) functions having constant relative risk-aversion elasticity. Nevertheless, the theorem suggests an extension of the stochastic dominance rules in a different direction.

The criterion of second-degree stochastic dominance (SSD) establishes (partial) preference ordering between uncertain prospects for all risk-averse agents, no matter how small or how large their risk aversion is.⁸ When all that is known about the agent is that he is risk averse, then one has no choice but to resort to the SSD criterion. Suppose, however, that one has some prior information on the range within which the risk-aversion coefficient can vary or on the lower limit that it can take. One may then be interested in the order between uncertain prospects that would be established by the agents having risk aversion within this range, or above that lower limit. The most extreme risk-averse agent is, of course, the one that ranks uncertain prospects on the basis of the maxmin rule. For this agent, the elasticity of risk aversion is $\delta = \infty$, and the order established by this criterion is, of course, complete. The SSD criterion, in contrast, is notoriously incomplete.

Between the two polar criteria of SSD that apply to any risk-averse agent and the maximin that applies only to the most extreme risk-averse agent, one can make as much use as possible of the available prior information by considering an "intermediate" criterion that can be defined as " δ -stochastic dominance," and which establishes preference ordering for all agents having (constant) relative risk-aversion elasticity larger than δ . This criterion can be formalized as follows:

Definition: Distribution F is said to be " δ -stochastic-dominant" over H if it is preferred over H by all risk-averse agents having relative risk-aversion elasticity larger than δ .

This definition offers an extension over the familiar SSD criterion only for $\delta > 0$. Obviously, the larger the δ the "more complete" (or less partial) the ordering. A straightforward extension of Theorem 3 is thus:

THEOREM 4: A necessary and sufficient condition for δ -stochastic dominance of F over H is

$$R^D(\delta, F, z) \leq R^D(\delta, H, z), \quad \text{for all } z \in [A, B]$$

with inequality holding for at least one z in that interval.

APPENDIX

This appendix summarizes the formulae of the two risk measures.

$$R = \sum p_i' \left[1 - \left(\frac{x_i}{z} \right)^{1-\xi} \right] = \sum p_i' \left[1 - \left(\frac{x_i}{\bar{x}_h} \right)^{1-\xi} \left(\frac{\bar{x}_h}{z} \right)^{1-\xi} \right]$$

$$= 1 - \left(\frac{\bar{x}_h}{z} \right)^{1-\xi} \cdot \left[\sum p_i' \left(\frac{x_i}{\bar{x}_h} \right)^{1-\xi} \right] = 1 - (1 - G)^{1-\xi} \cdot [1 - S]$$

where $S = \sum_{i=1}^h p_i' \left[1 - \left(\frac{x_i}{\bar{x}} \right)^{1-\xi} \right]$ and $G = \frac{z - \bar{x}_h}{z}$

$$R^D = \sum p_i' \left(\frac{f_i}{z} \right)^{1+\delta} = \left(\frac{\bar{f}}{z} \right)^{1+\delta} \cdot \sum p_i' \left(\frac{f_i}{\bar{f}} \right)^{1+\delta} = F(z) \cdot G^{1+\delta} \cdot [S^D + 1]$$

where $S^D = \sum p_i' \left[\left(\frac{f_i}{\bar{f}} \right)^{1+\delta} - 1 \right]$ and $G = \frac{\bar{f}}{z}$.

FOOTNOTES

¹Consider, as an illustration, either an increase in the highest value that x can obtain or a decrease in the lowest value that x can obtain. None of these changes will change the probability distribution of x and thus none will change the probability measure.

²For example, see Jack Meyer (1977).

³For a discussion on this concept, see David Newbery and Joseph Stiglitz (1981).

⁴Atkinson (1970) wrote the general functional form of constant-relative-inequality-aversion utility functions as

$$U(x) = A + B \cdot \frac{x^{1-\xi}}{1-\xi}, \quad \xi \neq 1$$

and

$$U(x) = \log_e (x), \quad \xi = 1$$

where ξ , the (constant) relative inequality aversion is constrained to be nonnegative. It should be noted, however, that the normalization $U(1) = 1$, or, for that matter, any normalization of the utility function, places a constraint on the values of A and B . The normalization $U(1) = 1$, for example, implies that, if $A = 0$, then $B = 1$ as suggested in equation (9). Without any such normalization, the measures of risk (in this paper) or the measure of inequality (in Atkinson's paper) can only be determined up to affine transformations. Similarly, the coefficient of relative risk aversion itself also remains undetermined unless the normalization is specified. If, for example, we choose $U(1) = b$, then this implies that ξ is determined from the equality

$b = A + B/(1 - \xi)$. This normalization thus determines, in turn, the admissible set from which ξ can be chosen so as to satisfy all the restrictions.

⁵Another difference is the choice of normalization for the utility function and the restrictions it places on the choice of ξ , as noted in footnote 4.

⁶Notice that, in our case,

$$\frac{dS}{dI} = (1 - \xi) \cdot [1 - I]^{-\xi}.$$

This, in turn, indicates that S and I will register the same directions for the change in spread if and only if $0 < \xi < 1$. The distinction between the two definitions will be more critical when applied to the measure of risk than to the measure of inequality. This can be illustrated as follows. Define an Atkinson-type measure of risk as

$$J = 1 - \frac{x_h^*}{z}.$$

If we multiply all the payoffs by λ (or multiply z by $1/\lambda$), we get $dJ(\lambda)/d\lambda = -[1 - J(1)]$, i.e., the measure of risk declines at a constant rate. In contrast, $dR(\lambda)/d\lambda = -(1 - \xi) \cdot \lambda^{-\xi} \cdot [1 - R(1)]$, i.e., the measure of risk defined in this paper declines at an increasing rate.

⁷To take an extreme case as an illustration, assume that all the payoffs fall below the target z . In that case any increase in z , however large, will not change, of course, the probability measure.

⁸see Joseph Hadar and William Russell, 1969; Giora Hanoch and Haim Levy, 1969.

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Chapter 4:

Non Parametric Test of the Expected Utility Hypothesis

Ziv Bar-Shira

Abstract

A nonparametric test of the expected utility is derived. It is shown that the expected utility holds if there exists a feasible solution to a system of linear inequalities. When a feasible solution exists, it can be manipulated to yield boundaries on the coefficient of absolute risk aversion. The test is applied to data on land allocations that are modeled as choices over lottery sets. The results that the expected utility hypothesis cannot be rejected in most of the cases and are in contrast to results obtained in laboratory experiments.

Non Parametric Test of the Expected Utility Hypothesis

1. Introduction

The expected utility hypothesis has been used extensively in most models of decisions under uncertainty. The expected utility property is a direct consequence of a number of axioms introduced by von Neumann and Morgenstern four decades ago. The empirical studies of decisions under uncertainty, that are based on the expected utility hypothesis, are valid only when the empirical data are consistent with the expected utility hypothesis. That is, the individuals' preferences have to obey the von Neumann-Morgenstern's axioms in order for the above analysis to be valid. In the early fifties economists, like Allais, started studying the validity question: is the above set of axioms consistent with reality? Allais constructed an example, a lab test, that shows that the way people rank lotteries is inconsistent with the expected utility hypothesis. During the following years this question was often addressed in the economic and the psychology literature. It was shown that in a large number of cases people make decisions that are inconsistent with the expected utility hypothesis (i.e. Kahneman and Tversky, MacCrimmon and Larsson). As a result of these contradictions economists introduced new theories that relax some of the axioms, mainly the independence axiom. The relaxation of the independence axiom caused some of the inconsistencies to disappear. Roughly speaking, the employed method was to make the decision function nonlinear in the probabilities (i.e. Kahneman and Tversky (1979), Quiggin (1982), Machina (1982), Fishburn (1983)). So far, most of the tests concerning the expected utility hypothesis were lab tests. They were based on questionnaires with hypothetical questions that were distributed in a class of students, for example.

The objective of the of this paper is to construct a test of the expected utility hypothesis which is based on a real data set, namely the actual decisions of input

allocation made by the farmers. By using that test, it will be possible to conclude whether the expected utility hypothesis is the appropriate assumption for analyzing decisions under uncertainty and behavior toward risk for a given data set.

The next section brings the motivation for the expected utility hypothesis and its axiomatic basis. The third section illustrates the expected utility property in the simplex and reviews the empirical shortcomings of the expected utility hypothesis. Section four explains how to model choice of land allocations as choice among different lotteries and then suggests some tests that check the reliability of the model. Section five describes the formalism of the test, for the three dimensional case and for the multidimensional case, and derives boundaries on the coefficient of absolute risk aversion. Finally, section 6 concludes the chapter with results and conclusions.

2. Assumptions

How does an individual rank uncertain prospects? For example, given the payoffs $(x_1, \dots, x_n)$ with the probabilities $(p_1^j, \dots, p_n^j)$ $j=1, \dots, m$, what is the most preferred prospect? More than three hundred years ago, when mathematicians started developing the modern probability theory, it was suggested that the attractiveness of a prospect j is given by its expected value, $\sum_i x_i p_i^j$. The higher the expected value of a prospect the more attractive it is. In 1728 Nicholas Bernoulli raised the following question concerning the expected value criteria, now known as the St. Petersburg paradox.¹ Suppose an individual is offered a lottery which gives 1 dollar if head comes at the first toss of a fair coin, 2 dollars if the first head comes at the second toss, 4 dollars if the first head comes at the third toss, etc. What is the sure gain that makes the individual indifferent to the above lottery ticket? Most individuals would indicate a finite amount

¹ For extended discussion of this paradox see Samuelson (1966).

of money, while the expected value of the lottery is infinite. It has been shown that even when the game is restricted to a finite number of tosses, say 1000, there is still a huge gap between the expected value of the lottery and what most individuals would be willing to pay for a single play of this game. The above paradox led Daniel Bernoulli (Nicholas' cousin) to propose that an individual who is engaged in ranking uncertain prospects takes into account more than just the expected value of the proposed prospect. Specifically, Bernoulli realized that an individual might evaluate a prize of 100 dollars as worth less than twice the worth of a 50 dollar prize. Hence, he suggested the expected utility criteria. That is, the value of prospect j is $\sum_i u(x_i) p_i^j$.

The relationship between people's preferences and a utility function (a mathematical function) will now be discussed. Consider the choice of some bundle $x \in X$, with no uncertainty, where X is the consumption set which is assumed to be closed and convex. The existence of a continuous utility function such that $u(x) \geq u(y)$ if and only if $x \succeq y$,² where $\succeq$ is the preference relation defined over the consumption set X , is guaranteed conditional on the following assumptions:

- Completeness.** For all x, y in X either $x \succeq y$ or $y \succeq x$ or both.
- Reflexivity.** For all x in X $x \succeq x$.
- Transitivity.** For all x, y, z in X if $x \succeq y$ and $y \succeq z$ then $x \succeq z$.
- Continuity.** For all y in X $\{x: x \succeq y\}$ and $\{x: y \succeq x\}$ are closed sets.
- Strong Monotonicity.** If $x \succeq y$ and $x \neq y$ then $x \succ y$

Note that although the utility function assigns a real number to any consumption bundle, it is not a cardinal scale but an ordinal one. This is because the absolute

² Proof may be found in Varian (1984) pp. 113.

magnitude has no meaning, only the relative magnitude matters. Hence, the utility function is not unique and any increasing monotonic transformation will do.

About four decades ago von Neumann and Morgenstern established the axiomatic basis for the more complex case, the case of uncertainty. In this case the choice is of some prospect $(p, 1-p; x_1, x_2) \in G$, where G is the prospect set, the set of all possible two outcome lotteries. The following assumptions were made on the individual's perception of a lottery.

A1) $(1, 0; x_1, x_2) = x_1$. Getting a prize with probability 1 is the same as getting the prize with certainty.

A2) $(p, 1-p; x_1, x_2) = (1-p, p; x_2, x_1)$. "Order" does not matter.

A3) $(q, 1-q; (p, 1-p; x_1, x_2), x_2) = (pq, 1-pq; x_1, x_2)$. An individual's perception of a lottery depends only on the net probability of receiving the various prizes.

Note that the assumption of only two outcome lotteries, is not restrictive at all since any multiple outcome lottery can be decomposed into a sequence of two outcome lotteries, when the outcomes are allowed to be lotteries as well. For example, $(1/3, 1/3, 1/3; x_1, x_2, x_3) = (2/3, 1/3; (1/2, 1/2; x_1, x_2), x_3)$. As before, it is assumed that the preferences defined on G are complete, reflexive, and transitive. The above three axioms are sufficient to ensure the existence of a continuous utility function which describes the individual's preferences. That is, $(p, 1-p; x_1, x_2) \succeq (q, 1-q; x_3, x_4)$ if and only if $u(p, 1-p; x_1, x_2) \geq u(q, 1-q; x_3, x_4)$. This utility function is unique up to an increasing monotonic transformation. The existence of a general utility function is not easily applicable for economic modeling of behavior under uncertainty (i.e. ranking prospects). For that purpose one is interested in a specific utility function which has the following property: $u(p, 1-p; x_1, x_2) = pu(x_1) + (1-p)u(x_2)$. That is, the utility of a prospect is the expected utility of its prizes. To guarantee that property there is a need

for two additional axioms.³

A4) $p \in [0,1]$, $(p, 1-p; x_1, x_2) \succeq x_3$ and $x_3 \succeq (p, 1-p; x_1, x_2)$ are closed sets for all x_1, x_2, x_3 in X

A5) $x_1 \succeq x_2$ if and only if $(p, 1-p; x_1; x_3) \succeq (p, 1-p; x_2, x_3)$ for all x_1, x_2, x_3 in X

The first axiom is a *continuity* one. The second is the famous *independence* axiom which was first formulated by Samuelson (1952). Although, the independence axiom is not an explicit part of the original set of axioms given by von Neumann and Morgenstern, it has been shown by Malinvaud to be an implicit assumption underlying their axioms. The following interpretation of the independence axiom, given by Samuelson (1966), helps one see its attractiveness. Consider an individual who prefers x_1 to x_2 . This individual is offered two lotteries $(p, 1-p; x_1, x_3)$ and $(p, 1-p; x_2, x_3)$. This is equivalent to being offered a toss of coin with probability $1-p$ of landing heads, if the coin lands heads the individual gets x_3 independent of his choice. If the coin comes tails it is only rational to assume that the original choice would be made.

3. Background and Motivation⁴

This section presents empirical evidence, obtained in laboratory experiments, against the expected utility hypothesis. The nature and the significance of this empirical evidence is best explained by using a unit triangle. Thus, this section starts with presentation of the expected utility in a simplex.

³ The axiomatic basis for the case of uncertainty was first given by von Neumann and Morgenstern. Their axioms are sufficient for the expected utility property. A slightly different version given, by Varian (1984), is presented here.

⁴ A good discussion of the motivation and background is in Machina (1987).

3.1. The expected utility in the simplex

The expected utility property may be illustrated in a triangle diagram. For that purpose, consider the set of all probability triples, (p_1, p_2, p_3) defined on fixed outcomes, $x_1 < x_2 < x_3$. Since the probabilities sum to one we can draw any lottery in the unit triangle in the (p_1, p_3) plane (figure 1). All northwest movements lead to stochastically dominating lotteries since any movement upward shifts some probability weights from x_2 to x_3 and any movement leftward shifts some probability weights from x_1 to x_2 . The individual's indifference curves can be found by solving the following equation:

$$\bar{u} = p_1 u(x_1) + (1-p_1-p_3)u(x_2) + p_3 u(x_3) \quad (1)$$

The solution leads to parallel indifference curves with slope of $(u(x_2)-u(x_1))/(u(x_3)-u(x_2))$. Thus it is enough to know one indifference curve in order to know the whole preferences over the entire triangle. The iso expected profit lines can be drawn in this triangle, as well. They are the solution of the next equation

$$\bar{x} = p_1 x_1 + (1-p_1-p_3)x_2 + p_3 x_3 \quad (2)$$

The solution shows that they are parallel lines with slope of $(x_2-x_1)/(x_3-x_2)$. Movement northeast along those lines keeps the expected profit constant while increasing the variance (it shifts probability mass from the center to the edges). Hence it represents mean preserving spread or pure increase in risk in the sense of Rothschild and Stiglitz. The individual's attitude toward risk depends on the relative slope of the indifference curves and the iso expected profit lines, where steeper indifference curves imply risk aversion and flatter imply risk seeking. This result is a consequence of the following relationship:

$$\frac{u(x_2)-u(x_1)}{u(x_3)-u(x_2)} > \frac{x_2-x_1}{x_3-x_2} \quad \text{implies} \quad \frac{u(x_2)-u(x_1)}{x_2-x_1} > \frac{u(x_3)-u(x_2)}{x_3-x_2} \quad (3)$$

which is equivalent to a concave utility function that represents risk aversion. The

case of risk seeking is similarly proven. Moreover, there is one to one relationship between the slope of the indifference curves and Arrow - Pratt coefficient of absolute risk aversion, $-U''/U'$. That is,

$$-\frac{U''}{U'} \equiv A \approx 2 \left[\frac{s-1}{s+1} \right]$$

where s is the slope of the indifference curves. This results is derived under the assumption that the outcomes are equally spaced and close to each other.

