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**Multi-Product Agricultural Supply
Response Estimation in Israel and the
United States**

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ABSTRACT

Alternative models were evaluated and used to estimate U.S., regional, and state multiple-product supply response in agriculture. Extensive interrelationships in product supplies and input demands were documented. Some commodities appeared to be produced by independent production processes, but the presence of constraining allocatable inputs (such as land) was shown to create a jointness in production. In such cases, multiple-product models are still needed to permit consistent estimation of product supply and/or input demand relationships, and input allocation demands can be identified with primal models but not with dual models. Commodity aggregation opportunities for consistent two-stage choice exist but appeared to be limited.

Quadratic programming input-output models of the economies of the United States and Israel were also developed in this study and used to address selected policy issues. Both models reflect state-of-the-art advancements in input-output analysis achieved under this study. The quadratic input-output models for Israel were both multi-regional and national in design and were used to examine the effects of taxation, import prices, and shifts in demand. Models developed for the United States were used to examine the effects of agricultural stabilization policies and the measurement of the economy's GNP. In addition, two new methods were developed: (1) for projecting technical coefficients beyond available published data, and (2) for estimating the coefficients of simultaneous product-demand input-supply functions subject to convexity.

RESEARCH REPORT

Introduction

Knowledge of the responsiveness of agricultural producers to change in product and factor prices is an essential component of applied research in agricultural economics. Recent developments have demonstrated our inability to predict producer response beyond a very narrow range of economic conditions. International food supplies have shifted quickly from surplus to shortage and back to surplus again. Energy costs have skyrocketed. Agricultural product and input prices far outside the range of prior experience have resulted. Our performance in anticipating commodity supply response in this environment has not been encouraging. Improved understanding of agricultural commodity response is necessary to: (1) guide government policy formulations by predicting both their commodity and intercommodity supply effects with consequent social benefits and costs, and (2) assist individual farmers to make rational investment and production decisions by providing more accurate annual price predictions.

Although much research has been devoted to agricultural supply response, most studies have focused on either aggregate (across commodities) supply or on own-price response for individual commodity supply (Askari and Cummings). Micro-level production decision studies, however, document the prominence of the multi-product firm. Only a few studies have estimated the impact of various alternative-product prices on commodity supply (Whittaker; Weaver; Shumway and Chang, 1980; Green).

Objectives

Four specific objectives were pursued in this research with the goal of improving our understanding of agricultural commodity supply relationships:

- a. Evaluate alternative multiple-commodity agricultural supply response models.
- b. Develop robust estimates of supply response parameters for major U.S. and Israeli agricultural commodities.
- c. Account for the relationship between sector interdependencies, sector engineering capacities, and the availabilities of primary inputs in a quadratic input-output model.
- d. Evaluate the net social payoff from government policies designed to reduce producers' exposure to income variability.

Evaluation of Alternative Supply Models

(Objective a)

Askari and Cummings cite 190 studies that have applied econometric models to time-series data in order to estimate agricultural supply response. Although much attention has been given to supply analysis, most prior work has been concerned with the development of single-commodity supply functions with little explicit regard for the effects of other commodity prices.

Some supply studies have included one or two alternative product prices as independent variables in their supply equations. For example,

in Just's detailed econometric analysis of eight California field crops, one or two alternative product prices were included in some of his empirical equations. In a few linear programming studies, the effect of an occasional alternative product price on the supply of a commodity has been derived (e.g., Colyer and Irwin; Helmers and Lagrone; Dean, Johnson, and Carter).

Except for one study in the 1960's (Powell and Gruen; Gruen, et al.), it appears that estimation of the cross-price supply parameters in a comprehensive way has been only a very recent research endeavor. In their pioneering work, Powell and Gruen imposed the assumption of a constant elasticity of transformation (CET) on the production function in order to reduce the number of supply parameters requiring estimation. Assuming profit-maximizing behavior, estimates of direct and cross elasticities of supply in Australia were developed for six commodities from time-series data (Gruen, et al.).

The Powell and Gruen multi-product CET supply model has been recently adapted and applied in two regional and one national supply studies in the U.S. Whittaker estimated direct and cross elasticities of supply for seven field crops in six regions using the CET and two alternative statistical models. Based on predictive accuracy and proportion of estimated parameters with expected signs, he judged the CET model to be best. Using a slightly different version of the CET model, Shumway and Chang (1980) estimated the parameters of supply for six Texas field crops, and Green estimated them for 13 U.S. field crops. Both of these latter studies focused on the stability of parameter estimates to model specification. A reasonably high degree of stability was found, but a

large proportion of estimated parameters had unexpected signs. Predictive performance was rather poor.

The only comprehensive derivation of direct and cross supply parameters using a programming model that has been reported was conducted by Shumway and Chang (1977). They derived all direct and cross-price parameters in the supply of 13 California field and vegetable crops. By construction, all parameters had the expected signs. Predictive performance was as good as with single-price-variable econometric models (which, in general, have been better than with multiple-price-variable econometric models). Alternative procedures for using linear programming results and time-series data to predict supply levels have been developed by Shumway and Talpaz. However, because parameter precision is not examined internally within a programming model, predictive accuracy is entirely dependent on the researcher's (perhaps artistic) ability to correctly specify the true production relationships within the model design.

Considerable recent attention in supply research has focused on dual models. For a given behavioral objective, Shephard and McFadden have demonstrated that there is a one-to-one duality between a production function (single or multiple product) and a dual function. For price-taking profit-maximizing firms, a convex indirect profit function linearly homogeneous in prices is dual to a concave production function. Estimation of one thus permits derivation of the other. If the production function is twice-continuously differentiable, the indirect profit function is also. The latter is a function of prices of variable inputs and products and quantities of fixed inputs and products. By the Envelope Theorem (Silberberg), its first derivatives with respect to prices are the

product supply and (negative) input demand equations which are homogeneous and symmetrical. A general class of dual models thus provides the same parsimony in parameter estimation as the CET model (which is locally dual to a production function in the same sense that the Rotterdam demand models are locally dual to a utility function). Empirical applications of dual models to multiple-product supply response in agriculture include the recent studies by Weaver for three commodity groups in North and South Dakota and by Lopez (1981) for two commodity groups in Canada.