3.2. Empirical evidence against the expected utility hypothesis

The most famous example of a contradiction to the expected utility property is the Allais paradox. Consider a choice of one prospect from the following pairs:⁵

a: (1M; 1) versus b: (0, 1M, 5M; 0.01, 0.89, 0.1)

and

c: (0, 5M; 0.9, 0.1) versus d: (0, 1M; 0.89, 0.11)

where M stands for a million dollars. Researchers, such as Allais, Raiffa, and MacCrimmon and Larsson have found that the great majority of the individuals who were asked to rank these prospects preferred a to b and c to d. Clearly, these choices are not consistent with the expected utility maximization since for any expected utility maximizer who prefers a to b the expression $\{-0.01u(0) + 0.11u(1) - 0.1u(5)\}$ is greater than zero, while this expression is less than zero for any expected utility maximizer who prefers c to d. To see how these choices violate the independence axiom, define e as (0, 5M; 1/11, 10/11) and rewrite the Allais paradox as:

aa: (1M, 1M; 0.11, 0.89) versus bb: (e, 1M; 0.11, 0.89)

and

⁵ Allais paradox is drawn as a decision tree at the end of this chapter (pp. 36).

cc: (e, 0; 0.11, 0.89) versus dd: (1M, 0; 0.11, 0.89)

The independence axiom states that preference of aa over bb implies preference of dd over cc, and vice versa. When the choices are as in the Allais paradox, it appears that the certainty equivalent of e is less than 1M when the alternative is 1M, while when the alternative is 0 the certainty equivalent of e is greater than 1M. In other words, the better the alternative (in a stochastically dominating sense) the more risk averse the individual is. This phenomenon, in general known as the *common consequence effect*, directly violates the independence axiom. Choices of a over b and c over d, however, do not imply choices of aa over bb and cc over dd when the compound prospect axiom does not hold. The ranking of the double letter pairs may be different from the ranking of the single letter pairs when the individual's perception of a lottery depends not only on the net probabilities. Thus, violation of the expected utility property does not imply violation of the independence axiom, although the reverse is true. When one draws the four prospects, a,b,c,d, in the unit triangle (figure 2) they form a parallelogram. Any expected utility maximizer whose indifference curves are steeper than the sides of the parallelogram would prefer a to b and d to c. The opposite choice would occur when the indifference curves are flatter than the sides of the parallelogram. For consistency with the choice as in the Allais paradox the indifference curves can not be parallel, they have to have, what Machina (1987) termed, the fanning out property (figure 3). Again, the figure shows that the stochastically dominating pair (a,b) is ranked with more risk aversion than the other pair (c,d).

It is important to mention that behavior as in Allais paradox appears also under different choice sets. A second example where people's choices are not consistent with expected utility maximization is known as the common ratio effect. It is drawn as decision tree on page 36. When faced with these decision problems, most people choose to have the \$1M prize with certainty over 98 percent chance of \$5M prize in the first case, and .98 percent chance of \$5M prize over 1 percent chance of \$1M prize

in the second case. As in the Allais paradox, indifference curves that fan out are consistent with this pattern of choices. However, in both cases of Allais paradox and the common ratio effect, it is sufficient for the indifference curves to fan out only a little bit. In other words, the indifference curves can be almost parallel and still consistent with behavior as in Allais paradox, or people can be almost expected utility maximizer and choose as in Allais paradox. Thus, the laboratory experiments show deviation from expected utility maximization but they do not show how deep the deviation is.

Recently, Conlisk pointed out that the inconsistency with expected utility maximization as in Allais paradox is a systematic one. That is, most of the people choose the pair (a,c) (in Figure 2), rather than choosing (a,c) and (b,d) with equal frequencies, when inconsistency with expected utility maximization is observed. Note that nonsystematic inconsistency with expected utility maximization (equal frequencies of (a,c) and (b,d)) is not much of a concern since similar objects are hard to distinguish, and thus people may get confused when faces with similar choices. Conlisk observed that the systematic inconsistency disappear when he moved the Allais paradox to the interior of the simplex. Moreover, when the segment d-c (Figure 2) was moved to the left upper corner of the simplex in such a way that point d is in the interior of the simplex and point c is on the line where $P_2=0$, Conlisk observed the same systematic inconsistency as in the original Allais paradox. However, the latter systematic inconsistency implies indifference curves that fan in rather than fan out. Conlisk explains his results by the existence of certainty effect, in the original Allais paradox people choose a over b because of the certainty effect. When all the points are in the interior of the simplex there is no certainty effect and systematic inconsistency disappear. In the third version of Allais paradox, the certainty effect ensures the choices of c over d and a over b. Consequently, systematic inconsistency exists but it implies fanning in indifference curves (when the indifference curves are straight lines).

4. The Data

The data set contains a cross section - time series sample on 97 farmers during ten years, 1973-82. It includes the following variables: allocation of land, yield, and revenue for five crops: Bell Peppers, Tomatoes, Onions, Melons, Eggplants. The data set also includes figures on total expenditure for water and other inputs. The cost for each crop can be recovered via a behavioristic approach described by Just *et al* (1989). The profit for each crop can be calculated by subtracting the recovered cost from the revenue figures.

4.1. Presentation of the farmers' decisions as lotteries

The first step of the analysis is presented next. Let $\Pi = (\pi_1, \dots, \pi_n)$ be the profits domain per unit of land for the farmer, where π_1 and π_n are the minimum and maximum profit per unit of land which might occur for some crop, respectively. Let P be an array of probability measure of dimension $n \times n \times n \times \dots \times n$ (J times, where J is the number of crops). An element of P , $p_{i_1, \dots, i_J}$, is the probability of having some specific combination of profits, say $(\pi_{i_1}, \dots, \pi_{i_J})$ where π_{i_j} is the profit per unit of land for crop j . In other words, P is the joint distribution function of profits.

Along with Muth's concept of rational expectations, the farmer's perceived joint distribution of the profits must coincide with the in-fact (real) joint distribution of the profits. An unbiased estimate of the real joint distribution function of the profits is the empirical actual frequencies of each payoff. Hence it is also an unbiased estimator of the in-mind (perceived) joint distribution of profits of the farmer. This approach of modeling profit expectations, however, is quite naive in the sense that constant expectations over time lead to constant decision over time. That is, the land allocation does not change over time. A quick glance at the data shows that this is not the case. Hence, there is a need to incorporate some mechanism which explains changes in the expectations over time which in turn explains the changes in land allocation among the

different crops. One possibility is to assume that the farmer is not completely rational and he is updating his expectation, concerning future profits, according to some moving average of the profit realizations in the last few periods. In other words, a series of good years might cause the farmer to expect another good year. In this case the farmer corrects his subjective profit distribution. That is, instead of giving equal weights to each realization he gives larger weights to the more recent realizations.

The second step is to transform any specific land allocation among the different crops to an ultimate lottery in terms of profit. This can be done by finding the total profit which is associated with each event. Formally, the payoff associated with some element of P , say $p_{i_1, \dots, i_J}$, is $\sum_{j=1}^J l_j \pi_{i_j}$, where l_j is the land allocated to crop j . This payoff corresponds to some outcome in the total profit domain $(\pi_1, \dots, \pi_N)$, say π_k , then setting the probability which is associated with it, p_k , to be equal $p_{i_1, \dots, i_J}$.

4.2. Some preceding Tests

Now, when any single decision made by the farmer is formulated as a lottery it is possible to answer the following questions⁶:

1. Does the farmer follow the first order stochastic dominance criteria? Equivalently, is his utility function increasing in its argument?
2. Does the farmer follow the second order stochastic dominance criteria? Is his utility function concave in its argument?
3. Does the farmer follow the third order stochastic dominance? In other words does he have decreasing absolute risk aversion?
4. What is the farmer's attitude toward pure increase in risk? Is there any available lottery which has higher expected value and equal or smaller variance?

⁶ For theoretical consideration of the stochastic dominance see Hadar and Russell.

Theoretically, the answer for the second and fourth questions must be the same. However the way the answer is constructed is different. In order to answer the above four questions one must define the set of all other lotteries available at some periods for some farmer. Any different land allocation gives a different lottery, hence one must find all lotteries that are associated with all *possible* different land allocations. Possible in the sense that the allocations obey the following constraints:

- C1. $\sum l_i \leq L$. There is a limited amount of land.
- C2. $\sum t_i l_i \leq T$. There is a limited amount of time
- C3. $\sum w_i l_i \leq W$. There is a limited amount of water.

Once all possible land allocations are known it is possible, as was explained in the previous paragraph, to find the set of all possible lotteries. The way to answer the first question is by checking whether there exists a lottery with $\sum_{i=1}^j p_i - p_i^* \leq 0$, for all j , with strict inequality at least for one j , where p^* indicates actually chosen lottery and p is any other available lottery which was not chosen. The existence of such a lottery shows that a stochastically dominating lottery was not chosen. The questions concerning higher order stochastic dominance can be similarly answered. Once one has the mean and the variance of each available lottery, a short search procedure will give the answer to the fourth question.

To continue the analysis there is a need for additional assumptions. Comparison of two decisions has to be based on the assumption that everything else is fixed, either along time, namely the individual's preferences do not change over time, or at one point of time, namely the preferences do not change across the population. There are arguments in favor of each of the above. The assumption of fixed preferences across the individuals might be based on the fact that the available data came from an Israeli Moshav - a cooperative village. This Moshav is quite isolated and stands as an environmental unit by itself. Consequently most of the individuals interactions are

within the Moshav and not with the outside environment. Furthermore, public opinion is a very powerful device which drives the individuals to behave according to it. Hence one can say that the preferences of everybody approach steady state preferences. Contingent on the rejection of the assets integration assumption (Markowitz (1952)), the fixed preferences over time assumption is composed of two other assumptions. The assets integration assumption says that the individual cares not only about the final wealth but also about the initial wealth position so that it makes a difference whether some wealth position was arrived at by loss of money or by gain of money. The first assumption is that the wealth does not change over time. At first glance this looks like a quite strong assumption. However, under the permanent income hypothesis of Friedman, where the individual considers all future incomes when evaluating his wealth, it looks more plausible. The second assumption is that the preferences defined over initial and final wealth (initial wealth and current prospects) are fixed. In other words, tastes are stable over time. Conditional on the fixed wealth hypothesis, the fixed preferences over time assumption might be empirically tested via the revealed preferences approach. It is proposed to use the fixed preferences over time assumption since the justifications for the first assumption are rather *local* ones, while the justifications for the second one are more *global* and likely to exist in general and not only in special cases. Furthermore, it is a testable hypothesis in a way which will be outlined below.

Let G_t be the set of all prospects available to the producer at time t , and g_t^* is the chosen prospect at that time. If g_t is any prospect in G_t , then g_t^* is directly revealed preferred to g_t , $(g_t^* R^o g_t)$. If we have a sequence of directly revealed preferred choices, say $(g_{t_1}^* R^o g_{t_2}^*), \dots, (g_{t_{n-1}}^* R^o g_{t_n}^*)$, then $g_{t_1}^*$ is revealed preferred to $g_{t_n}^*$, $(g_{t_1}^* R g_{t_n}^*)$. Data which were generated by a utility maximizing individual have to satisfy the weak axiom of revealed preference and the strong axiom of revealed preference. They are:

(WARP) If $g_{t_1} R^o g_{t_2}$ and $g_{t_1} \neq g_{t_2}$ then it is not the case that $g_{t_2} R^o g_{t_1}$.

(SARP) If $g_{t_1} R g_{t_2}$ and $g_{t_1} \neq g_{t_2}$ then it is not the case that $g_{t_2} R g_{t_1}$.

WARP and SARP are *necessary conditions* for utility maximization, where only (SARP) is a *sufficient condition* for utility maximization. Generally, violation of WARP implies a taste change or that the constant utility over time hypothesis must be rejected. Violation of WARP implies either that the preferences are not transitive or a taste change.

5. A test of the expected utility hypothesis

Checking the empirical validity of the expected utility hypothesis was done not only by lab tests, but also from a different direction. Lin, Dean and Moore, for example, compared the prediction of profit versus utility maximization in agricultural production. Robison appraised their method, as well as the reliability of a test constructed from responses to hypothetical questions. At a somewhat more theoretical level, Varian (1983) extended Afriat's theorem to the case of expected utility and found necessary and sufficient conditions for expected utility maximization. Green and Srivastava's work is a continuation of Varian's work. They, however, assume a complete market for contingent claims.

5.1. The three dimensional case

The three dimensional case is presented first because it has straightforward graphic interpretation in the triangle diagram. Consider a situation where the decision maker has to choose among lotteries defined on a fixed, ordered, and equally spaced outcome domain. The choice set in the first year, G_1 , includes the lotteries g_1^1 , g_1^2 and g_1^3 . g_2^1 , g_2^2 and g_2^3 belong to the choice set in the second year, G_2 , (figure 4). Suppose that g_1^2 and g_2^2 are the chosen lotteries in the first and second year,

respectively. Then there must be a straight line (an indifference curve) that separates the chosen lottery from all the other lotteries in each of the choice sets. If the utility function has a positive slope, then the indifference curves are restricted to have positive slope. Furthermore, the slope of the indifference curve in some year, say i , cannot be smaller than the slope of the line $g_i^2 - g_i^3$, or greater than the slope of the line $g_i^1 - g_i^2$. Once the range of the slope of the indifference curve is known for all the years, one can detect inconsistency with expected utility maximization by comparing the slope ranges. When the maximum slope in one year is smaller than the minimum slope in the other year the choices are inconsistent with expected utility maximization. There is no system of parallel indifference curves that indicates choice of l_1^2 and l_2^2 .

Numerical Example

Let p_i^j be the probability triples associated with the lotteries g_i^j of Figure 4. Their values for G_1 and G_2 are: $p_1^1 \equiv (.07, .73, .2)$, $p_1^2 \equiv (.17, .47, .36)$, and $p_1^3 \equiv (.33, .2, .47)$; and $p_2^1 \equiv (.33, .6, .07)$, $p_2^2 \equiv (.43, .4, .17)$, and $p_2^3 \equiv (.62, .1, .28)$. Visual examination clearly shows that preference of g_1^2 over g_1^1 and g_1^3 , and g_2^2 over g_2^1 and g_2^3 is consistent with expected utility maximization (i.e. there are straight lines with common slope that separate the chosen lotteries from all the other lotteries in their choice set). Algebraically, the expected utility maximization over G_1 implies

$$.07U(x_1) + .73U(x_2) + .2U(x_3) \leq .17U(x_1) + .47U(x_2) + .36U(x_3)$$

$$.33U(x_1) + .2U(x_2) + .47U(x_3) \leq .17U(x_1) + .47U(x_2) + .36U(x_3)$$

Subtracting the right hand side from both sides and writing in matrix form give

$$\begin{bmatrix} -.1 & .26 & -.16 \\ .16 & -.27 & .11 \end{bmatrix} \begin{bmatrix} U(x_1) \\ U(x_2) \\ U(x_3) \end{bmatrix} \leq 0$$

Expressing the second column of the probability matrix as minus the sum of the other two columns and dividing through by $U(x_3) - U(x_2)$ yield

$$\begin{bmatrix} -.1 \\ .16 \end{bmatrix} \left[\frac{U(x_1) - U(x_2)}{U(x_3) - U(x_2)} \right] \leq \begin{bmatrix} .16 \\ -.11 \end{bmatrix}$$

It immediately follows that the solution to the above two linear inequalities is any number in the closed interval, [-1.6, -.69]. Note that the solution to the above system equals the negative of the slope of the indifference curves, s , thus implying that s lies in the close interval [.69, 1.6]. Similarly, expected utility maximization over G_2 restricts the slope, s , to lie in the closed interval [.57, 1]. The intersection of the two allowable ranges is the set [.69, 1], because it is not an empty set the choices over G_1 and G_2 are consistent with expected utility maximization. Furthermore, utilizing the relationship between the slope of the indifference curves and the coefficient of absolute risk aversion (given in section 3.1.) shows that the latter, for wealth level x_2 , lies in the close interval [-.38, 0]. Which means, in this arbitrary example, risk seeking. Because an estimate of the coefficient of absolute risk aversion is given for only one level of wealth, no conclusion concerning trend can be derived.