Our subsequent evaluation of multiple-commodity supply models (Objective a) concentrates on the dual alternatives because of the theoretical coherence of the dual modeling procedures (see Pope; Lopez, 1982). The locally dual CET model is examined first. The relevance of more general dual models is addressed next, including theoretical extensions to deal with the allocatable fixed input case (a problem particularly characteristic of agricultural production) and more efficient computational methods for maintaining convexity (or concavity) in estimation of such systems. The final issue considered under this objective is geographic aggregation of regional product supply and input demand relationships.

The CET, A Locally Dual Supply Model

In the late 1960's, Powell and Gruen (P&G) made an important contribution to the production literature by developing a CES analog on the production possibilities surface and demonstrating how it permits estimation of a linear approximation to supply response along the surface. When applied to perfectly competitive firms, considerable estimation appeal occurred because of its parsimony in number of parameters requiring esti-

mation for an n-product system. Their supply model was in the spirit of the Rotterdam demand models in which the specified demand (supply) equations are intended to map only locally back to the utility (production) functions of the space in which agents maximize utility (profit).

This model has been used to estimate short-run commodity supply response in Texas (Shumway and Chang) and, more recently, in the U.S. (Shumway and Green). In the course of evaluating these empirical studies, an error in the original model was noted and corrected, and its serious consequence on estimates for systems consisting of three or more commodities was documented.

After assuming competitive equilibrium, the authors showed that the compensating small change in one product equals the negative change in a second product multiplied by the price ratio (P&G equation 18). This equation was used to derive the partial cross-price parameter in P&G equation (20) without noting that the earlier assumption of compensating change implied that all other output levels were held constant. This restriction was also ignored in the development of the supply model (P&G equation 27) where the cross-price parameters were specified assuming only that other product prices, aggregate input level, and technology were constant. No problem is created by this omission when we model firms that produce only two commodities. A misspecification is introduced, however, in models of firms with a potentially larger line of products unless (a) they are at a corner solution, (b) all other product prices are zero, or (c) other elasticities of transformation with respect to the product whose price changed are zero. The proof is in Shumway and Powell.

Two possible solutions for this problem might be considered. Either the recently developed CRESH/CRETH production system (Vincent, Dixon, and Powell) could be used, or the following changes could be made in the CET linear supply model:

a. Choose a log linear supply system rather than one that is linear in variables.

b. Use Allen partial elasticities of transformation (Allen, p. 504) rather than direct partial elasticities of transformation. This permits us to write the supply equation for commodity i (for details of the derivation, see Shumway and Powell):

$$(1) \quad \log y_{it} = \phi_{it} + \sum_{j \neq i} r_{ij} s_j (\log p_{jt} - \log p_{it}),$$

where y_{it} is output of commodity i in period t , ϕ locates the position of the production possibilities surface. r_{ij} is the elasticity of transformation between commodities i and j , s_j is the share of j in total revenue, and p is expected product price.

Equation (1) permits linear estimation of a system of supply equations having the same parameter parsimony as in P&G's original model. It also maintains the CET hypothesis but does not suffer from the error noted earlier. When applied to Texas field crop data for the years 1946-76, substantial differences from the original model estimates were found both in transformation elasticities and mean supply elasticities (Shumway and Powell). The range of corrected transformation elasticities was much wider (-2.96 to + 4.75), and the corrected supply elasticities were generally smaller in absolute magnitude (75% between ± 0.2) than the

uncorrected estimates. The corrected supply elasticities were considerably closer to those obtained from a less restrictive dual model of the same commodities (Shumway, 1983).

A General Class of Dual Models

While the corrected CET supply model is dual at a point to a production possibilities surface characterized by a constant elasticity of transformation, other dual models are globally or locally dual to a less restrictive production function where input levels may or may not be exogenously determined. For a price-taking firm, a concave production function is sufficient to assure that a profit-maximizing set of product and input levels exists. Concavity of the production function is also sufficient for a one-to-one duality with an (indirect) profit function which is convex and linear homogeneous in prices. If the production function is twice-continuously differentiable in quantities, the profit function is twice-continuously differentiable in prices of the same inputs and/or outputs. In this case, the matrices of second partial derivatives are symmetrical since they are invariant to order of differentiation.

These duality implications are two-way, so a convex, linear homogeneous, and twice-continuously-differentiable profit function also implies a concave twice-continuously-differentiable production function. The duality relationships apply locally even if the production function is only locally concave. Because the product supply and (negative) input demand functions are first derivatives in prices of the profit function, properties of the profit function are also imbued in them. They are homogeneous of degree zero in prices and are symmetrical.

Due to the two-way duality which extends also to higher-order deri-

vatives, considerable information about the production function can be obtained from estimates of the profit function or supply and demand functions. Tests for nonjoint production functions (which permit legitimate independent examination of a commodity's physical and economic production relationships), separable production functions (which permit commodity aggregation for consistent two-stage choice), homothetic production functions (which permit the use of functional forms that maintain straight-line expansion paths out of the origin), and various types of technical change can all be conducted by estimating the dual model (e.g., profit function and/or product supply and input demand functions).

A thorough examination of these duality implications between the production function and the profit function was explored in this study (Nash). Tests about the production function based on estimates of a translog profit function were presented in detail. Tests based on the generalized Leontief, quadratic, and quadratic mean of order p were presented in Nash, Shumway, and Pope, and additional approximation tests were developed in Shumway (1983).

Allocatable Fixed Inputs and Modeling Implications

It is clear that interdependent production processes, of which there are many forms, generally result in a joint production technology. Another cause is the presence of a constraint on an allocatable input (see Shumway, Pope, and Nash). Since many agricultural firms produce more than one product and since quantity (although not necessarily quality) of land is virtually always distinguishable by product to which it is allocated, this case is particularly representative of agriculture.¹ At the firm level, available agricultural land is often fixed over short to

intermediate adjustment periods. At the regional level, it is essentially fixed over very long adjustment periods, except as land is removed due mainly to forces external to the agricultural sector.

Even if there is no technical interdependence in production, the presence of a constraining allocatable input generally causes joint production and requires that a multiple-product modeling framework be used for analysis. If production is joint only because allocatable inputs are constraining, separate production, cost, and profit functions cannot be written for each product when the allocation constraint is internalized. This result is the same for any cause of joint production. However, in this case separate profit equations can be maximized as a system subject to the resource constraint. It is possible to identify input demand allocations by estimating the individual production functions, deriving first-order equations from the constrained joint profit equation, and solving simultaneously for the input demands. However, it is not possible to identify the input demand allocations from a dual approach because the derivatives must be taken from a joint indirect profit function. Consequently, when it is important to extract the allocation equations, the dual approach is not useful if an allocatable input is in fact fixed and constraining.

Maintaining Convexity (Concavity) in Estimation

For many functional forms, homogeneity and symmetry conditions can be maintained in estimation by normalization and linear restrictions. Unless the parameter estimates satisfy convexity (or concavity) conditions, nonlinear restrictions are required to maintain this property when estimates fully subject to the theory are sought. Several procedures have

been developed to maintain convexity (e.g., Lau's Cholesky factorization and Hartley, Hocking, and Cooke's convex programming), but they are extremely costly computationally for moderate-sized systems of equations.

An alternative constrained least-squares algorithm was developed by converting the problem into an exterior penalty function. The variable metric algorithm (Fletcher and Powell) modified with the golden-section line search (Talpaz) was used. This procedure was more efficient and obtained estimates with smaller sum-squared errors in application than did the Cholesky factorization (Talpaz, Shumway, and Alexander). It was used to obtain parameter estimates fully consistent with the theory for field crop production in Texas (Talpaz, Shumway, and Alexander).²

Geographic Aggregation

If the same input supply and product demand relationships assumed at the regional level (i.e., exogenous prices) apply at the national level and if there are no other sources of externalities, national product supply and input demand equations can be derived from regional estimates by horizontal summation. Because product supplies and demands appear to be largely recursive in nature, the assumption of exogenous expected product prices appears to be a reasonable approximation of reality even at the national level. However, input supplies and demands are not as recursive, so simultaneity effects leading to endogenous input prices are more likely at the national than at the regional level. While most inputs can move freely from one region to another in response to economic conditions, movement is not so easy among nations.

This fact creates the potential of simultaneity bias in directly-estimated national equations when input prices are treated as exogenous

in those estimations. It also suggests that greater confidence might be placed in geographic aggregations of the regional estimates if appropriate adjustment is made for upward-sloping national input supply curves. Such adjustment of the product supply and input demand elasticities may be accomplished as follows: Let

$$(2) \quad X^A = f(P),$$

$$X_i^B = f(P_i), \quad i = r+1, \dots, m,$$

where superscript A is national input demand or product supply; superscript B is national input supply; P is (P^1, P^2) ; P^1 is $(p_{r+1}, \dots, p_m)$: prices exogenous at the regional level but endogenous at the national level, i.e., variable input prices; P^2 is $(p_1, \dots, p_r, p_{m+1}, \dots, p_n)$: exogenous prices at the national level, i.e., output prices, and quantities of other exogenous variables, i.e., $p_{m+1} \equiv x_{m+1}, \dots, p_n \equiv x_n$; X is $(x_1, \dots, x_m)$: output and variable input quantities. Taking the total derivative of any x_i^A , we have

$$(3) \quad dx_i^A = \sum_{j=1}^n \frac{\partial x_i^A}{\partial p_j} dp_j,$$

$$(4) \quad \frac{dx_i^A}{dp_k | P^2} = \sum_{j=1}^n \frac{\partial x_i^A}{\partial p_j} \cdot \frac{dp_j}{dp_k | P^2} \quad \text{for any } k \text{ in } P^2,$$

where $\hat{P}^2$ is the complement of P_k in P^2 . The elasticity of x_i^A with respect to P_k when $\hat{P}^2$ is constant is:

$$\begin{aligned}
 (5) \quad E_{x_i P_k | \hat{P}^2}^A &= \frac{dx_i^A}{dp_k | \hat{P}^2} \cdot \frac{P_k}{x_i} = \sum_{j=1}^n \frac{\partial x_i^A}{\partial p_j} \cdot \frac{P_j}{x_i} \cdot \frac{dp_j}{dp_k | \hat{P}^2} \cdot \frac{P_k}{P_j} \\
 &= \sum_{j=1}^n E_{x_i P_j | \hat{P}}^A \cdot E_{P_j P_k | \hat{P}^2}
 \end{aligned}$$

where E is the elasticity of the first variable with respect to the second, and $\hat{P}$ is the complement of P_j in P . The matrix $E_{P_j P_k | \hat{P}^2}$ consists of the following elements:

a) It is 0 for $j \neq k$,	$\forall j = 1, \dots, r, m+1, \dots, n,$
b) It is 1 for $j = k$,	$\forall j = 1, \dots, r, m+1, \dots, n,$
c) It is $E_{P_j x_j}^B \cdot E_{x_j P_k \hat{P}^2}^A$	$\forall j = r+1, \dots, m.$

Equation (5) can then be rewritten as:

$$(6) \quad E_{x_i P_k | \hat{P}^2}^A = \sum_{j=r+1}^m E_{x_i P_j | \hat{P}}^A \cdot E_{P_j x_j}^B \cdot E_{x_j P_k | \hat{P}^2}^A + E_{x_i P_k | \hat{P}}^A$$

If we let E_{At} be the $n \times (n-m+r)$ matrix of $E_{x_i P_k | \hat{P}^2}^A$ (i.e., total national elasticities of product supplies and input demands with respect to the exogenous variables), E_A be the $n \times n$ matrix of $E_{x_i P_j | \hat{P}}^A$ (partial national elasticities of product supply and input demand derived from

horizontal summation of the regional equations), E_B be an $n \times n$ matrix of zeros except for the $(m-r) \times (m-r)$ diagonal submatrix of $E_B^{P,x}$ (input supply flexibilities), and E_A is the $n \times (n-m+r)$ submatrix of E_A , then (6) can be rewritten in matrix form as:

$$(6a) \quad E_{AT} = E_A E_B E_{AT} + E_A$$

which reduces to

$$(7) \quad E_{AT} = (I - E_A E_B)^{-1} E_A$$

The final result is a matrix of national product supply and input demand elasticities with respect to product prices and other exogenous variables, adjusted for the (previously unaccounted for) upward-sloping input supply functions. This aggregation procedure was used by Saez and Shumway to derive estimates of U.S. supply elasticities from econometrically-estimated parameters for ten regions.

Estimation of Supply Response Parameters

(Objective b)

Based on the foregoing evaluation of alternative estimation methods, data bases were developed and empirical estimation conducted to determine comprehensive product supply and input demand relationships for the U.S. and ten regions, field crop supply and input demand relationships for

Texas, and land price relationships in Iowa. These applications are presented in that order in this section.

Data Development

The reliability of results obtained from estimated models depends to a great extent on the soundness of the data used. Consequently, considerable effort was devoted to data development.