5.2. The multidimensional case

Generalization to the multidimensional case is the next objective. Now, the indifference curves are hyperplanes. The indifference hyperplanes have to be parallel for consistency with expected utility maximization, on one hand. On the other side, the indifference hyperplanes have to be consistent with the choice in any given year. That is, the hyperplane has to separate the chosen lottery from all the other lotteries in the choice set. This restricts the slopes of the hyperplane to some specific ranges. If any of the ranges is disjoint with some other range then inconsistency is observed. A mathematical derivation of these conditions is given next.

Let G_a be the set of all lotteries of the form (x, p) where x is an increasingly ordered vector of outcomes and p is a vector of probability measure defined on x , both of dimension J . Let G_i a sub set of G_a consisting of the choice set faced by the

individual at time t , usually G_t is smaller ⁷ than G_a because of some resource constraints. G_t is assumed to be convex. It is important to understand that any two lotteries are different only in their probability measure, the outcomes are the same. In other words, different lotteries have different probability distribution functions over common outcome space. Let I be the number of lotteries in G_t and, with out loss of generality, assume that the first lottery is the chosen one. Thus, the results of the expected utility maximization at time t ,

$$\max_{(x, p^1) \in G_t} \sum_{j=1}^J U(x_j) p_j^i,$$

is (x, p^1) . It, then, follows that at any time t : ⁸

$$U(x_1) p_1^i + \dots + U(x_J) p_J^i \leq U(x_1) p_1^1 + \dots + U(x_J) p_J^1 \quad i = 2, \dots, I.$$

Subtracting the r.h.s. from both sides and writing in matrix form yield

$$\begin{bmatrix} p_1^2 - p_1^1 & \dots & p_J^2 - p_J^1 \\ \vdots & \ddots & \vdots \\ p_1^I - p_1^1 & \dots & p_J^I - p_J^1 \end{bmatrix} \begin{bmatrix} U(x_1) \\ \vdots \\ U(x_J) \end{bmatrix} \leq 0 \quad (1)$$

Because the elements of each row in the above probability matrix sum to zero, this system of homogeneous linear inequalities can be written as

$$\begin{bmatrix} p_2^2 - p_2^1 & \dots & p_J^2 - p_J^1 \\ \vdots & \ddots & \vdots \\ p_2^I - p_2^1 & \dots & p_J^I - p_J^1 \end{bmatrix} \begin{bmatrix} U(x_2) - U(x_1) \\ \vdots \\ U(x_J) - U(x_1) \end{bmatrix} \leq 0 \quad (2)$$

Dividing each linear inequality by $U(x_2) - U(x_1)$ yields equivalent non-homogeneous system of linear inequalities of the following form

⁷ Smaller in the sense that $G_t \subset G_a$ and not $G_a \subset G_t$.

⁸ Note that the superscript t is omitted when redundant.

$$\begin{bmatrix} p_3^2 - p_3^1 & \dots & p_J^2 - p_J^1 \\ \vdots & \ddots & \vdots \\ p_3^I - p_3^1 & \dots & p_J^I - p_J^1 \end{bmatrix} \begin{bmatrix} (U(x_3) - U(x_1))/(U(x_2) - U(x_1)) \\ \vdots \\ (U(x_J) - U(x_1))/(U(x_2) - U(x_1)) \end{bmatrix} \leq \begin{bmatrix} p_2^2 - p_2^1 \\ \vdots \\ p_2^I - p_2^1 \end{bmatrix} \quad (3)$$

or in short $Ps \leq y$. The solutions set⁹ for the system in (3) is a convex polytope, it is also not vacuous because of the convexity assumption on G_t . The next proposition summerizes the findings so far.

Proposition

Under the maintained hypothesis of constant preferences over time, an individual is an expected utility maximizer if the intersection set of the solutions sets for (3) over t is not a null set.

5.3. Boundaries On Coefficient of Absolute Risk Aversion

The Arrow-Pratt coefficient of absolute risk aversion (ARA), $-U''/U'$, conveys information on the individual's willingness to undertake a constant gamble at different levels of wealth. Thus, it is plausible to assume decreasing ARA, or equivalently, the wealthier the individual the more likely that uncertain project will be undertaken. Next shown is the one to one correspondence between the solution for (3), s , and the ARA coefficients.

Let x be evenly spaced where the distance between two adjoint outcomes is unitary and let the units be small, then a good approximation for the ARA coefficient at wealth level x_i is

$$A_i = -2 \frac{(U(x_{i+1}) - U(x_i)) - (U(x_i) - U(x_{i-1}))}{(U(x_{i+1}) - U(x_i)) + (U(x_i) - U(x_{i-1}))} \quad i=2, \dots, I-1 \quad (4)$$

where A_i denotes the ARA coefficient at wealth level x_i ; and let s_i be the i 'th element of the solution vector s . Then, by simple substitution one can verify that the

⁹ A general method for solving linear inequalities is given in the appendix.

following relations hold

$$A_2 = -2 \frac{s_1 - 2}{s_1} \quad (5)$$

$$A_3 = -2 \frac{s_2 - 2s_1 + 1}{s_2 - 1}$$

$$A_i = -2 \frac{s_{i-1} - 2s_{i-2} + s_{i-3}}{s_{i-1} - s_{i-3}} \quad i=4, \dots, I-1.$$

It follows that the transformation in (5) can be used to calculate simultaneous boundaries on the coefficients of ARA, A_i . The available information, as conveyed by the data, implies that the A_i 's are equally likely to be at any point inside the boundaries. In other words, the A_i 's are uniformly distributed between the boundaries. Consequently, unbiased point estimates for the A_i 's are the mass center of the convex polytope formed by the boundaries. Practically, the average of all the vertices of the convex polytope gives the desired estimates. Now, the point estimates can be tested for trend, for example if the estimated A_i 's get smaller as i increases, there is empirical evidence in favor of decreasing ARA.

6. Results and Conclusions

The methodology suggested in the previous sections for testing the expected utility hypothesis was applied to the data set. For the empirical purpose, the actual land allocations made by the farmer over ten years, has taken as the set of all possible land allocations. The test was preformed in two stages. In the first it was checked whether there is straight hyperplane that separates the chosen lottery for all other lotteries belonging to its choice set. This stage was preformed for the ninth and tenth year. In the second stage, it was checked whether the intersection set of the allowable ranges of the slopes of the separating hyperplanes is empty or not. The first stage was

performed by checking existence of feasible solution to the system in (3) for the ninth and the tenth year separately. The second stage was performed by checking existence of feasible solution for the ninth and tenth year simultaneously. All computations were performed by using the Fortran programming language and the Nag library, the latter was used to check existence of feasible solution for a system of linear inequality.

Prior to performing the test, all possible land allocation were presented as feasible lotteries. For this purpose, the domain of total profits was divided to eight equal sub domains, and then the empirical density was estimated. Two schemes for forming expectations have been used: i. adaptive expectations, by mean of assigning geometrically declining weights to past realization of profits;¹⁰ ii. naive or rational expectation, by mean of assigning equal weights to past realization of profits. In both the ninth and the tenth year, the past eight years were assumed to be in the information set. As the last step before performing the test, it was checked whether the chosen lottery is not first order stochastically dominated by any other lottery in its choice set.

It was found that in 7 out of 194 choices the chosen lottery is first order stochastically dominated by at least one other lottery in its choice set for the case of naive/rational expectations. For the adaptive, expectation only one choice did not meet the first order stochastic dominant criterion. For the naive rational expectations, four farmers (farmers 8, 9, 14, and 26) failed the first stage of the test, indicating that the indifference curves are not straight hyperplanes. For the adaptive expectations, also four farmers (farmers 9, 14, 26, and 31) failed the first stage of the test. For the naive/rational expectations, farmers 4, 12, 17, 21, 25, 31, 40, 48, 77, and 88 failed the second stage of the test. For the adaptive expectations, farmers 4, 12, 17, 21, 25, 39, 40, 72, 83, and 92 failed the second stage of the test. The same number of farmers failed the first stage of the test, and the same number of farmer failed the second stage

¹⁰ The are .24, .19, .15, .12, .10, .08, .06, .05 for the most recent to the least recent, respectively.

of the test. In the first stage 3 out of 4 failed the test for both expectation schemes. In the second stage 6 out of 10 failed the test for both expectation schemes. Thus, I conclude that the first order stochastic dominant is very sensitive to the expectation scheme, while both stages of the test are not. It should be mentioned that failing the first order stochastic dominant criterion does not imply rejection of the expected utility hypothesis. For both expectation scheme, four out of 194 choices were inconsistent with straight hyperplane indifference curves. Moreover, for both expectation schemes, 10 out of 93 farmers showed inconsistency with parallel hyperplane indifference curves. Hence, I conclude that the phenomenon of inconsistency with straight hyperplane indifference curves is less likely to appear than the phenomenon of inconsistency with parallel hyperplane indifference curves. However, the extent to which individuals exhibit inconsistency with expected utility maximization in laboratory experiments is significantly larger than the extent to which individuals show inconsistency with expected utility maximization in reality. It is suggested that the factor causing the differences between laboratory experiments and reality is that in the former artificial limited situations are presented to individuals, these limited situations are rare in reality. Thus, the growing concerns about the reliability of expected utility based models should be dismissed.

Appendix: Solving System of Linear Inequalities

The starting point for actually solving system of linear inequalities is the following theorem.¹¹

The system of linear inequalities

$$Py \leq s \quad (1)$$

has no solution if and only if the equations

$$P'z = 0 \quad b'z = -1 \quad (2)$$

have a nonnegative solution.

One can find whether the equations in (2) have nonnegative solution by solving the canonical linear programming of

$$\min q \quad \text{subject} \quad P'z = 0 \quad b'z - q = -1 \quad (3)$$

where y and q are required to be nonnegative. Now if the minimum value of q is zero then the equations in (2) have nonnegative solution and the system in (1) has no solution. On the other hand, if the minimum value of q is strictly positive then the solution of the dual problem provides a vector s and number λ such that

$$Ps + \lambda b \leq 0 \quad 0 \leq -\lambda \leq 1.$$

Hence, $s/(-\lambda)$ solves the original problem in (1).

¹¹ Proof of the theorem and numerical example are in Gale.

Allais Paradox

Common Ratio Effect

Thin lines are iso-expected profit lines.
Thick Lines are indifference curves
This case represents risk aversion.

Thick lines are indifference curves.

Indifference curves that fan out.

The dark area indicate allowable range of the indifference curves.
In this case the choice is consistent with the expected utility maximization.

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Chapter 5:

Estimation of Farmers' Risk Attitude: An Econometric Approach

Z. Bar-Shira, R. E. Just, D. Zilberman

Abstract

An econometric procedure for estimation of Arrow-Pratt coefficients of risk aversion is derived. The model, of farmers allocating land among different crops and time between leisure and labor, allows for testing Arrow's hypotheses of decreasing absolute risk aversion and increasing relative risk aversion. The empirical results support these hypotheses.

Estimation of Farmers' Risk Attitude: An Econometric Approach

1. Introduction

The Arrow-Pratt (Arrow (1971), Pratt (1964)) measures of risk aversion has been discussed quite extensively in the theoretical, as well as in the empirical, economic literature. However, there is not much empirical evidence concerning their magnitude or their attributed properties. The two most widely used measures of risk aversion are the measure of absolute risk aversion and the measure of relative risk aversion. The first question at hand is about the signs and the magnitudes of these measures, namely, are the individuals risk averse and up to what degree. The second question is about the effect of wealth changes on the two measures of risk aversion. Arrow hypothesized that the measure of absolute risk aversion is decreasing with wealth, where the intuitive explanation is that the wealthier the individual the greater his willingness to undertake an uncertain project of a certain scale. Arrow also hypothesized that the measure of relative risk aversion is increasing with wealth, where the intuition is that when wealth and risky project are increased by the same proportion the willingness to undertake the project is decreasing. The third measure of risk aversion is the measure of partial risk aversion (Menezes and Hanson, and Zeckhauser and Keeler), it is demonstrated below that the behavior of the measure of partial risk aversion under wealth changes or risky income changes is determined by the measures of absolute and relative risk aversion, when the measure of relative risk aversion is increasing with wealth. The behavior of the measures of risk aversion is not only interesting for itself, it also has a crucial rule in signing many comparative static results concerning behavior under uncertain conditions (i.e. Sandmo, Just and Zilberman). The objective of this paper is to provide a simple method of estimation of the measures of risk aversion, and to statistically test their behavior under wealth changes.

The literature has attempted to provide empirical evidence of individuals' risk attitude. The experimental psychology literature initiated these attempts by conducting laboratory experiments in which a series of hypothetical questions concerning choice between risky alternatives was presented, for example, to a class of students (an example of such experiments is in Kahneman and Tversky). These laboratory experiments were designed in such way that the only factor affecting the decision maker is his risk attitude, thus isolating the effect of risk from other factors that may influence the decision in a real world situations. In economics, these attempts were made mostly in the agricultural sector where the individuals' risk attitude has very important policy implications. Dillon and Scandizip (1978) have used the experimental approach for elicitation of risk attitude of subsistence farmers in northeast Brazil. The main disadvantage of the experimental method is that the individuals might not have the incentive to reveal their true preferences. In order to overcome this disadvantage, Binswanger (1980, 1981, and 1984) conducted a fairly large scale experiment in rural India. Unlike the experimental approach with hypothetical payments, Binswanger's experiment used payments that were both real and significant, the highest expected payoff for a single decision exceeded the monthly income of an unskilled worker. However, even the experimental approach with significant payments is not disadvantage free, the individuals might have utility from the gamble itself and the individuals might have preference for particular probabilities. The econometric approach retains neither of the disadvantages of the experimental approaches because it uses the actual decisions made by the individuals. Moscardi and de Janvry (1977) used an econometric approach where the decision makers follow the safety-first rule.¹ Antle (1987, 1989) and Bardsley and Harris (1987) used an econometric approach for elicitation of risk attitude where the decision makers are expected utility maximizers. While Newbery and

¹ Binswanger shows that predictions derived by these models are inconsistent with the experimental behavior.

Stiglitz (1981), Robison (1982), Pope (1982), Hazell (1982) and Binswanger (1982) give wide discussion of the relevant literature, we will discuss next the more closely related works (by Antle, and Bardsley and Harris).

Antle (1987) suggests a general model in which the joint distribution of profits and risk attitudes in a producer population can be used to estimate the moments of the risk attitude distribution. Estimation of the general model is feasible via *generalized method of moments*, for empirical purpose however a moment based approach was applied to Indian data. Antle's framework, however, does not reveal information on the properties of the estimated measures of risk aversion. That is, the effects of change in wealth or income on the measures of risk aversion are not tested empirically. Bardsley and Harris developed a simple model relating debt and assets portfolio choice of the farm. The production decision is made implicitly by choosing the optimal mean variance combination on the efficiency frontier. Bardsley and Harris applied their model to Australian grazing data to examine the behavior of the partial risk aversion coefficient under changes in wealth and income. Their model, however, does not allow direct conclusion on the effects of wealth change on the measures of absolute and relative of risk aversion.