A 34-year (1946-79) annual time-series of quantities, prices, and/or receipts and expenditures for twelve comprehensive product categories and six variable input categories was compiled from published and unpublished sources (mainly the U.S. Department of Agriculture) for the U.S. and ten regions. Quantities of two quasi-fixed input categories, real estate and family labor, and data on government commodity programs, monthly precipitation and average temperatures were also compiled for the same years. More than 700 variables were included in the computerized data base from which variables used in the product supply - input demand estimation were developed (Ospina).

Similar data were collected for six Texas field crops, three variable inputs, and two quasi-fixed inputs for the state supply-demand analysis. Livestock data were not obtained at the state level.

A final cross-sectional and time-series data base was collected for districts of another state, Iowa, in order to examine the causes of differences over time and space in the price of a fixed input, farm land, in a predominantly agricultural state. The data base included annual land prices for nine districts for the period 1967-79, a productivity measure, corn, soybean and hog prices, and the consumer price index.

Regional and U.S. Product Supply and Input Demand Relationships

A comprehensive product supply and input demand system consisting of as many as eight product categories and six input categories was estimated for the U.S. and each of ten regions. For efficient estimation, both contemporaneous correlation and symmetry restrictions across equations warranted estimating the equations as a system. This was accomplished using Zellner's generalized least squares on seemingly unrelated equations. This method has properties asymptotically equivalent to maximum likelihood.

Two functional forms of the profit function were examined, the translog and normalized quadratic. Both are flexible in the sense of being second-order Taylor's series expansions around a point. The normalized quadratic performed better on both of two measures: *ex post* predictive accuracy and consistency of parameter signs with theoretical expectations for price-taking profit-maximizing firms. The results presented in this section are based on the normalized quadratic formulation.

While statistical performance varied by region, R^2 levels were consistently very high (≥ 0.96), mean square errors were low, and standard errors were low relative to parameter estimates for a large portion of the parameters. Regional own-price elasticities for product supplies and input demands are reported in Table 1 for 1978. The complete matrix of 1978 U.S. supply and demand elasticities from the direct U.S. estimates are reported in Table 2. The 1978 U.S. elasticities derived from the regional product supply and input demand estimates and national estimates of input supply elasticities are reported in Table 3. Because of simultaneity bias underlying the estimates in Table 2 and because theoretic-

Table 1. Estimated Regional Product and Input Own-Price Elasticities at 1978 Prices

Output/ Input	North East	Lake States	Corn Belt	Northern Plains	Appalachia	South East	Delta States	Texas Oklahoma	Mountain	Pacific
LIVSTK	-.456	-.329	.006	.432	.122	.101	.154	-.199	.419	-.253
FEEDGR	.135	.022	.011	.098	.242	.265	-4.351	-.030	-.137	.219
FOODGR	.282	.004	.628	-.098	b/	a/	.014	.057	.008	.223
COTTON	c/	c/	1.494	c/	-.018	-.185	.456	.377	.596	.405
OILCROP	a/	.220	.053	.545	-.033	.011	-.365	.019	a/	a/
HAYFOR	2.343	1.438	3.052	-3.248	-.097	.224	-.580	.338	.222	.741
VEGFRT	-.271	-.167	.190	.264	.196	-.356	-.732	.134	-.112	-.160
TOBACCO	-.531	a/	a/	c/	-.302	1.351	c/	c/	c/	c/

AGRCHEM	.240	-.027	-.373	.011	-.056	-.057	-.237	-.353	-.329	-.477
FSL	-1.066	.068	.250	-.415	-.521	-.195	-.101	.151	-.288	-.268
MPMCH	-.132	.140	-.134	.055	.163	.578	-.001	.535	-.023	.398
TXINT	-.008	.075	-.126	-.192	-.307	-.105	-.061	-.069	.210	.000
HLABOR	-.320	-.148	-.086	-.340	-.306	-.892	-1.394	-.883	-.209	-.839
MISCEL	-.084	-.466	.794	-1.217	-.954	-.850	-2.164	-1.692	.420	-.287

a/ Product was included with vegetable-fruit-sugar crops due to its small share in the region.

b/ Food Grains were aggregated with Feed Grains; the former represents a small share in the region.

c/ Commodity is not produced in the region.

Row codes: LIVSTK is livestock quantity, FEEDGR is feed grains quantity, FOODGR is food grains quantity, COTTON is cotton quantity, OILCROP is oil crops quantity, HAYFOR is hay-forage quantity, VEGFRT is vegetable-fruit-sugar crops quantity, TOBACCO is tobacco quantity, AGRCHEM is agricultural chemical quantity, FSL is feed-seed-livestock purchases quantity, MPMCH is mechanical power-machinery quantity, TXINT is taxes-interest quantity, HLABOR is hired labor quantity, and MISCEL is miscellaneous input quantity.

Table 2. Directly - Estimated U. S. Product Supply and Input Demand Equations

Variable	Supply Equations						Demand Equations						
	LIVSTK	FEEDGR	FOODGR	COTTON	OILCROP	VEGFRT	TOBACCO	AGRCHEM	FSL	MPMCH	TXINT	HLABOR	MISCEL
INTERCP	30658.77	7246.68	5817.67	-4304.22	-13393.89	8528.16	-662.13	5291.55	-3916.26	31798.10	-9342.14	2243.47	-2490.49
	(9040.44)	(8205.20)	(6039.13)	(6536.10)	(4760.41)	(3554.40)	(2922.15)	(3606.95)	(1898.90)	(9699.66)	(2104.04)	(8974.17)	(1370.53)
LIVSTKPR	-1880.17	111.34	-836.06	444.26	997.12	-833.68	-382.22	133.08	400.83	-1398.69	-97.12	49.90	-445.79
	(887.79)	(394.08)	(378.76)	(359.59)	(375.32)	(364.40)	(350.89)	(308.32)	(237.88)	(732.07)	(307.70)	(225.32)	(175.93)
FEEDPR	111.34	797.05	120.91	249.51	-203.93	-353.90	256.10	271.71	-14.38	-286.99	-120.38	612.08	14.75
	(352.47)	(448.25)	(238.93)	(256.26)	(235.30)	(149.93)	(160.21)	(138.80)	(71.34)	(357.17)	(83.90)	(169.36)	(53.23)
FOODPR	-836.06	120.91	586.97	-16.18	-346.11	-407.56	83.59	52.53	112.88	-808.08	68.76	623.79	36.20
	(352.61)	(248.68)	(300.82)	(216.30)	(201.69)	(156.56)	(159.97)	(142.71)	(80.65)	(353.07)	(92.13)	(146.96)	(57.36)
COTTPR	444.26	249.51	-16.18	704.34	118.89	-161.53	-152.32	-188.92	-15.23	-310.74	40.76	-703.59	-114.72
	(321.62)	(256.26)	(207.81)	(300.68)	(186.52)	(139.08)	(139.80)	(119.80)	(67.33)	(329.46)	(78.35)	(145.15)	(50.12)
OILCRPR	997.12	-203.93	-346.11	118.89	1498.07	539.82	-413.82	-268.85	-343.68	-147.24	-32.44	-6.92	-84.85
	(349.40)	(244.91)	(201.69)	(194.14)	(290.61)	(191.64)	(183.40)	(172.21)	(91.83)	(389.29)	(117.34)	(123.41)	(68.53)
VEGFPR	-833.68	-353.90	-407.56	-161.53	539.82	-1116.59	244.89	-352.71	-1.93	1926.53	213.81	241.99	-251.70
	(364.40)	(167.62)	(168.17)	(155.49)	(205.86)	(418.34)	(289.75)	(247.27)	(176.60)	(527.73)	(215.25)	(95.25)	(134.62)
TOBACPR	-382.22	256.10	83.59	-152.32	-413.82	244.89	-570.11	416.51	426.44	-513.61	-53.99	27.67	239.74
	(326.67)	(166.75)	(159.97)	(145.51)	(183.40)	(269.74)	(361.77)	(213.21)	(130.02)	(480.00)	(168.02)	(89.36)	(97.50)
AGCHPR	133.08	271.71	52.53	-188.92	-268.85	-352.71	416.51	1088.30	790.29	301.33	-536.64	-463.93	202.46
	(308.32)	(155.18)	(153.29)	(133.94)	(184.98)	(247.27)	(229.02)	(293.48)	(152.56)	(420.21)	(207.27)	88.49	(104.78)
FSLPR	400.83	-14.38	112.88	-15.23	-343.68	-1.93	426.44	790.29	-634.39	-783.53	-313.87	162.63	74.17
	(237.88)	(79.76)	(86.63)	(75.27)	(98.64)	(176.60)	(139.66)	(152.56)	(226.38)	(307.34)	(222.10)	(48.41)	(114.15)
MPMCHPR	1398.69	-286.99	-808.08	-310.74	-147.24	1926.53	-513.61	301.33	-783.53	421.90	-149.34	788.43	-1373.72
	(732.07)	(399.33)	(379.26)	(368.35)	(418.17)	(527.73)	(515.61)	(420.21)	(307.34)	(1150.10)	(389.09)	(224.23)	(220.21)
TXINTPR	-97.12	-120.38	68.76	40.76	-32.44	213.81	-53.99	-536.64	-313.37	-149.34	1261.13	-70.13	351.77
	(307.70)	(93.81)	(98.97)	(87.59)	(126.04)	(215.25)	(180.48)	(207.27)	(222.10)	(389.09)	(402.05)	(59.43)	(169.76)
HLABRPR	49.90	612.08	623.79	-703.59	-6.92	241.99	27.67	-463.93	162.63	788.43	-70.13	-292.18	125.69
	(225.31)	(189.35)	(157.86)	(162.28)	(132.56)	(95.25)	(95.98)	(88.49)	(48.41)	(224.23)	(59.43)	(180.69)	(36.48)
MISCPR	-445.79	14.75	36.20	-114.72	-84.85	-251.70	239.74	202.46	74.17	-1373.72	351.77	125.69	1207.06
	(175.93)	(59.52)	(61.62)	(56.03)	(73.61)	(134.62)	(104.73)	(104.78)	(114.15)	(220.21)	(169.76)	(36.48)	(124.06)

Table 2. (Continued)

Variable	Supply Equations					Demand Equations							
	LIVSTK	FEEDGR	FOODGR	COTTON	OILCROP	VEGFRT	TOBACCO	AGRCHEM	FSL	MPMCH	TXINT	HLABOR	MISCFL
FAMLABR	-2.73 (.34)	1.60 (.37)	1.05 (.24)	-.95 (.33)	.53 (.22)	.78 (.13)	-.09 (.18)	-.71 (.13)	.19 (.07)	.30 (.35)	.56 (.10)	-2.90 (.28)	.10 (.06)
REALEST	1.91 (.72)	-2.13 (.82)	-1.41 (.58)	1.03 (.66)	.75 (.46)	-.71 (.29)	.31 (.29)	-.10 (.15)	-.26 (.15)	-2.64 (.75)	-.04 (.18)	1.46 (.70)	-.10 (.11)
TEMP	-167.61 (103.29)	24.56 (53.97)	11.24 (23.57)	65.58 (23.49)	9.27 (22.18)	-45.65 (26.92)	2.68 (9.02)	-31.86 (145.27)	-106.53 (80.98)	76.78 (401.97)	-104.14 (85.57)	956.19 (382.04)	72.76 (56.85)
PREC	706.38 (356.58)	-367.80 (277.21)	27.95 (222.78)	411.45 (219.97)	185.21 (149.42)	115.39 (133.32)	15.43 (104.47)	-6.30 (39.64)	50.62 (22.51)	-272.63 (109.91)	40.82 (24.18)	57.20 (108.35)	22.44 (16.14)
TIME	-146.72 (51.89)	418.13 (62.14)	203.27 (36.97)	-85.12 (56.18)	246.77 (34.63)	100.72 (23.56)	-13.66 (35.12)	-207.60 (22.46)	5.41 (12.94)	-244.66 (56.05)	59.36 (17.03)	-641.19 (44.47)	1.03 (9.64)
DIVP		-196.96 (674.99)	-275.01 (128.20)	1302.08 (1821.82)									
ESDY		1022.33 (328.38)	-415.01 (214.61)	-510.03 (223.50)	2389.77 (421.74)		-239.27 (174.13)						
ES_DY		-597.59 (196.57)		147.74 (111.38)	-2285.55 (393.20)		263.90 (207.21)						

R^2 for system = .993; MSE = 3.94 with 261 DFS. Standard errors are in parentheses.
Variable input quantities are measured in negative units.

Column codes:

LIVSTK is livestock quantity, FEEDGR is feed grains quantity, FOODGR is food grains quantity, COTTON is cotton quantity, OILCROP is oil crops quantity, VEGFRT is vegetable-fruit-sugar crops quantity, TOBACCO is tobacco quantity, AGRCHEM is agricultural chemicals quantity, FSL is feed-seed livestock purchases quantity, MPMCH is mechanical power-machinery quantity, TXINT is taxes-interest quantity, HLABOR is hired labor quantity, and MISCFL is miscellaneous input quantity.