Antle (1989) discusses the structural econometric approach versus the nonstructural one. The structural econometric approach uses optimality conditions of input choice to infer on both technology and risk attitude, while the nonstructural approach uses the distribution of net returns to estimate the risk attitude only. Structural modeling requires more data and more assumptions than the nonstructural modeling. However there is no free lunch, the less information one puts into the modeling the less information one gets in return. Thus, when one needs to estimate behavioral relationship for predicting input use or conducting policy analysis, the appropriate approach should be the structural one. Furthermore, one should be aware that when modeling structural real-life-situation by nonstructural model, the risk and other factors affecting

the decision process are collapsed into the risk factor. Thus, nonstructural modeling may yield biased estimates of the risk attitude, problem which initiated the experimental approaches. Antle's estimation (1987) of risk attitude is based on structural modeling while his current work provides estimates of the risk attitude via nonstructural modeling. A nonstructural approach is also utilized by Bardsley and Harris. The model presented here, however, can be viewed as a combination of both approaches, it requires only general data and still allow for some behavioral inference and separation of risk and other factors affecting the decision process.

Another relevant concern is variation in the risk measure which is likely to occur over the sample space. This variation can be due to one or more of the following sources: i) variation of risk attitude among individuals, namely, interpersonal changes; ii) variation of risk attitude over time, namely, intrapersonal changes; and iii) variation of wealth, when the estimated measure is not constant. Antle assumes random utility approach in order to accommodate variation in risk attitude over time and individuals, While Bardsley and Harris explain variation in risk attitude by wealth changes. We explain changes in risk attitudes by wealth changes under the maintained hypothesis that all individuals have the same risk attitude. In other words, variation in risk attitude is explained by movement along the utility function rather than by movement of the utility function. Some of the empirical work (Dillon and Scandizzo, Moscardi and de Janvry, Binswanger) concerning elicitation of risk attitude has tried to relate variation in risk attitude to variation in socioeconomic characteristics among individuals or over time. The success of those attempts should not necessarily be interpreted in favor of the random utility approach. Actually, in some situations, it is quite reasonable to assume that differences in socioeconomic characteristics among individuals affect the individual's wealth level and thus his risk attitude, rather than directly affect the risk attitude. In the case studied here, the empirical background suggests similarities between individuals, however differences in wealth are observed. Thus as suggested

by the theory, we consider wealth as a primary factor affecting risk attitude, while differences in individuals' preferences over monetary outcomes are assumed to be negligible. Empirical evidence, supporting the assumption of homogeneous risk attitudes, is due to Antle (1989), he writes "One interpretation of these results is that risk attitudes are homogeneous within villages".

The method of estimation presented below employs a model which describes a real life situation. A situation in which small farm owners, who form a village, are allocating fixed amount of land among agricultural activities and fixed amount of time among leisure and farm work. Thus, farmers derive utility from both monetary wealth and leisure, so that their objective function is a two argument utility function. By choosing different land allocation the farmers are choosing different lotteries, in this sense the farmers' problem is the same as the investors' problem who have to decide on their asset portfolio. The technology is assumed to be a constant returns to scale and fixed proportion one. Although in general these are restrictive assumptions, they are appropriate for the purpose of estimating risk attitude among small farm owners. The plausibility of the constant return to scale assumption is due to the fact that, in the short run, total scale of the farm is constant because of either the time or the land constraint. Hence, increasing scale of some activities must be accompanied by decreasing scale of other activities. Consequently, there is reallocation of all inputs including managerial input. Thus, the increase in size of some activities can be done without loss of efficiency that is usually caused by increasing all inputs but the managerial one. The fixed proportion assumption has been statistically found, by Just *et al.*, to be the most appropriate one for the case of a village consisting of small farm owners. Their main justification is that farmers follow norms, established by the extension service, in allocating their inputs among the various agricultural activities.

This chapter proceeds as follow. The first section discusses the relationships between the three measures of risk aversion and their theoretical responses to change

in income or wealth. The second section outlines the model and derives some conclusions on the effect of risk on production. The third section gives empirical background and describes the data. In the fourth section, the estimation procedure is outlined. The fifth section presents the results and compares them to other results that have been obtained by other researchers. Finally, the paper concludes with short summary and conclusions.

2. Measures of Risk Aversion and Their Properties

A general measure of risk aversion is the Pratt's risk premium, ρ , it is defined as

$$U(W_0 + E(\pi) - \rho(W_0, \pi)) = E(U(W_0 + \pi)),$$

where U is von Neumann - Morgenstern utility function, W_0 indicates nonstochastic initial wealth, π denotes stochastic income, and E is the expectation operator. Positive risk premium implies that the expected income is preferred to the expected utility of random income, which is the definition of risk aversion. The risk premium can be interpreted as the maximum amount which one would pay to replace a random income with its expected value. Because the utility function is monotone, the risk premium can be written as

$$\rho(W_0, \pi) = W_0 + E(\pi) - U^{-1}(EU(W_0 + \pi)).$$

It says that the risk premium is the difference between the expected wealth, $W_0 + E(\pi)$, and its certainty equivalent, $U^{-1}(U(W_0 + \pi))$. The importance of the measures of absolute, relative, and partial risk aversion arise when considering the effects of initial wealth change and income change on the magnitude of the risk premium, also the sign of the risk premium can be inferred by the signs of the measures of risk aversion.

Arrow and Pratt first introduced the measure of absolute risk aversion, it is defined as

$$A(W) \equiv \frac{U''(W)}{U'(W)}$$

where W indicates total wealth and U'' and U' indicate the second and first derivative of the utility function, respectively. The measure of absolute risk aversion is the appropriate one to describe situations in which total wealth has fixed stochastic part - income, and variable nonstochastic part - initial wealth. Arrow pointed out that it is natural to hypothesis that individuals' willingness to undertake a certain risky project is greater when they are wealthier. In other words, wealthier individuals would have greater amount of risky assets in their portfolio. Thus, the measure of absolute risk aversion should be decreasing with wealth. For further illustration it is useful to consider the risk premium. Pratt, and Menezes and Hanson show, by different methods, that if the measure of absolute risk aversion is decreasing (increasing) with wealth then the risk premium is decreasing (increasing) with wealth. Economically it means that the wealthier the individual the smaller the maximum amount he would pay for insurance against fixed risk.

In situations where both the stochastic and the nonstochastic components of the wealth are changing at the same proportions the appropriate measure is the measure of relative risk aversion, it is given by

$$A(W) \equiv -\frac{U''(W)}{U'(W)} W.$$

Arrow's hypothesis is that when both the initial wealth and the risky project are increased by the same proportion the individual's willingness to undertake the risky project is smaller. In other words, wealthier individuals would hold smaller proportion of risky assets in their portfolio. For this case, Pratt, and Menezes and Hanson show that proportional increase in both wealth and income, will result in more than

proportional increase (decrease) in the risk premium if the measure of relative risk aversion is increasing (decreasing) with wealth. The intuitive hypothesis of increasing relative risk aversion is also supported mathematically. Assuming bounded utility functions.² Arrow shows that the measure of relative risk aversion must be greater than unitary for arbitrarily large wealth and less than unitary for arbitrarily small wealth.³ Thus, continuous monotone measure of relative risk aversion must be increasing with wealth and must be equal one for some wealth between zero and infinite. Another difference between the two measures is that while the measure of relative risk aversion is a pure number, the measure of absolute risk aversion is affected by the wealth units. Consequently, the former is universally comparable between different studies while the latter is not.

The third measure of risk aversion being considered is the measure of partial risk aversion, it is given by

$$P(W_0, \pi) \equiv \frac{U''(W_0 + \pi)}{U'(W_0 + \pi)} \pi.$$

Like the measure of relative risk aversion, this measure is units free. The measure of partial risk aversion is the appropriate one to describe situations in which initial wealth is fixed and income is changed proportionally. In this case there is one to one relationship between the risk premium and the measure of partial risk aversion, increasing partial risk aversion implies that proportional increase in risky income would result in more than proportional increase in the risk premium.

The measure of partial risk aversion is related to the measure of absolute risk aversion as follow

² Bounded utility assumption is necessary to eliminate what is known as St. Petersburg paradox (Economic of Uncertainty and Information, PP. 7-8).

³ This result follows immediately by denoting the upper bound of the measure of relative risk as $\bar{R}$, and then integrating twice the expression $U''/U' \geq -\bar{R}/W$.

$$P(W_0, \pi) = A(W_0 + \pi)\pi.$$

Differentiation of both sides with respect to W_0 gives

$$\frac{\partial P}{\partial W_0} = A' \pi,$$

hence decreasing absolute risk aversion implies decreasing partial risk aversion with respect to initial wealth. The measure of partial risk aversion is also related to the measure of relative risk aversion, its relationship is given by

$$P(W_0, \pi) = R(W_0 + \pi) \frac{\pi}{W_0 + \pi}.$$

Differentiation of both sides with respect to π yields

$$\frac{\partial P}{\partial \pi} = R' \frac{\pi}{W_0 + \pi} + R \frac{W_0}{(W_0 + \pi)^2},$$

hence increasing relative risk aversion imply increasing partial risk aversion with respect to π . The opposite, however, does not hold, one may have increasing partial risk aversion with respect to π and decreasing relative risk aversion at the same time.

The previous discussion shows that information concerning the behavior of the measures of absolute and relative risk aversion (when the latter is increasing) is sufficient to determine the behavior of the measure of partial risk aversion, but not the other way around. We now show that the elasticity of the measure of absolute risk aversion with respect to wealth⁴ determines the behavior of both measures of absolute and relative risk aversion. The elasticity of the measure of absolute risk aversion with respect to wealth is defined as

$$\epsilon_W^A \equiv A' \frac{W}{A}.$$

Assuming risk aversion, this elasticity is negative (zero, positive) when the measure of

⁴ The concept of elasticity of risk aversion is due to Just and Zilberman (1983)

absolute risk aversion is decreasing (constant, increasing). In addition, it is smaller (greater) than unitary in absolute value when the measure of relative risk aversion increasing (decreasing). It follows that $0 < \epsilon_W^A < -1$ is equivalent to decreasing absolute risk aversion and increasing relative risk aversion, when $\epsilon_W^A = -1$ the measure of relative risk aversion is constant. The analysis in this section shows that the elasticity of the measure of absolute risk aversion with respect to wealth determines the behavior of all three measures of risk aversion.

3. The Model

The model consists of I individuals who can allocate some fixed amount of land, $\bar{L}$, among J crops, each with net return π_j . Furthermore, labor is also a scarce input because hired labor is not available and all the labor input used on the farm has to be supplied by the owner and his family. Although, in this study the absence of labor market is due to the fact that farmers obey the principle of *self employment*,⁵ absence of labor market is a typical situation in developing countries. The absence of labor market emphasizes the importance of time allocation between leisure and agricultural activities in the decision process. Thus, each farmer is assumed to be an expected utility maximizer, with utility function that is defined upon wealth, W , and leisure, T_l . The decision problem can be written as

$$\max_{L, T_l} E[U(W, T_l)] = \max_{L, T_l} E[U(W_0 + \pi' L, T_l)] \quad (1)$$

subject to:

$$\bar{L} - \sum_{j=1}^J T_j L_j = T_l \quad (1.a)$$

⁵ The self employment principle is one of the five principles which define Moshav - an agricultural settlement in Israel (for more details see Zusman).

$$\sum_{j=1}^J L_j \leq \bar{L} \quad (1.b)$$

where L is a vector of land allocation, W_0 is an initial wealth, π is a vector of net profits, T is a vector whose typical element T_j is the time required to cultivate one unit of land planted with crop j , $\bar{T}$ is the available time, and E is the expectation over the distribution of profits. The first constraint (1.a) is the time constraint, it says that total amount of the available time minus time spent on farm activities equals to time devoted to leisure. The second constraint (1.b) is the capacity constraint, it says that the total cultivated land is less than or equal to total amount of the available land.

Substitute the time constraint (1.a) into the objective function (1) and assume that the capacity constraint (1.b) is not binding.⁶ Then, the first order conditions are

$$\frac{\partial E[U]}{\partial L} = E[\pi \cdot U_1] - E[T \cdot U_2] = 0 \quad (2)$$

where U_1 and U_2 are the partial derivatives with respect to the first and the second argument, respectively.

The interpretation of the first order condition is straightforward, one more unit of time allocated to either leisure or one of the crops will generate on average the same utility increasement. Thus, in the absence of labor market, the cost of labor is determined by its opportunity cost in terms of leisure. In addition, the cost of labor is endogenous to the decision maker, and hence the optimal production scheme is different than the optimal decision when the cost of labor is exogenous (as in a situation where hired labor is available).

Another factor that affects the optimal decision is risk. Following Sandmo (1971) it can be shown that $(\bar{\pi}_j/T_j) \cdot U_1 > U_2$ for a risk averse individual. Thus, in the case of decreasing marginal utility from leisure the individual consumes more leisure under

⁶ Note that, in this study, the observed land allocations sum to less than the total land, $\bar{L}$.

uncertainty than under certainty. Furthermore, in this model under certainty the individual grows only one crop, the crop for which the profit per unit of time is maximum, and leisure is consumed until the point where its marginal utility is equal to the marginal utility from one more unit of time allocated to the chosen crop. Under uncertainty, however, the farmer grows more than one crop, and leisure is consumed until the point where its marginal utility is less than the marginal utility from the lowest profit per unit of time. Thus, it appears that risk averse farmers diversify risks by choosing a crop portfolio. This led researchers to model the choice of crop portfolio as choice of asset portfolio. Generally, the approach is to find the efficient frontier in the mean-variance plane by quadratic programming, and then to deduce on the magnitude of the risk aversion coefficient from the slope of the frontier at the chosen point. The approach of econometric estimation has the advantage of allowing better statistical inferences.

A Taylor series expansion of U_1 around expected wealth, $\bar{W} = W_0 + \bar{\pi}'L$ (where $\bar{\pi} = E[\pi]$), for non-random leisure yields

$$U_1 = \bar{U}_1 + \bar{U}_{1,1}(\pi - \bar{\pi})'L = \bar{U}_1[1 + \frac{\bar{U}_{1,1}}{\bar{U}_1}(\pi - \bar{\pi})'L] \quad (3)$$

where $\bar{U}_1$ and $\bar{U}_{1,1}$ are U_1 and $U_{1,1}$ evaluated at $\bar{W}$, respectively. Substituting (3) into (2) and dividing by $\bar{U}_1$ obtains

$$\begin{aligned} \bar{\pi} - T \frac{\bar{U}_2}{\bar{U}_1} + \frac{\bar{U}_{1,1}}{\bar{U}_1} E[\pi(\pi - \bar{\pi})']L &= 0 \\ = \bar{\pi} - T \frac{\bar{U}_2}{\bar{U}_1} + \frac{\bar{U}_{1,1}}{\bar{U}_1} \Phi L &= 0 \end{aligned} \quad (4)$$

where Φ is the covariance matrix of net profits from each crop. It is worthwhile to mention that $T\bar{U}_2/\bar{U}_1$ represents the perceived wage rate per unit of land.⁷ The

⁷ Note that the units of $T\bar{U}_2/\bar{U}_1$ are $\frac{\text{time}}{\text{land}} \cdot \frac{\text{utility}}{\text{time}} / \frac{\text{utility}}{\text{money}} = \frac{\text{money}}{\text{land}}$.

perceived wage rate reflects the perceived opportunity cost of time in terms of leisure, translated to money worth by the marginal rate of substitution between leisure and money. In this form, the first order conditions are identical to the usual optimality conditions under uncertainty, where profits minus costs are equated to the risk premium.

Now let the measure of absolute risk aversion, A , to vary with W

$$A \equiv -\frac{\bar{U}_{1,1}}{\bar{U}_1} = A(W) \quad (5)$$

An unrestricting functional form is: $A = \alpha \bar{W}^\beta$, it has the advantage of not imposing risk seeking or risk averse behavior and it allows nonlinear changes in the risk attitude with respect to wealth. α greater than zero implies risk aversion, smaller than zero implies risk seeking. β , is the elasticity of absolute risk aversion with respect to W , that is

$$\frac{\partial A}{\partial W} \frac{W}{A} = \beta. \quad (6)$$

As demonstrated above, β determines the behavior of the measures of risk aversion with respect to wealth change. Substituting the functional form for the measure of absolute risk aversion into the first order conditions gives

$$\bar{\pi} - T \frac{\bar{U}_2}{\bar{U}_1} - \alpha \bar{W}^\beta \Phi L = 0,$$

which, after some obvious algebraic manipulations, can be written as

$$\Phi L = \frac{1}{\alpha} \bar{W}^{-\beta} (\bar{\pi} - T \frac{\bar{U}_2}{\bar{U}_1}). \quad (7)$$

Since L is observed directly and Φ and $\bar{\pi}$ can be estimated from the available data. This form of relationship would be estimable if $\bar{W}$ and $T \bar{U}_2 / \bar{U}_1$ were directly observed. To overcome this problem, one needs estimates for W_0 and T . We suppose

that the individuals follow linear consumption function, hence their saving is proportional to their profit. As a consequence, the initial wealth at each time point is the sum of all past profits multiplied by the marginal propensity to save. Formally

$$W_{0,t} = \sum_{i=1}^{t-1} (1+r)^{t-i} b \cdot (\pi' L)_i \quad (8)$$

where r is the interest rate and b is the marginal propensity to save. Furthermore, we use past profits, that have occurred up to fifteen years before the beginning of the data set, by the mean of all available profits. The perceived wage rate depends on the expected profit, the higher the profit per unit of land the higher the perceived wage per unit of land. The perceived wage rate can also vary with the farmer's characteristics, hence it can be expressed as function of profit and the farmer characteristics. A functional form that keeps the simplicity of the structure and still allows enough flexibility is:

$$T \frac{\bar{U}_2}{\bar{U}_1} = \bar{\pi} - \bar{\pi} \left[\sum \psi_i c_i \right] \quad (9)$$

where c_i is the i th characteristic and ψ_i is the corresponding coefficient. Substitution of equation 9 into eq. 7 yields estimable functional form

$$\Phi L = \frac{1}{\alpha} (W_0 + \bar{\pi}' L)^{-\beta} \bar{\pi} \left[\sum \psi_i c_i \right] \quad (10)$$

Taking the natural logarithm of both sides yields

$$\ln \Phi L = \ln \frac{1}{\alpha} - \beta \ln \left[W_0 + \bar{\pi}' L \right] + \sum \psi_i c_i \cdot \ln \bar{\pi} + \varepsilon, \quad (11)$$

where ε is a random disturbance reflecting measurement errors in the data.

4. Empirical Background and Data

The methodology of the previous section was applied to estimating the risk attitude of individual farmers in southern Israel. The data were collected by the accounting office of Moshav Ein-Yahav. This cooperative settlement is located in the Arava region of Israel. The Arava includes the plains between the Red Sea and the Dead Sea. It is an arid region with high minimum and maximum temperatures making it an off-season producer of winter vegetables for local and export market. The Arava region, other than a few additional settlements, is not very populated. Each farm is privately controlled, and its economic life depends on its profitability. Outputs marketing and inputs purchasing are done cooperatively because of scale advantages. The Moshav members accept the principles of self-employment and mutual collaboration as the Moshav ideological basis. The social behavioral norms in this kind of settlement are very stringent, new candidates for membership has to meet these norms, and the majority of the members have to approve their admittance. All these strict rules lead to a very homogeneous population in terms of preferences. Hence it is reasonable to consider the differences in preferences as a secondary (and differences in wealth as primary) factor affecting the risk attitude.

The data set has two parts. The first contains some socioeconomics variables in the following order: diligence, thrift, management, experience, spouse work, agricultural knowledge, motivation, land quality, education, and risk seeking. These variable that describe the farmers' characteristics were constructed by a mean of a Delphi panel assembled from the Moshav leaders and the farm advisors. The variables are rating from one to five representing consensus of the panel. The higher the number the better the attribute, for example the higher the number - the better the management ability, the higher the land quality, or the greater the spouse contribution. The second contains cross-section time series sample on 101 farmers for 10 years (1973-1982). It has aggregated variables consisting of total water use, total water cost, and expenditure

on other inputs such as pesticides, fertilizers, and cultivating materials. The arid conditions and the remoteness of outside water make it one of the most critical inputs. Thus, drip irrigation, a very efficient way to use water, is the dominant technology. Disaggregated data on cultivated land, yield, and revenue are available by crop for bell peppers, tomatoes, onion, eggplants, and melons. Almost all farmers grow tomatoes, bell peppers, and melons. A smaller number also grow eggplants and onions. The accounting office, in most of the cases, does not record input expenditure by crop but it does record total input expenditure. One can use a behavioristic approach to recover the input expenditure per crop (Just *et al.*), and then to recover the profit for each crop per unit of land. Two main factors cause the opportunity cost of land to be zero. First, there is no market for land because the land is owned by the government and renting or selling land is illegal. Second, the land allocated to the farmer is sufficient amount such that the self-employment principle binds the time constraint before the land constraint become binding.

5. The Estimation

The decision process is based on subjective expectations of future profit as well as on expected variance of future profit. Future profit expectations are based on previous year profits, when these data are available. It is very likely to have some missing values for previous year profits since no farmer grows every crop every year. A farmer who has such missing values may use estimated values while forming his future profit expectation. The estimated values are based on all available information. That is, the farmer may look at profit of farms which are similar to her own in terms of size, location, and owner's wealth, and then decide what profit she would have made. One can estimate past profit missing values by regressing profit on year and farmer dummies. That is

$$\pi_{i,t} = \sum_{i=1}^T \gamma_i^y \delta_i^y + \sum_{i=1}^I \gamma_i^f \delta_i^f \quad (12)$$

Then, the estimated profit value is

$$\hat{\pi}_{i,t} = \hat{\gamma}_i^y + \hat{\gamma}_i^f$$

where $\hat{\gamma}$ are the estimated parameters from equation (12).

Future profit expectations tend to be adaptive in nature, simply because a farmer's profit stream is highly correlated with itself over time. The farmer's expectations can be estimated by a weighted average of actual profit (or its prediction when the actual value is missing) over past years, where the coefficient sum to one and decline at a geometric rate (more on the optimality of this procedure see Just (1977)). The actual estimation was based on just the past three years profit, because most of the explanatory power is in the profit of the last few years and because of insufficient data. Mathematically the estimated expected profit for farmer i at time t is

$$\hat{\pi}_{i,t} = \theta^* \sum_{k=1}^3 \theta^k \pi_{i,t-k} \quad \text{where } \pi_{i,t} = \begin{cases} \pi_{i,t} & \text{when } \pi_{i,t} \text{ is available} \\ \hat{\pi}_{i,t} & \text{when } \pi_{i,t} \text{ is not available} \end{cases} \quad (13)$$

and $\theta^* = 1/\sum_{k=1}^3 \theta^k$. The estimated expected variance is given by

$$\hat{\Phi}_{i,t} = \theta^* \sum_{k=1}^3 \theta^k (\pi_{i,t-k} - \hat{\pi}_{i,t-k})(\pi_{i,t-k} - \hat{\pi}_{i,t-k})' \quad (14)$$

A simple grid search may used to identify maximum likelihood estimate of θ .

The endogenous variable L appears on both sides of equation (11), hence resulting simultaneity problem. We have used an instrumental variable approach to resolve this problem. The instrument are consistent of the first and second moments of the profit distribution function, that is the means and the components of the variance matrix, in our case 20 variables all together. Because there is no component of the variance matrix on the right hand side of equation (11), the equation is identified.⁸

⁸ The machine built in routine for simultaneous equations would have not worked in this case because the predicted land allocations have to be positive and to sum to one. Furthermore, a built in routine would have computed predicted values of total profit, $\pi'L$, instead of land allocations, L .

To Resolve the simultaneity problem we ran an instrumental logit model. That is,

$$\log \frac{S_k + \delta}{S_1 + \delta} = \alpha + \sum_{j=1}^J \beta_j m_j + \sum_{j=1}^J \sum_{i \geq j}^J \gamma_{ij} v_{ij} \quad k=2...5 \quad (15)$$

where S_k is the share of land allocated to crop k , and δ equals $\frac{1}{2L}$. We used the minimum chi square method to estimate this logit model (Maddala (1983)), namely we minimized the weighted sum of square error where the weights are $(\bar{L} S_k S_1)^{\frac{1}{2}}$. Consequently, the resulting estimates are BLUE. Note that equation (15) implicitly assumes that the decision process is a sequential one where in the first step the farmer decides on total amount of land to be cultivated and in the next step he decides on the allocation of cultivated land among the five crops. Next, we estimated equation (11) by OLS where the land allocations on the right hand side are replaced by their predicted values obtained from the instrumental logit model.

6. The Results

The ratio S_k/S_1 is expected to increase in m_k and decrease in m_1 because the higher the profit of a crop the greater the amount of land allocated to this crop. This ratio is also expected to decrease in v_{kk} and increase in v_{11} because for risk averse individual the higher the variance of a crop the smaller the amount of land allocated to that crop. Table I shows the results of the four instrumental logit regressions, out of 16 signs only one has significant opposite sign and only three have insignificant opposite sign.

Table II brings the regression results of equation (11). The first and the most important finding is that the elasticity of the measure of absolute risk aversion equals -0.3. It implies decreasing absolute risk aversion on one hand and increasing relative risk aversion on the other hand. The derived behavior of the measure of partial risk

aversion is straightforward, it is increasing with income and decreasing with wealth. The results reported in table II were obtained for $r = .04$ which was the real interest rate during the observed time, and assigning the marginal propensity to save, b , value of .25, this value was confirmed by the Moshav leaders as realistic one. Furthermore, we performed sensitivity analysis to verify how robust the results are to changes in the marginal propensity to save. Table III shows the estimated values of α and β for different values of b . For $b = .15$ the estimated values for α and β are 9.95 and .38, respectively. For $b = .35$ the estimated values for α and β are 11.67 and .26, respectively. It appears that the estimates for α and β are qualitatively robust to changes in b , namely the derived conclusions on the behavior of the measures of risk aversion with respect to wealth or income changes are the same.

It is interesting to compare our results to other available results. A straightforward comparison is possible with Binswanger (1981), Binswanger estimation of the elasticity of the measure of absolute risk aversion with respect to wealth is -.32, this is surprisingly close to our finding. A less straightforward comparison is possible with Bardsley and Harris who report results for three different zones, their estimation of the elasticity of the measure of partial risk aversion with respect to income and wealth are in the ranges of 0.129 to 0.194 and -.312 to -.642, respectively. One can easily verify that the following relationships hold (a) $\epsilon_{W_0}^P = \epsilon_W^A \frac{W_0}{W}$ that is the elasticity of the measure of partial risk aversion with respect to initial wealth equals the elasticity of the measure of absolute risk aversion with respect to wealth multiplied by the share of nonstochastic wealth in total wealth; (b) $\epsilon_{\pi}^P = \epsilon_W^A \frac{\pi}{W} + 1$ that is the elasticity of the measure of partial risk aversion with respect to the random income equals the elasticity of the measure of absolute risk aversion with respect to wealth times the share of the random income in total wealth. Thus, our finding that the elasticity of the measure of absolute risk aversion equals -.3 is equivalent to that the elasticities of the measure of

partial risk aversion with respect to initial wealth and random income are -.24 and .94, respectively.⁹ While qualitatively Bardsley and Harris results are consistent with ours it appears that the magnitude of their estimated elasticity of the measure of partial risk aversion with respect to random income is a way below ours. We used the inverse relationships of (a) and (b) above to check the credibility of Bardsley and Harris results for the Pastoral Zone. The share of the random income in total wealth at the median points for the Pastoral Zone is .05. Using this share and their -.471 estimate of the elasticity of partial risk aversion with respect to wealth yields -.495 as an estimate for the elasticity of absolute risk aversion with respect to wealth. However using the elasticity of the measure of partial risk aversion with respect to random income yields -16 as an estimate for the elasticity of the measure of absolute risk aversion with respect to wealth. Thus, inconsistency does exist in Bardsley and Harris estimation of the elasticities of the measure of partial risk aversion with respect to wealth and income. This inconsistency is resulting from failing to restrict the relationship between the two elasticities, as implied by the theory, during estimation.

Figures 1 and 2 show the distribution of expected wealth and initial wealth, respectively. Their medians are 168232.4 dollars and 134019.3 dollars, respectively, these values are pretty close to reality. Figure 3 shows the distribution of the absolute risk aversion, its median is 0.00000589. Figure 4 shows the distribution of the relative risk aversion, its median is .09917. Figure 5 shows the distribution of the partial risk aversion, its median is 0.158. Antle (1987) estimated the measure of partial risk aversion to be in the range of .19 to 1.77, Binswanger reports similar results, his estimated measure of partial risk aversion lies between .32 to 1.72 for the majority of the individuals. Antle (1989) reconfirms his findings from 1987, where he reports that the mean of the partial risk aversion is 1.11 for one village and 1.14 for the other. We find the

⁹ Note that, as reported below, the share of initial wealth in total wealth is 0.8 at the median points.

measure of partial risk aversion to be in the range from .05 to .25 for most of the individuals. Thus, our sample of Israeli farmers exhibits lower degree of partial risk aversion than of the Indian farmers. Bardsley and Harris found that the measure of partial risk aversion at the median point is .072, .099, and .696 for three different zones. Hence, in two of the zones the Australian farmers exhibit similar degree of partial risk aversion to that of the Israeli farmers, while in the third zone the degree of partial risk aversion is similar to that of the Indian farmers. Our estimated measure of relative risk aversion is distributed around 1 and we accepted the hypothesis that its mean is not different from 1 at 1% significant level. Thus, our estimated measure of relative risk aversion compiles with Arrow's Hypothesis. Figures 6 through 10 bring the distribution of the perceived wage in dollars per unit of land for the five crops. The peppers get the highest perceived wage with median of 1337.9; next are the tomatoes with median of 1098.4; third are the onions with median of 755.5; fourth are the melons with median of 405.6; and the lowest are the eggplants with median of 366. This results are compatible with reality were the peppers and tomatoes are high labor crops, and eggplants and melons are low labor crop.

The effect of the characteristics on the perceived wage rate can be decompose to direct effect and indirect effect. The direct effect is due to the impact of the characteristics on the time required to cultivate one unit of land. The indirect effect comes through the marginal rate of substitution between leisure and money, when less time is devoted to farm activities more time is devoted to leisure and the marginal rate of substitution goes down. Hence both effects are in the same direction. It is expected that diligence, management, motivation and education (to a less extent) will have positive coefficient because they are likely to reduce the time required to cultivate one unit of land. When land quality and labor are substitute factors also the land quality is expected to have a positive coefficient. Experience has a negative coefficient, one likely explanation is that experience is associated with older age, and older people

work more slowly. Spouse work may increase the time required to cultivate one unit of land because two people working together are not as twice efficient as one person working alone. Thrift should have positive coefficient because thrifty person tends to waste less time (or to save more time). Agricultural Knowledge increases the perceived wage, it may be the case where people who work harder and longer get higher yields and hence associated with those with better agricultural knowledge.

We found that total wealth is positively correlated with the risk seeking variable constructed by the Delphi panel (with correlation coefficient of .41). The main criterion guiding the Delphi panel members in evaluating the risk seeking attribute is the farmer willingness to adopt new technology such as new variety, new pesticide, new fertilizer, etc. The positive correlation means that the richer the farmer the less risk averse he is, hence this finding is consistent with the evidence for decreasing absolute risk aversion presented above. It is not clear, however, to what extent richer farmers are less conservative or nonconservative farmers are richer.

7. Summary and Conclusions

We developed a methodology for assessing the effect of wealth changes on the measures of absolute, relative, and partial risk aversion. It has been demonstrated that the behavior of all three measures is determined by single elasticity, the elasticity of absolute risk aversion with respect to wealth. Our main findings are that the measure of absolute risk aversion is decreasing with wealth, the measure of relative risk aversion is increasing with wealth, and the measure of partial risk aversion is increasing in risky income and decreasing with nonstochastic initial wealth. Thus, this findings suggest empirical evidence that supports Arrow's hypotheses.

The degree of risk aversion, as measured by the measure of partial risk aversion, is lower for the Israeli farmers compare with the Indian farmers. Their elasticity of

absolute risk aversion with respect wealth, however, is almost the same. Thus, differences in risk attitude between the two groups of farmers are due to wealth differences rather than differences in preferences over nonrandom outcomes (i.e. the shape of the utility function). The Australian farmers exhibit similar risk attitudes to these of the Israeli farmers in two of the zones, and similar to these of the Indian farmers in the other zone.