Row codes:

LIVSTKPR is livestock price, FEEDPR is feed grains price, FOODPR is food grains price, COTTPR is cotton price, OILCRPR is oil crops price, VEGFPR is vegetable-fruit-sugar crops price, TOBACPR is tobacco price, AGCHPR is agricultural chemicals price, FSLPR is feed-seed-livestock purchases price, MPMCHPR is mechanical power-machinery price, TXINTPR is taxes-interest price, HLABRPR is hired labor price, MISCPR is miscellaneous inputs price, FAMLABR is family labor quantity, REALEST is real estate quantity, TEMP is temperature, PREC is precipitation, TIME is time trend, DIVP is diversion payments, ESDY is the dummy for effective support price, and ES_DY is the interaction between ESDY and expected price. All price variables are divided by the price of hay-forage.

Table 3. Derived U. S. Product and Input Price Elasticities at 1978 Prices After Adjusting for Upward-Sloping Input Supply Functions

Output/ Input	Supply Elasticity with Respect to the Price of							
	LIVSTKPR	FEEDPR	FOODPR	COTTPR	OILCRPR	HAYFRPR	VEGFRPR	TOBACPR
LIVSTK	-.040	.004	-.043	.019	.043	.025	.074	.008
FEEDGR	.010	.016	-.015	.019	-.012	.028	-.005	.034
FOODGR	-.327	-.069	.008	.037	.010	.228	.027	.001
COTTON	.193	.093	.036	.302	-.058	.045	-.174	.023
OILCRP	.158	-.022	.004	-.030	-.006	.001	.003	-.066
HAYFOR	.189	.099	.180	.026	.012	.388	-.081	-.038
VEGFRT	.288	-.023	.011	-.073	.014	-.047	-.201	.024
TOBACCO	.185	.388	-.001	.065	-.311	-.086	.083	-.107
AGRCHEM	-.135	-.209	-.045	.189	.139	.302	.160	.067
FSL	-.011	-.062	-.099	.084	.077	.273	.047	.020
MPMCH	.176	.157	.095	-.034	-.089	-.105	-.088	-.015
TXINT	-.060	-.003	-.038	.008	.006	.020	.117	-.000
HLABOR	-.078	-.004	.001	.061	-.006	.053	-.007	.037
MISCEL	.170	.058	.015	.019	.010	-.056	.017	-.008

Column codes: LIVSTKPR is livestock price, FEEDPR is feed grains price, FOODPR is food grains price, COTTPR is cotton price, OILCRPR is oil crops price, HAYFRPR is hay-forage price, VEGFRPR is vegetable-grain price, and TOBACPR is tobacco price.

Row codes: LIVSTK is livestock quantity, FEEDGR is feed grains quantity, FOODGR is food grains quantity, COTTON is cotton quantity, OILCRP is oil crops quantity, HAYFOR is hay-forage quantity, VEGFRT is vegetable-grain quantity, TOBACCO is tobacco quantity, AGRCHEM is agricultural chemicals quantity, FSL is feed-seed-livestock purchases quantity, MPMCH is mechanical power-machinery quantity, TXINT is taxes-interest quantity, HLABOR is hired labor quantity, and MISCEL is miscellaneous input quantity.

cally unexpected regional parameters were not arbitrarily adjusted prior to aggregating to the U.S. elasticities in Table 3, it is expected that these two sets of elasticities bracket the true elasticities in most cases.

Texas Field Crop Product Supply and Input Demand Relationships

A careful examination of the supply of six field crops (cotton, sorghum, wheat, corn, rice, and hay) was conducted for Texas. Three sets of estimates were obtained: (a) short-run supply response along the CET production possibilities surface for the period 1946-76, (b) product supply and variable input demand relationships for the period 1957-79 based on a normalized quadratic profit function with symmetry restrictions maintained, and (c) product supply and variable input demand relationships for the latter case with symmetry and convexity restrictions maintained. The first two sets of parameters were estimated as seemingly unrelated systems of equations by Zellner's generalized least squares. Homogeneity and symmetry restrictions were maintained in both while convexity restrictions were also maintained in the final set. However, the final set of estimates did not account for the contemporaneous correlation among equations, which proved to have an important effect on the estimates. The penalty-augmented nonlinear algorithm (Talpez, Shumway, and Alexander) was used to obtain the final set of estimates.

Variable inputs in the last two cases included fertilizer, hired labor, and machinery operating inputs used in the production of the six field crops. Quasi-fixed inputs were land and family labor. Extensive dual tests on the structure of production (nature of the production func-

tion) were conducted using the second model.

The empirical estimates for the six short-run supply equations from the first model (Equation 1) are reported in Table 4. These estimates based on the corrected CET formulation are substantially different from the earlier estimates from the uncorrected CET formulation (Shumway and Powell). Elasticities of supply are much closer to the elasticities obtained from the more general dual model which are reported below.

The second and third sets of estimates were based on the following system of product supply and input demand equations derived from the normalized quadratic restricted profit function:

$$(8) \quad x_{it} = b_i + \sum_{j=1}^{m-1} b_{ij} p'_{jt} + \sum_{j=m+1}^n b_{ij} x_{jt}, \quad \forall i = 1, \dots, m-1.$$

where x_{it} ($i=1, \dots, m-1$) is the quantity of the i^{th} netput (positive for an output, negative for a variable input) in year t , p'_{jt} is the (expected) price of the j^{th} netput divided by the price of the m^{th} netput in year t , x_{jt} ($j=m+1, \dots, n$) is the quantity of the j^{th} fixed input or other exogenous variable in year t . The system consisted of $m-1$ equations where m is the number of different outputs plus variable inputs. An equation was estimated for each except the numeraire netput, $i=m$.

In the second set of estimates, linear symmetry restrictions were maintained so the seemingly unrelated system was linear in parameters, and Zellner's generalized least-squares was used to obtain parameter estimates (as in the first set of estimates). The estimated parameters

Table 4. CET Loglinear Supply Model Estimates, Texas, 1946-76.^a

Commo- dity	Unit (million)	Intercept	CET Price Variables ^b					Shift Variables				
			Corn	Cotton	Hay	Rice	Sorghum	Wheat	Land Input	Lagged Output (Technology)	Weighted Diversion Payment ^c	Weather Index
Corn	bu.	-1.537 (1.515)		-.649 (.262)	.857 (.455)	-.486 (.726)	-2.963 (1.859)	4.746 (2.551)	-.616 (.551)	.671 (.182)	-.0013 (.0007)	.871 (.206)
Cotton	lb.	3.762 (1.012)	-.649 (.262)		.309 (.173)	.077 (.211)	-.593 (.159)	-.114 (.202)	.180 (.214)	-.059 (.075)	-.0032 (.0007)	.838 (.106)
Hay	ton	-1.316 (1.826)	.857 (.444)	.309 (.169)		.106 (.801)	.305 (.291)	.119 (.388)	-1.015 (.510)	.307 (.165)		1.327 (.285)
Rice	cwt.	-3.156 (2.162)	-.486 (.709)	.077 (.206)	.106 (.801)		-.091 (.359)	-.030 (.779)	.167 (.354)	.665 (.122)		.799 (.273)
Sorghum	bu.	-2.261 (1.703)	-2.963 (1.859)	-.593 (.159)	.305 (.298)	-.091 (.368)		-1.112 (1.001)	-1.327 (.432)	.699 (.081)	-.0018 (.0006)	1.662 (.312)
Wheat	bu.	-8.369 (1.568)	4.746 (2.551)	-.114 (.202)	.119 (.398)	-.030 (.798)	-1.112 (1.001)		1.097 (.454)	.383 (.091)	-.0001 (.0002)	1.636 (.195)

^aEstimated standard errors are in parentheses.^bPrice variables are defined as in equation (1). The parameter estimates reported in these columns are the corrected Allen elasticities of transformation.^cBecause diversion payments were not available in all years, this variable was not converted to logarithms.

are reported in Table 5. All own-price parameters have the expected sign, but the Hessian matrix of the profit function in prices is not positive semidefinite. Consequently, the estimated profit function is not convex. Elasticities of supply computed at 1979 prices are reported in Table 6.

Since the parameter estimates were not fully consistent with an underlying convex profit function, duality relationships must be interpreted cautiously. Several dual tests of the production structure were conducted, some of which were approximate tests developed as part of this project. Various functional forms were used because not all could be tested with the normalized quadratic form. Nonjointness, homotheticity, and weak separability in subsets were tested to determine whether

- (a) some outputs were produced independently,
- (b) the production function could be represented by straight-line expansion paths out of the origin, and/or
- (c) some outputs or inputs could be aggregated for consistent two-stage choice.

The results of each of the tests are reported in Table 7. If duality relationships hold, the following inferences apply to Texas field crop production over the 1957-79 period (based on a .01 significance level):

- (a) Only wheat is produced independently. The other five crops are jointly produced and interactions among them are substantial.
- (b) Both the input expansion path and the output expansion path are straight lines out of the origin.
- (c) Cotton, sorghum, and corn production can be aggregated, and their production choices can be consistently examined in two stages.

The final estimates are based on the second model with convexity

Table 5. Parameter Estimates, Normalized Quadratic Product Supply and Input Demand, Texas, 1957-79.^a

Variable	Supply Equations						Demand Equations ^b	
	Cotton	Sorghum	Wheat	Corn	Rice	Hay	Fertilizer	Hired Labor
	(M lb) ^c	(million bu)				(M cwt)	(M ton)	(M lb)
								(T worker) ^d
Intercept	-110.5	8.81	.028	1.59	1.028	.006	89.7	3.57
(T)	(50.9)	(7.11)	(4.386)	(8.46)	(.515)	(.141)	(17.8)	(1.49)
Normalized prices:								
Cotton	563.1	-151.8	1.44	58.4	-5.53	.296	-29.0	-5.01
(\$/Tlb/PO)	(159.1)	(19.9)	(12.80)	(20.5)	(1.50) ^e	(.469)	(60.9)	(4.78)
Sorghum	-151.8	33.3	-5.57	-7.98	1.87	-.016	-11.9	5.77
(\$/Tbu/PO)	(19.9)	(12.6)	(3.91)	(9.34)	(.68)	(.197)	(27.0)	(1.80)
Wheat	1.44	-5.57	9.72	7.03	.620	.008	-8.48	-.147
(\$/Tbu/PO)	(12.80)	(3.91)	(2.87)	(3.56)	(.293)	(.088)	(11.61)	(.884)
Corn	58.4	-7.98	7.03	1.71	-3.99	-.186	-3.30	-2.07
(\$/Tbu/PO)	(20.5)	(9.34)	(3.56)	(12.20)	(.70)	(.207)	(31.55)	(1.56)
Rice	-5.53	1.87	.620	-3.99	.868	.081	-22.1	-.340
(\$/Tcwt/PO)	(1.58) ^e	(.71)	(.309)	(.74)	(.082)	(.017)	(2.7)	(.138)
Hay	.296	-.016	.008	-.186	.081	.005	-1.36	-.030
(\$/Tton/PO)	(.494)	(.207)	(.093)	(.219)	(.017)	(.007)	(.82)	(.040)
Fertilizer	-29.0	-11.9	-8.48	-3.30	-22.1	-1.36	1376.9	-20.7
(\$/Tlb/PO)	(67.4)	(29.9)	(12.83)	(34.88)	(2.8)	(.86)	(146.4)	(6.0)
Hired Labor	-5.01	5.77	-.147	-2.07	-.340	-.030	-20.7	1.18
(\$/worker/ year/PO)	(5.29)	(2.00)	(.978)	(1.73)	(.145)	(.042)	(6.0)	(.56)
Family Labor	2.18	-.316	-1.20	-.930	-.289	-.036	-3.00	-.938
(T worker) ^d	(6.93)	(.856)	(.60)	(1.043)	(.067)	(.018)	(2.34)	(.191)
Land	21.7	12.5	12.0	16.1	1.25	.128	5.48	1.97
(M acre)	(36.4)	(4.8)	(3.4)	(4.9)	(.34)	(.099)	(12.26)	(1.15)
Time	55.6	-4.56	-.200	-.942	-.517	-.003	-45.8	-1.81
(year)	(25.9)	(3.60)	(2.236)	(4.291)	(.261)	(.072)	(9.1)	(.76)
Commodity - Specific								
Variables:								
Weather	22.1	2.28	1.59	1.12	.144	.044		
(WI)	(1.4)	(.42)	(.17)	(.21)	(.025)	(.006)		
Diversion								
payments	-6.45	-.019	.0002	.021				
(\$/TU)	(.82)	(.035)	(.0081)	(.048)				

^aNormalized quadratic functional form. Standard errors are in parentheses. $R^2 = .996$; MSE = 1.01 with 106 DFS.