Figure 1. The Wealth Distribution

Figure 2. The Initial Wealth distribution

Figure 3. The Absolute Risk Aversion Distribution

Figure 4. The Relative Risk Aversion Distribution

Figure 5. The Partial Risk Aversion Distribution

Figure 6. The Perceived Wage Rate Distribution for Peppers

Figure 7. The Perceived Wage Rate Distribution for Tomatoes

Figure 8. The Perceived Wage Rate Distribution for Onions

Figure 9. The Perceived Wage Rate Distribution for Melons

Figure 10. The Perceived Wage Rate Distribution for Eggplants

Table I
Parameter Estimates

Dependent Variable	Independent Variable	DF	Parameter Estimate	Standard Error	T for H: Parameter=0	Prob > T
$\frac{S_2}{S_1}$	m_1	1	-1.2E-05	3.6E-06	-3.3	0.001
	m_2	1	6.0E-06	2.5E-06	2.4	0.017
	v_{11}	1	6.4E-09	3.4E-08	0.18	0.84
	v_{22}	1	8.2E-08	2.6E-08	3.0	0.0025
$\frac{S_3}{S_1}$	m_1	1	-3.8E-06	5.1E-06	-0.75	0.45
	m_3	1	-9.5E-07	5.2E-06	-0.18	0.85
	v_{11}	1	7.0E-08	4.8E-08	1.47	0.14
	v_{33}	1	-1.7E-08	9.8E-08	-0.17	0.86
$\frac{S_4}{S_1}$	m_1	1	-1.1E-05	4.8E-06	-2.38	0.017
	m_4	1	5.5E-06	7.0E-06	0.74	0.45
	v_{11}	1	-2.0E-08	4.6E-08	-0.43	0.66
	v_{44}	1	-4.1E-08	1.7E-07	-0.23	0.81
$\frac{S_5}{S_1}$	m_1	1	-3.8E-06	4.4E-06	-0.87	0.38
	m_5	1	4.1E-06	4.7E-06	0.89	0.37
	v_{11}	1	-1.1E-08	4.1E-08	-0.28	0.77
	v_{55}	1	-2.7E-07	1.2E-07	-2.16	0.03

Table II
Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H: Parameter=0	Prob > T
Intercept	1	11.13	1.84	6.03	0.0001
Elasticity of ARA	1	-0.30	0.12	-2.55	0.0108
experience	1	-0.18	0.059	-3.14	0.0017
Spouse Work	1	-0.15	0.03	-4.49	0.0001
Diligency	1	0.008	0.004	1.68	0.09
Thrift	1	0.028	0.005	5.88	0.0001
Management	1	0.013	0.005	2.401	0.016
Agricultural-Knowledge	1	-0.24	0.05	-4.36	0.0001
Motivation	1	0.01	0.005	2.019	0.043
Land Quality	1	0.006	0.005	1.19	0.23
Education	1	0.001	0.002	0.58	0.55

R-Square 0.9947
Number of Observations 843

Table III

Sensitivity Analysis

b	.15	.20	.25	.30	.35
α	9.95	10.67	11.13	11.44	11.67
β	.38	.33	.30	.27	.26

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Chapter 6:

Uncertainty and Farmers' Labor Supply

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Abstract

We examine farmer's labor supply under uncertainty. The effects of increases in autonomous income, the mean of the random variable, risk, and risk aversion are investigated for different types of risk. The analysis shows mostly unambiguous results for owner-labor only farms. Unambiguous results for labor-hiring farms are obtained under the assumption of nondecreasing absolute risk aversion with owner labor.

Uncertainty and Farmers' Labor Supply

1. Introduction

It is widely recognized that agricultural producers often face high risks affecting their income. This problem is especially important for family farms, which tend to be small and may have less access to capital and insurance markets. Family farms are characterized by the fact that a significant share of the labor input to the production process is supplied by the owner and his family. The allocation of farmers' time between leisure and labor and the allocation of labor between the farm and outside jobs are problems that have been part of an on-going discussion in agricultural economics literature (*e.g.* Dawson (1984), Gebauer, Regev, and Zilberman *et al.*). Less attention has been given to the analogous problems under uncertainty. Farmers' decisions regarding labor supply may be entirely different in the presence of risk and risk aversion. Hence, predictions of the effects of governmental policies based on theory of behavior under certainty may be misleading. Econometric studies (*e.g.*, Antle, Bardsley and Harris, and Bar-Shira *et al.*) show that farmers, indeed, exhibit risk aversion and that high risks are present; therefore, more research in this area is justified. The objective of this paper is to study the effects of uncertainty on labor supply decisions of farmers.

The traditional labor supply problem under uncertainty was examined in several studies starting with Block and Heineke. These authors showed that an increase in the mean of autonomous income decreases labor supply, while an increase in the risk of this income increases the supply of labor. Recently, Dardanoni showed that if the effect of an increase in labor supply on the measure of relative (or partial) risk aversion has the same sign as the effect of a wage increase on labor supply under certainty, then the effect of an increase in the risk of the wage on labor supply has the

opposite sign.

These two papers deal with situations where the economic agent is a *wage taker*. However, self-employed individuals, including farmers, affect the average and marginal return of their labor by the choice of labor supply. Thus, in the latter case the distribution of the return to labor is partially endogenous and the problem needs a different treatment. Moreover, with regard to farmers there are new problems that require examination – for example, the effect of uncertainty on other inputs (an obvious one is hired labor) and output level.

Zilberman and Just examined this problem in context of technology adoption. Dawson (1988) assumed an additively separable utility function defined on income and leisure, and analyzed the effect of uncertain output price on both family labor supply and the demand for hired labor. It was shown that the introduction of uncertainty reduces total labor and output, which is essentially the result of Sandmo.

Our model could be viewed as a generalization of the models in Block and Heineke and Dawson (1988). We proceed by endogenizing the distribution of the rate of return to labor in the former and by generalizing the functional form of the utility in Dawson's model. Moreover, we extend the latter analysis by considering more general forms of risks and by investigating the impact of a mean preserving spread in the distribution of the random variables. Finally, we investigate the impact of an increase in risk aversion on labor supply – a question which has not yet been addressed in the economic literature.

The paper proceeds as follows. In section 2 we present a model of farmers' labor supply under income uncertainty. The model is general enough to account for different types of income risk (*e.g.* yield risk, price risk, risky autonomous income) and alternative assumptions regarding the use of hired labor as a substitute for the owner labor. The third section studies the impact of an increase in both risk and risk aversion on the labor-supply in owner-labor only farm, when the risky parameter is

additively separable from the labor input. The latter assumption is relaxed in section 4 where we examine the effects of nonadditive risks. In sections 5 and 6, we relax the former assumption and consider the case when hired labor is available and used. In section 7 we examine some empirical evidence regarding the impact of uncertainty on the labor supply. Finally, section 8 concludes the paper with a short summary.

2. A Model of Farmers' Labor-Supply Decisions Under Uncertainty

The discussion in this section consists of two parts. First, we introduce the notations, objective function, and the necessary and sufficient conditions for a local maximum of the objective function. The following subsection presents a detailed discussion of various types of risk and their properties.

2.1. The Objective Function and Conditions for a Maximum

The farmer's total income is given by $Y = y + \pi$, where y is autonomous income and π is the farm profits. Profits are given by¹

$$\pi = \pi(L, K, I) = pQ(L, K) - wl^h - T,$$

where K denotes a fixed input and T is fixed cost. L denotes total labor input which is the sum of owner labor, I , and hired labor, l^h . Finally, p and w denote the output price and the wage rate, respectively. The production function, $Q(L, K)$, is assumed to be well-behaved under both certainty and uncertainty. Uncertainty is introduced via a random variable ε , which affects profits either through the production function, prices, or fixed cost. This formulation of profits assumes perfect substitution between hired labor and owner labor. This assumption is plausible in the case where the owner also works in cultivating his land, so that his labor on the farm is not limited to

¹ For simplicity, hereafter the autonomous income, y , is suppressed by the fixed cost term T .

managerial activities.

The farmer chooses l and L , *ex ante*, so as to maximize the expected value of his von Neumann-Morgenstern utility function defined over income and leisure, l^e , subject to time and income constraints. That is,

$$\max_{l \geq 0, L \geq l} \int U[Y, l^e] dF(\epsilon),$$

$$\text{such that, (i) } 1 = l + l^e$$

$$(ii) Y = \pi(L, K, l, \epsilon)$$

where 1 is total available time. $U(Y, l^e)$ is assumed to be twice differentiable, nondecreasing, and quasi concave in Y and l^e . The subjective cumulative distribution function of ϵ , F , is assumed to have finite moments. The Kuhn-Tucker necessary conditions for a local maximum of the above objective function are:

$$(1) EU_Y \pi_l - EU_{l^e} \leq 0$$

$$(2) l \cdot [EU_Y \pi_l - EU_{l^e}] = 0$$

$$(3) EU_Y \pi_L \leq 0$$

$$(4) L \cdot [EU_Y \pi_L] = 0.$$

It follows that if (1) holds with strict inequality, then $l = 0$. If (3) holds with strict inequality, then $L = l$. The corner solution of $L = l$ is of a special importance, since it defines a class of farms that do not hire labor – a common phenomenon among family farms (*e.g.* G. Thijssen). The sufficient conditions for a local maximum are:

$$a_{11} \equiv E[U_{YY}(\pi_l)^2 + U_Y \pi_{ll} - 2 \cdot U_{Yl^e} \pi_l + U_{l^e l^e}] < 0$$

$$a_{22} \equiv E[U_{YY}(\pi_L)^2 + U_Y \pi_{LL}] < 0$$

$$B \equiv a_{11} \cdot a_{22} - (a_{12})^2 > 0$$

where

$$a_{12} \equiv E[U_{YY}\pi_L\pi_l + EU_Y\pi_{Ll} - U_{Yl}\pi_L].$$

2.2. Risk Types: Definitions and Properties

The analysis of labor supply under uncertainty requires distinction among several types of income risks. We classify different risks according to their effects on the level of profits and on the value of marginal product of labor. Additional criteria which are important for the analysis are the curvatures of both profit and marginal profit functions with respect to the risky parameter.

The first type of risk we consider is *additive risk*; it is the case where the profit function is additively separable in the labor input and the risky parameter. Thus, profits equal

$$\pi = \pi^1(L, K, l) + \pi^2(\varepsilon, K),$$

which implies that both $\pi_{l\varepsilon} = 0$ and $\pi_{L\varepsilon} = 0$. As the notation suggests profits may be either linear or nonlinear in ε . An example of additive risk that has been widely discussed in the literature is risky autonomous income, i.e., $y = \bar{y} + \varepsilon$. Other examples include uncertain price of other inputs and uncertainty with regard to fixed cost. This last example may arise where a fixed factor of production, such as land, is subject to uncertain tax.

The second type of risk considered is a *linear nonadditive risk*. We model linear nonadditive risk with the Just and Pope (1977, 1978) stochastic production function, thus

$$\pi = p[g(L, K) + \varepsilon h(L, K)] - wl^h - T.$$

Note that the labor input and the random parameter are not separable, hence $\pi_{L\varepsilon} \neq 0$. By letting g be equal either zero or h , this formulation reduces to one of a multiplicative risk (e.g. uncertain output price). A multiplicative formulation restricts the random variable ε to affect profit and the value of marginal product of labor in the same

direction. The general formulation allows the random variable to affect the profit and the value of marginal product of labor in opposite directions. The economic interpretation of this property is that inputs may be risk reducing as well as risk increasing, while multiplicative risks increase with any input.

We classify linear nonadditive risks as follows

$$(R1) \quad \text{sign}(\pi_\epsilon) = \text{sign}(\pi_{L\epsilon}) > 0 \quad (\text{e.g. output price risk})$$

$$(R2) \quad \text{sign}(\pi_\epsilon) = \text{sign}(\pi_{L\epsilon}) < 0 \quad (\text{e.g. hail risk})$$

$$(R3) \quad \text{sign}(\pi_\epsilon) \neq \text{sign}(\pi_{L\epsilon}) > 0 \quad (\text{e.g. pest risk})$$

$$(R4) \quad \text{sign}(\pi_\epsilon) \neq \text{sign}(\pi_{L\epsilon}) < 0 \quad (\text{e.g. precipitation risk})$$

The labor is a risk increasing input under output price and hail risks, and it is a risk decreasing input under pest and precipitation risks. It is obvious that both the value marginal product of labor and the profits increase with output price. Hence, a risky output price is described by *R1*. Hail damage reduces both profits and the value marginal product of labor. Hence uncertain hail damage is modeled by *R2*. Pest attack would reduce output and hence profits. However, since labor applied to monitoring or spraying pesticides reduces the damage caused by the pests, the pest attack increases the value marginal product of labor (e.g., Talpaz and Borosh, Feder). It is therefore modeled with *R3*. Greater precipitation may substitute for irrigation and hence reduce cost and increase profits. It also makes the marginal labor used for irrigation less valuable, hence the value marginal product of labor decreases. Thus, it is captured by *R4*.

Finally, a *nonlinear nonadditive risk* is considered. This is the case where the risky parameter is neither additively nor multiplicatively separable from the labor input, i.e.,

$$\pi = pQ(L, \epsilon, K) - wl^h - T.$$

An example is uncertainty regarding the quality of hired labor. Note that, while in the previous risk models profits were linear in the risky parameter, in this case it does not. This means that $\pi_{\epsilon\epsilon} \neq 0$, a property which happens to be crucial to the analysis below. The nonlinear nonadditive risk model is less tractable, yet adds little in terms of number of risk types that can be modeled. Hence we limit the discussion mostly to linear risks.

3. Owner-Labor Only: the Additive Case

In this section, we consider the case where the farmer's labor is the only labor input in the production process. This case could arise due to (1) absence of a labor market due to specialization and inability of hired labor to substitute for the owner labor; (2) noneconomic factors that prevent employment of hired labor;² and (3) situations in which the return to labor in the farm is not high enough to hire labor—a situation which is a corner solution to our model.

Next we study the effects of increasing risk, risk aversion, and a shift in the mean of the random variable on the level of the owner labor. The importance of these effects arises in view of governmental interventions that are common in agriculture.

3.1. Intrapersonal Comparisons: Shift in the Mean of Income

We consider the effect of a change in the mean of ϵ when all other central moments remain constant. Formally, let $\epsilon = \bar{\epsilon} + \theta$ where θ is deterministic. Totally differentiating the first-order condition with respect to θ and l , and evaluating the result at $\theta = 0$, we obtain

² For example, a significant share of the Israeli farmers are cooperative members who, due to ideology, honor the principle of *no hired labor*.

$$\frac{\partial l}{\partial \theta} = \frac{E(U_{Yl}) - E(U_{YY}\pi_l)}{a_{11}}.$$

Assuming that leisure is a normal good under certainty, the right-hand side of the equation is negative and hence labor supply decreases in the mean of ε . Thus, a lump-sum transfer to farmers, such as an introduction of a tax break for land, would decrease the labor supply and output. Note that these results would be obtained under certainty, so in essence this is not a new result but a proof of its preservation under uncertainty.

3.2. Increase in Risk

We turn now to the question of the effect of a change in risk which has important implications for stabilization policies. An increase in risk is modeled by the well-known scheme of mean-preserving increase in risk (Rothschild and Stiglitz). For this end, we replace ε by $\theta(\varepsilon - \bar{\varepsilon}) + \bar{\varepsilon}$. An increase of θ increases the variance of ε , but leaves its mean unchanged. Differentiating the necessary condition with respect to θ and l and evaluating the result at $\theta = 1$, we obtain:

$$\frac{\partial l}{\partial \theta} = \frac{COV[\varepsilon, (-U_{lY} + \pi_l U_{YY}) \partial Y / \partial \varepsilon]}{a_{11}}.$$

Thus, the direction of the effect of an increase in risk on owner labor is the same as

$$\text{sign} [(-U_{lY} + \pi_l U_{YY}) \left(\frac{\partial Y}{\partial \varepsilon} \right)^2 + (-U_{lY} + \pi_l U_{YY}) \frac{\partial^2 Y}{\partial \varepsilon \partial \varepsilon}].$$

Assuming linear additive risks (e.g. risky autonomous income), the second term in the above expression vanishes. Two assumptions are required to show that the first term is positive. The first is that leisure is a normal good. The second is that the measure of absolute risk aversion ($A \equiv -U_{YY}/U_Y$) is nonincreasing in the owner labor (i.e. $\partial A / \partial l < 0$, (Dardanoni)). This result is easy to verify by differentiating A with respect to l and utilizing the assumption that leisure is a normal good.³

³ An alternative set of assumptions that is sufficient for the second term to be positive is

The intuitive explanation for this is that the farmer hedges against the increase in risk by increasing his labor supply. In other words, if part of the farmers' income becomes more risky and this part is additively separable from the labor input, then the farmer increases his effort and hence the mean of his income to compensate for the increase in its dispersion. It is important to note that this behavior is not due to risk aversion alone—rather, it is also required that leisure is a normal good *and* that risk aversion decreases with labor. The latter ensures that with increasing efforts to increase the mean of income, the aversion to income risk decreases and thus the farmer perceives the increase in uncertainty as less harmful.

An additive risk parameter may enter profits nonlinearly. An example is uncertainty regarding the taxation of a fixed input such as land, where ε is some parameter of the tax function. If income is concave in ε ($\partial^2 Y / \partial \varepsilon \partial \varepsilon < 0$), then the riskier ε is, the smaller is the mean of income and the larger is the variance of income. As a result, the riskier ε is, the smaller is the certainty equivalent of income, and the above intuition carries over to explain an increase in the farmer's labor supply. If income is a convex function of ε the increase in the risk of ε increases the mean of income. This effect tends to increase the certainty equivalent of income, while the increase in the riskiness of the income tends to decrease it. Without more information on preferences and technology the total effect is indeterminate.

3.3. Interpersonal Comparisons: Increase in Risk Aversion

In the discussion below we study how behavior of farmers varies with aversion to risk. These behavior patterns have important implications for support policies when attention is being paid to the distributional effects. Farmers with farms which differ in size may well exhibit different risk attitudes due to differences in initial wealth, access

rate ($U_{Y\pi} < 0$). Note that these assumptions may hold even if A is increasing in l .

to capital markets, *etc.* If, indeed, this is the case, then support policies that reduce risk (*e.g.*, stabilization of water supply, development of resistant species, *etc.*) or income transfers to farmers may affect farms of dissimilar sizes differently.

A convenient and general approach for comparisons between individuals with different risk attitudes was introduced by Diamond and Stiglitz. Since the effects of an increase in risk aversion have not been examined (to the best of our knowledge) in the context of labor supply decisions, and because the approach is very general and could easily applied to other problems as well, we describe.

We begin with a definition of an increase in risk aversion. Consider a family of utility functions $U(\epsilon, \theta)$, where ϵ is a random variable which affects utility and θ is some parameter that increases risk aversion. Then, θ must satisfy the following relationship:

$$-\frac{\partial^2 \log U_{\epsilon}}{\partial \epsilon \partial \theta} > 0.$$

That is, the index of risk aversion is increasing with θ . Diamond and Stiglitz showed that this is equivalent to the requirement that the risk premium increases with θ . Next, we present a corollary (Diamond and Stiglitz) regarding the relationship of a control variable α and an increase in θ .

Corollary: Let $\hat{\alpha}(\epsilon)$ be the level of the control variable which maximizes the utility function when ϵ is deterministic, *i.e.*, the solution to

$$\max_{\alpha} U(\alpha, \epsilon).$$

Let $\alpha^*(\theta)$ be the level of the control variable that maximizes the equivalent problem when ϵ is random, *i.e.*, the solution to

$$\max_{\alpha} \int V(U(\alpha, \epsilon), \theta) dF(\epsilon),$$

where V is some cardinalization of U .⁴ If an increase in θ represents an increase in risk aversion in the sense of the above definition, then α^* increases (decreases) with θ if $\hat{\alpha}$ decrease (increases) with ϵ .

⁴ Note that V is separable in U and θ to ensure that a change in θ does not change the indifference curves.

Intuitively, an increase in risk aversion is equivalent to a decrease in certainty equivalent of ϵ . The above corollary has immediate implications for the labor supply problem under linear additive risk. Under certainty labor supply decreases with any linear additive parameter, hence a more-risk averse farmer will work more under additive risk. As a consequence a stabilization policy that is established in order to stabilize some additive risk will decrease the output of risk averse farmers, while the output of risk neutral farmers will remain unchanged.

4. Owner-Labor Only: the Nonadditive Case

It is common wisdom that the wage rate is one of the more stable prices in the economy. Moreover, examples where the labor decision is made *ex ante*, before the realization of the wage, are rare since labor contracts usually specify the wage rate. However, uncertain return to the owner labor is common in the case of self-employed individuals in general and of farmers in particular.

4.1. Shifts in the Mean of Nonadditive Risk

The first question to be considered is the effect of a change in the mean of ϵ when all other central moments remain constant.⁵ The formal analysis is similar to the one of section 3.1, and we obtain:

$$\frac{\partial l}{\partial \theta} = - \frac{E[\pi_{l\epsilon} U_Y]}{a_{11}} - \frac{E[-U_{lY} \pi_{\epsilon} + \pi_l \pi_{\epsilon} U_{YY}]}{a_{11}}.$$

The increase in the mean of ϵ consists of a substitution effect (the first term) and a wealth effect (the second term). The substitution term is of the same sign as $(\pi_{l\epsilon})$. Under the assumption that leisure is a normal good, the wealth term is opposite in sign

⁵ Note that the effect of change in autonomous income is the same as in the case of additive risk.

to (π_e) . Thus, for the risk types $R1$ and $R2$ ($\pi_{Le} = \pi_e$), the two effects are in opposite directions, and the total effect on the owner labor is indeterminate. However, if $(\pi_{Le} \neq \pi_e)$, then the two effects are in the same direction, and the total effect is positive and negative for risk types $R3$ and $R4$, respectively.⁶

4.2. Increase in Nonadditive Risk

We turn now to examine the effect of risk increase, which is modeled similarly to 3.2 to obtain:

$$\frac{\partial l}{\partial \theta} = \frac{COV[\epsilon, (-U_{lY} + \pi_l U_{YY})\pi_e + U_Y \pi_{le}]}{a_{11}}$$

Under the assumption that the risk is linear, the sign of the above covariance is equal to

$$sign[\pi_e^2(U_{YYY}\pi_l - U_{lYY}) + 2U_{YY}\pi_e\pi_{le}].$$

Under the plausible assumptions that have been discussed in 3.2, the first term in the above expression is positive. The second term has the opposite sign of the product $\pi_e\pi_{le}$. Thus, for risk types $R3$ and $R4$, an increase of linear nonadditive risk increases labor supply and output.

This result is supported by solid intuition. The assumption that the profit function is linear in the risky parameter implies that the increase in the variability of the uncertain parameter increases the variability of profit without changing its mean. Hence, the certainty equivalent of the farmer's income decreases, and the intuition concerning an increase in autonomous income is carried over to the explain the effect of an increase in risk on the labor supply.⁷

⁶ It is worth mentioning that regardless the type of the risk it can be shown, that the qualitative effects of the increase in the mean of the risky parameter are identical to the effects of an increase in this parameter under certainty. Thus, prediction of behavior under uncertainty could be drawn from information on behavior under certainty.

⁷ When $sign(\pi_{le}) = sign(\pi_e)$, then it is still possible to sign this covariance under special cir-

4.3. Interpersonal Comparisons: Increase in Risk Aversion

Using the corollary presented in 3.3, the effect of risk aversion on labor supply can be inferred from behavior under certainty. If labor is a risk reducing input *i.e.* $\pi_{\varepsilon} \neq \pi_{l\varepsilon}$, then the rule is simple: $\pi_{l\varepsilon} < 0$, implies that the more risk averse is the farmer the larger is the labor supply and output, we get the opposite result if $\pi_{l\varepsilon} > 0$. If $\pi_{\varepsilon} = \pi_{l\varepsilon}$ (*i.e.* labor is a risk increasing input), then the results are indeterminate.

5. Labor-Hiring Farms: Additive Risk

We turn now to the question of the impact of additive risk on both owner labor supply and the level of hired labor. When hiring labor under certainty, the farmer separates his decision about the total labor employed in the farm from his decision about consumption of leisure. The optimality condition concerning total labor then equates the value marginal product of labor to its wage. Introduction of additive risk preserves the separation and the optimality condition is unchanged.⁸ To see this, rearrange the third necessary condition to get

$$\pi_L E[U_Y] = 0.$$

Positive marginal utility of income implies that

$$pQ_L(L) = w,$$

which is the labor allocation rule under certainty. Since total labor is determined by the certainty rule, the farmer's problem may be broken into two stages. The first one involves the decision about total labor and is not affected by changes in the distribution of the random variable ε or the risk attitude. The second stage is the

cumstances mentioned in the introduction, as suggested by Dardanoni.

⁸ This result was, first, pointed out in a recent study of separability in household production models (Fabella 1988).

determination of the owner labor for a given optimal total labor. Hired labor is determined by the difference between the two optimal values.

The necessary condition for the solution of the second stage is:

$$EU_Y w = EU_{l''}.$$

The comparative statics of the effects of changes in the distribution of ϵ and risk aversion on l can be preformed as in 3.2 and 3.3 when the only difference is that π_l is replaced by w . The qualitative results with regard to l remain the same, and the effect on hired labor is the same in magnitude as the effect on l but opposite in direction.

6. Labor-Hiring Farms: The Nonadditive Case

Contrary to introduction of additive risk, the introduction of nonadditive risk affects the total labor in the farm. Rearranging the third necessary condition we get:

$$-E\{U_Y[(pQ_L - w) - E(pQ_L - w)]\} = E[U_Y E(pQ_L - w)],$$

where the expectation on the left-hand side is the covariance of marginal utility of income with the net value of marginal product of labor.

Under the assumption that $\text{sign}(\pi_\epsilon) = \text{sign}(\pi_{L\epsilon})$, risk aversion ($U_{YY} < 0$) implies that the covariance on the left-hand side is negative. Assuming concavity of pQ_L in ϵ , the negativity of the covariance and Jensen's inequality imply

$$pQ_L(\bar{\epsilon}) \geq E(pQ_L) > w.$$

Therefore, introduction of risk with the characteristics mentioned above causes the farmer to decrease the level of total labor and output. In particular, note that the above result is applicable to linear risks of $R1$ and $R2$ types.

If $\text{sign}(\pi_\epsilon) \neq \text{sign}(\pi_{L\epsilon})$ and pQ_L is a convex function of ϵ , then a similar analysis shows that under uncertainty more labor is employed and more output is produced.

Included in this last case are risks of $R3$ and $R4$ types.

6.1. A Framework for Comparative Statics

A single framework for the comparative static analysis that follows is used. Let θ denote a small change in one of the following: autonomous income, mean of the random variable, variance of the random variable, or risk aversion. Total differentiation of the first order conditions (1) and (3) with respect to total labor, owner labor, and θ ; and solving for $\partial L / \partial \theta$ and $\partial l / \partial \theta$ yield the following

$$\frac{\partial l}{\partial \theta} = \frac{-z_1(\theta)a_{22} + z_2(\theta)a_{12}}{B} \quad \frac{\partial L}{\partial \theta} = \frac{-z_2(\theta)a_{11} + z_1(\theta)a_{12}}{B}, \quad (5)$$

where z_1 and z_2 are the derivatives of the first order conditions (1) and (3) with respect to θ , respectively. When analyzing different problems, only z_i , $i = 1, 2$ are different. The terms involving a_{11} and a_{22} represent direct effect, and the terms involving a_{12} represent indirect effect. Moreover, the sign of a_{12} determines the effect of owner labor on total labor and vice versa.⁹

As can be seen from the equations in (5), the sign of a_{12} is crucial for the analysis. It is shown below that its sign is determined by the following relationship

$$\frac{dA}{dl} = \frac{\partial A}{\partial Y^w} - \frac{\partial A}{\partial l^e}.$$

Block and Heineke assumed constant absolute risk aversion with respect to leisure ($\partial A / \partial l^e = 0$). Dardanoni postulates that $dA / dl \leq 0$, i.e. the absolute measure of risk aversion is decreasing in the owner labor. This assumption is implied by the two assumptions that A is decreasing in income and increasing in leisure, but the opposite is not true. Although the assumption of decreasing absolute risk aversion in owner labor is plausible, we cannot rule out the possibility that the opposite is true. The

⁹ Implicit differentiation of the first order condition (1) shows this result.

main justification is that having more leisure is perceived by the individual as being richer (as it says time is money), so that $\partial A / \partial l^e < 0$ is plausible. It follows that income and leisure affect the absolute risk aversion in opposite directions, so that it is possible for the absolute risk aversion to increase in owner labor.

Assuming decreasing absolute risk aversion in owner labor and risk types $R1$ and $R2$ we find that a_{12} is positive. To see this, define N as

$$N \equiv \frac{U_{YY}w - U_{Yl^e}}{U_Y}.$$

The derivative of N with respect to income and of A with respect to owner labor are identical. Thus, assuming decreasing absolute risk aversion in owner labor implies that N decrease in income. Let ε^* be the value of ε that equates π_L to zero, and let N^* be the value of N corresponding to ε^* . Assuming risk types $R1$ and $R2$, one can follow the technique introduced by Sandmo to show

$$E[(U_{YY}w - U_{Yl^e})\pi_L] > -N^* \cdot EU_Y \pi_L = 0.$$

The right hand side vanishes by the first order conditions. Hence, the left-hand side, a_{12} , is positive.

For risk types $R3$ and $R4$ ($\text{sign}(\pi_e) \neq \text{sign}(\pi_{L_e})$), it can be similarly shown that $a_{12} \leq 0$. Under the assumption that the measure of absolute risk aversion is increasing in owner labor, for each risk type the sign of a_{12} is reversed.

6.2. The Effect of a Change in The Autonomous Income

We turn now to investigate the impact of an increase in the autonomous income. Thus, in this case, $\theta = y$ and the effects of a rise in the autonomous income on owner labor and total labor are given by the equations in (5), where

$$z_1 = EU_{YY}w - EU_{Yl^e} \quad z_2 = EU_{YY}\pi_L.$$

We start with signing the expression for $\partial l / \partial \theta$, the strategy is to sign each term

separately.¹⁰ The first term, $-z_1 a_{22}$, is negative because both a_{22} and z_1 are negative, the former by the second-order condition and the latter by the assumption that leisure is a normal good. The sign of z_2 depends on whether the labor is risk reducing or risk increasing input. Assuming that the absolute risk aversion is decreasing in income, z_2 is positive (negative) when labor is a risk increasing (decreasing) input (Sandmo). The following table summarizes this information.

risk type	z_1	z_2	$\frac{a_{12}}{dA/dl} < 0$	$\frac{a_{12}}{dA/dl} > 0$
R1, R2	-	+	+	-
R3, R4	-	-	-	+

Assuming that the measure of absolute risk aversion is decreasing in owner labor, the above results together with the equations in (5) show that the effects of a rise in the autonomous income on owner labor and total labor are ambiguous, for all risk types. However, the alternative assumption that $dA/dl \geq 0$ yields unambiguous results. An increase in the autonomous income decreases owner labor for all types of risk. Total labor increases (decreases) when labor is a risk increasing (decreasing) input. Consequently, the sign of

$$\frac{\partial l^h}{\partial \theta} = \frac{\partial L}{\partial \theta} - \frac{\partial l}{\partial \theta}$$

is unambiguously positive for the risk types R1 and R2.

It appears that three factors determine the effect of a rise in the autonomous income on total labor. The decrease in risk aversion caused by the greater wealth causes a direct effect (captured by $-z_2 a_{11}$) to increase (decrease) total labor if labor is a risk increasing (decreasing) input. Then, there is an indirect effect through owner labor. Assuming that leisure is a normal good, the increase in autonomous income

¹⁰ Note that no algebraic simplification is helpful here.

decreases the owner labor (as indicated by z_1). When $dA/dl > 0$, this decreases the risk aversion of the farmer, and hence increases (decreases) total labor (as indicated by a_{12}) if labor is a risk increasing (decreasing) input. Thus, the direct effect resulting from decreasing absolute risk aversion in income is at the same direction as the indirect effect resulting from both assumptions that leisure is a normal good and that absolute risk aversion is increasing in owner labor. However, when the measure of absolute risk aversion is decreasing in owner labor, the direct effect and the indirect effect are in opposite directions and no unambiguous results are obtained. A similar intuition supports the results regarding owner labor.

It is possible to sign the effect of an increase in the autonomous income on the total labor under alternative assumptions. Substituting the expressions for a_{11} , a_{12} , z_1 , and z_2 into the expression for $\partial L/\partial \theta$ and rearranging we get:

$$\frac{\partial L}{\partial \theta} = \frac{[EU_{Yl} \cdot w - EU_{l \cdot l}] \cdot EU_{YY} \pi_L}{B} - \frac{[EU_{YY} w - EU_{Yl}] \cdot EU_{Yl} \pi_L}{B}$$

Assuming that, under certainty condition, the effect of availability of additional unit of time (i.e. release from jury duty) on owner labor is positive, the sign of the first term equals $\text{sign}(z_1)$. The assumption that the partial derivative of absolute risk aversion with respect to leisure is constant (Block and Heineke) implies $EU_{Yl} \cdot \pi_L = 0$. As a result, the effect of an increase in the autonomous income on total labor is positive (negative) when labor is risks increasing (decreasing).

6.3. Change in the Mean of The Nonadditive Risk

In this section we use the equations in (5) to find the effects of a change in the mean of a nonadditive risk. In this case, ε is replaced by $\varepsilon + \theta$, and z_1 and z_2 are given by

$$z_1 = EU_{YY} \pi_\varepsilon w - EU_{Yl} \pi_\varepsilon \quad z_2 = EU_{YY} \pi_\varepsilon \pi_L + EU_Y \pi_L \varepsilon$$

The analysis is confined to linear risks, and it requires distinction between the four types of risk mentioned in section 2.2. The following table shows the signs of z_1 , z_2 , and a_{12} .

risk type	z_1	z_2	a_{12} $dA/dl < 0$	a_{12} $dA/dl > 0$
R1	-	+	+	-
R2	+	-	+	-
R3	+	+	-	+
R4	-	-	-	+

Similar to the analysis in 6.2, the assumption that $dA/dl < 0$ yields ambiguous results for all types of linear risk. When the measure of absolute risk aversion is increasing in owner labor the effects of an increase in the mean of the random variable are as follows.

risk type	$\partial l / \partial \theta$	$\partial L / \partial \theta$	$\partial l^h / \partial \theta$
R1	-	+	+
R2	+	-	-
R3	+	+	?
R4	-	-	?

For risk types R1 and R4 ($\pi_\varepsilon \geq 0$), the results are qualitatively identical to the results of the previous section. Intuitively the increase in the mean of the random variable is perceived as an increase in income. For risk types R2 and R3, $\pi_\varepsilon < 0$, the intuition is reversed. Note that this is different from the situation in owner-labor only farms, where a change in the mean of the random variable creates a substitution effect (the price of leisure changed). In hired labor farms the opportunity cost of leisure is given by the constant w , and hence a shift in the mean of ε is interpreted as a change in income alone.

6.4. Increase in Nonadditive Risk

In this section we use the modeling of an increase in risk introduced in section 3.2. Here, an increase in θ represents a mean preserving spread in the distribution of ε . The effects of an increase in risk on total labor and owner labor are, once again, given by the equations in (5), where now z_1 and z_2 are

$$z_1 = E[(\varepsilon - \bar{\varepsilon})(U_{YY}w - U_{Yl^*})\pi_\varepsilon] \quad z_2 = E[(\varepsilon - \bar{\varepsilon})(U_{YY}\pi_L\pi_\varepsilon + U_Y\pi_{L\varepsilon})].$$

The term z_1 is the covariance between ε and $(U_{YY}w - U_{Yl^*})\pi_\varepsilon$. Differentiating the latter expression with respect to ε and assuming a linear risk, we get

$$\text{sign}(z_1) = \text{sign}((U_{YY}w - U_{Yl^*})\pi_\varepsilon^2).$$

Thus, under the assumptions that the absolute risk aversion is decreasing in income and that the marginal utility of leisure is increasing with income but at a decreasing rate, z_1 is positive for any linear risk.

The sign of z_2 is positive when $\pi_\varepsilon \neq \pi_{L\varepsilon}$, and negative when $\pi_\varepsilon = \pi_{L\varepsilon}$. To see this, we rewrite z_2 as

$$\frac{\pi_\varepsilon}{\pi_{L\varepsilon}} E[U_{YY}\pi_L\pi_{L\varepsilon}(\varepsilon - \bar{\varepsilon})] + E[U_Y\pi_{L\varepsilon}(\varepsilon - \bar{\varepsilon})].$$

substituting $\pi_L - \bar{\pi}_L$ for $\pi_{L\varepsilon}(\varepsilon - \bar{\varepsilon})$ in to the first term we get¹¹

$$\frac{\pi_\varepsilon}{\pi_{L\varepsilon}} (EU_{YY}\pi_L^2 - EU_{YY}\pi_L\bar{\pi}_L) + E[U_Y\pi_{L\varepsilon}(\varepsilon - \bar{\varepsilon})]$$

By noting that the sign of the last term is the same as the covariance between ε and $U_Y\pi_{L\varepsilon}$ and hence equals $\text{sign}(U_{YY}\pi_\varepsilon\pi_{L\varepsilon})$, it can be seen that the sign of all three terms is positive when $\pi_\varepsilon \neq \pi_{L\varepsilon}$, and negative when $\pi_\varepsilon = \pi_{L\varepsilon}$. The following table summarize the signs of the different terms determining the effect of an increase in risk on

¹¹ Similar technique to this one was used by Pope and Kramer.

owner labor and total labor.

risk type	z_1	z_2	$\frac{a_{12}}{dA/dl} < 0$	$\frac{a_{12}}{dA/dl} > 0$
R1	+	-	+	-
R2	+	-	+	-
R3	+	+	-	+
R4	+	+	-	+

Substitution of these signs into the equations in (5) yields unambiguous results when absolute risk aversion is increasing in owner labor. These are summarized in the table below.

risk type	$\partial l / \partial \theta$	$\partial L / \partial \theta$	$\partial l^h / \partial \theta$
R1	+	-	+
R2	+	-	-
R3	+	+	?
R4	+	+	?

Note that these results are exactly opposite to the results obtained for an increase in autonomous income. To see why, note that a utility function which is concave in income implies a concave utility function in the random variable. However, when π_ϵ is negative an individual who is risk averse in income seeks risk in ϵ . Thus, an increase in risk is perceived as a decrease (increases) in the certainty equivalent of the random variable, ϵ , when π_ϵ is positive (negative). It follows that, regardless to the sign of π_ϵ , there is a decrease in the certainty equivalent of income.

6.5. Increasing Risk Aversion under Nonadditive Risk

In this section we consider greater aversion to nonadditive risk. Consider a family of utility functions indexed by θ , where θ is an index of risk aversion. That is $V(U(\epsilon, L, l), \theta)$. By total differentiation of the first order conditions for the model rewritten in terms of V we find that the effects of an increase in risk aversion on total labor and owner labor are given by the equations in (5), where the expressions for z_i $i = 1, 2$ are:

$$z_1 = EV_{U\theta}U_l = EV_{U\theta}(U_Y w - U_{l'}) \quad \text{and} \quad z_2 = EV_{U\theta}U_L = EV_{U\theta}(U_Y \pi_L)$$

By a method similar to Diamond and Stiglitz (Theorem 4), it is possible to show that z_i $i=1,2$ is positive (negative) if $U_j \geq (\leq) 0$ for $\epsilon \leq \epsilon_j^*$ and $U_j \leq (\geq) 0$ for $\epsilon \geq \epsilon_j^*$, $j = L, l$. In other words, if U_L and U_l have the single crossing property and it crosses form above (below).¹²

The sign of z_1 is the same as sign π_ϵ . To see this note that

$$U_{l\epsilon} = (U_{YY}w - U_{Yl'})\pi_\epsilon \begin{matrix} < \\ > \end{matrix} 0 \quad \text{if} \quad \pi_\epsilon \begin{matrix} > \\ < \end{matrix} 0$$

Thus, U_l crosses from above (below) when π_ϵ is positive (negative). It follows that the sign of z_1 is the same as the sign of π_ϵ and opposite to the sign of $U_{l\epsilon}$.

Moreover, since for linear risk, π_L vanishes only for a single value of ϵ , $U_L = U_Y \pi_L$ has the single crossing property as well. It follows that, if $\pi_{L\epsilon}$ is positive (negative), then U_L crosses form below (above). Thus, the sign of z_2 is opposite to the sign of $\pi_{L\epsilon}$.

The following table gives the signs of z_1 , z_2 , and a_{12} for the various types of risks.

¹² As asserted by Diamond and Stiglitz, one way to assure that z_j $j=1,2$ is unambiguously signed is to require that $U_{j\epsilon}$ $j=l, L$ are uniformly signed at $U_L = 0$ and $U_l = 0$, i.e. at the optimal levels of l and l under certainty.

risk type	z_1	z_2	$\frac{a_{12}}{dA/dl} < 0$	$\frac{a_{12}}{dA/dl} > 0$
R1	+	-	+	-
R2	-	+	+	-
R3	-	-	-	+
R4	+	+	-	+

Once again, if the measure of absolute risk aversion is increasing in owner labor, the equations in (5) yield unambiguous result which are given in the table below. As before the reversed assumption yields ambiguous results.

risk type	$\partial l / \partial \theta$	$\partial L / \partial \theta$	$\partial l^h / \partial \theta$
R1	+	-	-
R2	-	+	+
R3	-	-	?
R4	+	+	?

The qualitative results of this section are opposite in sign to the results obtained when analyzing the effect of an increase in the mean of the random variable (section 6.3). To see why, note that an increase in aversion to the riskiness of ϵ is perceived as a decrease in the certainty equivalent of ϵ .¹³ Thus, the effect of an increase in risk aversion is qualitatively identical to that of a decrease in the mean of the random variable.

¹³ Note, that an increase in risk aversion, in the above sense, causes increase (decrease) in the concavity of the utility function in the random variable when π_ϵ is positive (negative)

7. Empirical Evidence

The objective of the current section is to examine how significant are the effects of risk and risk aversion on labor supply decisions of owners of owner-labor only farms. We assume a small multiplicative risk, and derive a first order¹⁴ Taylor expansion of the first order condition (1):

$$p \bar{E}Q_L(1 - \beta) = \frac{\bar{U}_{l^*}}{\bar{U}_Y},$$

where $\beta = P \cdot \alpha^2$. That is, β is the product of the coefficient of partial risk aversion, P , and the square of the coefficient of variation, α^2 ; $\bar{U}_Y$ and $\bar{U}_{l^*}$ are U_Y and U_{l^*} evaluated at the optimal labor supply, l^* and at the mean of the random parameter, $\bar{E}$. Three immediate comparative statics results are worth mentioning. The larger are the degree of risk aversion, the risk and the share of profits (the risky income) in total income, the smaller is the labor supply of the farmer. Estimates of β provide evidence regarding the significance of risk and risk aversion on the certainty equivalent of output price. This, combined with estimates of the elasticity of farmers' labor supply with regard to output price, η , enable a quantitative assessment of the impact of risk and risk aversion on the supply of labor.

Newbery and Stiglitz provide empirical evidence concerning the magnitudes of the coefficient of partial risk aversion and the coefficient of variation. The coefficient of variation is in the range of 15-50 percent, and the partial risk aversion is in the range of .3 to 1.72. Thus, the estimated β is in the range of 1 to 43 percent. The meaning of this finding is that the risky price is perceived as nonrandom price which is 1 to 43 percent lower than the expected value of the risky price. Evidence concerning the elasticity of labor supply with respect to output price is reported by Thijssen and

¹⁴ The risks assumed to be small in the sense that the deviations of the random variable from its mean are arbitrary close to zero. The error term in the approximation is of second order in these deviations.

Lopez, who independently show that the labor supply is very inelastic, $\eta=0.12$ (Lopez) and $\eta=0.22$ (Thijssen). Thus, even though the impact of risk and risk aversion on the perceived price and hence on welfare may be significant, the impact on the labor supply is not.

8. Conclusions

The paper examines the effects of uncertainty on farmers' labor supply and the demand for hired labor. The problem was considered under several types of income risk and for two types of farms: owner-labor only and labor-hiring farms. It is worthwhile to mention that neither of the farm types is a special case of the other. The case of labor-hiring farms displays an interaction effect between total labor and owner labor, where the case of owner-labor only farms leads to a substitution effect resulting from the fact that the opportunity cost of leisure depends on its consumption.

In the presence of additive risk, we showed that in owner-labor only farms an increase in the expected value of income decreases labor supply, while it increases with risk and risk aversion.

For the same farm type and in the presence of nonadditive risks, it was shown that the result depends on the exact specification of the risk model. When labor is a risk increasing input, the results are ambiguous since the substitution effect is in opposite direction to the income effect. However, we derive unambiguous results when labor is a risk decreasing input. The introduction of a more violent pest, for example, or an increase in its riskiness increases labor supply and output. However, in presence of this type of risk, the larger the risk aversion the smaller the labor supply. When considering precipitation risk, we see that more rain reduces the labor supply on one hand. On the other hand an increase in the variability of rain or an increase in risk aversion, for this risk type, increases the labor supply.

For all types of risks, it was shown that the effect of an increase in risk aversion is in the opposite direction from the effect of the underlying parameter on labor supply under certainty.

Additive risk does not change the level of total labor in labor-hiring farms. The qualitative effects on the owner labor supply is the same as in owner-labor only farms. Since total labor is not affected by the risk in this case, the effects on hired labor are of the same magnitude but opposite in direction.

Unambiguous results for labor-hiring farms, in presence of nonadditive risk, are obtained under the assumption of increasing absolute risk aversion in owner labor. It has been shown that, in presence of linear nonadditive risk, the effect of an increase in autonomous income on owner labor is negative regardless of the exact specification of risk. The effect on total labor is negative when labor is a risk reducing input; when labor is a risk increasing input the effect is positive. An intuitive explanation supports these results. It has been shown, formally as well as intuitively, that the effect of an increase in linear nonadditive risk on the decision variable is opposite in direction to the effect of an increase in autonomous income. Moreover, in the presence of linear nonadditive risk, the effect of an increase in risk aversion is opposite in direction to the effect of an increase in the mean of the random variable; which in turn has qualitatively the same effect as an increase in autonomous income for risk types $R1$ and $R4$ ($\pi_e > 0$). For risk types $R2$ and $R3$ ($\pi_e < 0$) the effect of an increase in the mean of the random variable is opposite in direction to the effect of an increase in autonomous income. Thus, the intuition behind the effect of a change in the mean of the random variable, its variance, or risk aversion follows from the intuition behind the effect of a change in autonomous income.

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DESCRIPTION OF COOPERATION

The empirical work on the economic evaluation of the drought compensation program was conducted by Nir Becker and Hecktor Barak under the supervision of Prof. David Bigman and reported in their M.Sc. Theses, submitted at the Department of Agricultural Economics, the Hebrew University of Jerusalem. A shorter version of their reports has been presented at the 1988 meeting of the International Association of Agricultural Economics and published in the Occasional Papers No.5 of the Association. This version is included here as Chapter 1. Dr. Chaim Tzaban was very helpful in advising on the application of the biological model of wheat growing which was the basis for this work.

The three chapter (4, 5, and 6) on farmers' attitude towards risk were part of the Ph.D. thesis, submitted by Dr. Ziv Bar-Shira at the Department of Agricultural and Resource Economics, the University of California at Berkeley. Professor David Zilberman was the major dissertation adviser, and the members of this thesis committees were Professor Peter Berck, Professor James Chalfant, Professor George Judge, and Professor Leo Simon. Professor Richard Just assisted in the econometric estimation of the farmer's risk attitude, and Professor Eitan Hochman contributed valuable comments. Israel Finkelstein initiated the idea to investigate the problem of farmers' labor supply under uncertainty, and made many helpful comments on the other chapters.

As part of their joint work on this project, the two principal investigators worked together during the academic year 1987/88, in which Professor David Bigman spent a sabbatical at Berkeley. The various chapters in this report reflect this joint work.

AN EVALUATION OF THE RESEARCH ACHIEVEMENTS

The research was the first in-depth evaluation of the Drought Compensation Program that is implemented by the Government of Israel in the Northern Negev. In recent years, this program has often been criticized as undesirable on economic grounds if not outright wasteful. The research indicates, however, that, in the absence of any agricultural insurance programs, the use of these lands for agricultural production might have been drastically reduced - even though this production is clearly desirable for the economy. The research also made a number of recommendations aimed at reducing the waste.

The research also outlines the framework for similar studies of other stabilization programs in Israel or in other countries.