^bVariable input quantities are measured in negative units.

^cUnit codes: M = million, PO = operating capital inputs price index (1910-14 = 100), T = thousand, U = unit of output (i.e., lb., bu., cwt., ton), WI = weather index (actual yield/predicted yield).

^dQuality adjusted to 1972 equivalent.

^eAlthough the symmetry restriction gives the same parameter estimate, for example, on cotton price in rice supply as on the price of rice in cotton supply, the standard errors computed in seemingly unrelated regression algorithms are approximate and differ slightly between equations.

Table 6. Product Supply and Input Demand Elasticities, Texas, 1979.^a

Output or Input	Elasticity with Respect to the Price of						
	Cotton	Sorghum	Wheat	Corn	Rice	Hay	Fertilizer
							Hired Labor
							Capital Operating Inputs
Cotton	.25	-.74	.01	.52	-.28	.06	.03
Sorghum	-.26	.62	-.18	-.27	.36	-.01	.05
Wheat	.003	-.14	.43	.33	.16	.01	.05
Corn	.11	-.17	.26	.07	-.88	-.16	.01
Rice	-.04	.15	.09	-.59	.72	.26	.38
Hay	.01	-.01	.01	-.16	.39	.10	.13
Fertilizer	-.01	-.03	-.04	-.01	-.55	-.13	-.70
Hired labor	-.02	.27	-.01	-.18	-.16	-.06	.20
Capital operating Inputs	-.05	.05	-.57	.29	.24	-.07	-.15
							-.01
							-.37

^aBased on normalized quadratic parameter estimates reported in Table 5.

Table 7. Technology Tests, Texas, 1957-79.

Test	Functional Form	Items	F-Statistic	Critical Value
Nonjointness	Normalized Quadratic			
		All outputs	7.60	$F_{15,106}^{.01} = 2.23$
		Wheat and hay	3.66	$F_{9,106}^{.01} = 2.62$
		Wheat	1.75	$F_{5,106}^{.01} = 3.21$
Homotheticity	Translog	Fertilizer, hired labor	1.07	$F_{2,78}^{.01} = 4.92$
		All outputs	1.33	$F_{6,78}^{.01} = 3.07$
		Cotton, sorghum, corn, rice	0.61	$F_{4,78}^{.01} = 3.59$
		Cotton, sorghum, corn	0.57	$F_{3,78}^{.01} = 4.07$
		Cotton, sorghum	1.96	$F_{2,78}^{.01} = 4.92$
Weak Separability	Quantity Ratio	Fertilizer, hired labor:		
		Fertilizer/hired labor	7.96	$F_{8,11}^{.01} = 4.74$
		Cotton, sorghum, corn, rice:		
		Rice/cotton	1.81	$F_{6,11}^{.01} = 5.07$
		Rice/sorghum	1.73	$F_{6,11}^{.01} = 5.07$
		Rice/corn	12.34	$F_{6,11}^{.01} = 5.07$
		Corn/cotton	0.44	$F_{6,11}^{.01} = 5.07$
		Corn/sorghum	0.67	$F_{6,11}^{.01} = 5.07$
		Sorghum/cotton	0.78	$F_{6,11}^{.01} = 5.07$
		Cotton, sorghum, corn:		
		Corn/cotton	0.38	$F_{7,11}^{.01} = 4.89$
		Corn/sorghum	0.96	$F_{7,11}^{.01} = 4.89$
		Sorghum/cotton	0.76	$F_{7,11}^{.01} = 4.89$
		Cotton, sorghum:		
		Sorghum/cotton	1.52	$F_{3,11}^{.01} = 4.74$

also maintained. Since this requires nonlinear restrictions across equations, the Talpaz-Shumway-Alexander nonlinear search algorithm was used. Contemporaneous correlation among equations was not accounted for and variables were scaled, so the results reported by Talpaz, Shumway and Alexander are not immediately comparable with those in Table 5.

Value of a Fixed Input, Land

To explain temporal and spatial differences in farm land prices in predominantly agricultural regions of a state where most farms produce multiple products, the following model based on a multiple-product dual derivation was estimated using 1967-77 time-series data for five districts of Iowa:

$$\begin{aligned}
 (9) \quad L_{dt} = & a_0 + a_1 [b \text{ PCORN}_{t-1} + (1-b)\text{PCORN}_{t-2}] \\
 & + a_2 [b \text{ PSOY}_{t-1} + (1-b)\text{PSOY}_{t-2}] \\
 & + a_3 [b \text{ PHOG}_{t-1} + (1-b) \text{ PHOG}_{t-2}] \\
 & + a_4 \text{ YCORN}_{t-1} + a_5 \text{ CPI}_{t-1} + a_6 \text{ CSR}_d
 \end{aligned}$$

where L_{dt} is farm land price in the d^{th} district in year t , PCORN is Iowa farm-level corn price, PSOY is Iowa farm-level soybean price, PHOG is Iowa farm-level hog price, YCORN is Iowa corn yield, CPI is the U.S. consumer price index, and CSR_d is the corn suitability rating (a productivity measure) for the d^{th} district.

Expected product prices were represented as an adaptive function of

the previous two years' prices. The lag structure was based on preliminary tests in which longer lags and different lag structures among commodities were rejected at the 5% significance level. To determine the total effect of product price changes on land prices under the possibility that input prices are endogenous, input costs were not included in the model. The lagged structure on price required the use of nonlinear regression, and the Marquardt algorithm was used.

The empirical estimates, with standard errors in parentheses, are as follows:

a_0	= -6.4365	(1.021)
a_1	= .4184	(.1784)
a_2	= .4852	(.1076)
a_3	= .3711	(.1658)
a_4	= .8879	(.1620)
a_5	= .5406	(.1661)
a_6	= 1.0544	(.1089)
b	= .3611	(.1018)

R^2 was .987. All parameter estimates had the expected sign and were significant at the 1% level of significance. These variables explain nearly all the spatial and temporal variability in land prices for these five districts over the data period. They also predict beyond the data period more accurately than models reported by other researchers. Although a very simple structure, this dual multiple-product model of land prices performed very well.

Sector Interdependencies and Capacities

The approach taken to capturing sector interdependencies and engineering capacities in this study involved the use of various configurations of input-output models. This section presents the theoretical framework of the input-output models developed in this study and the research results from their application in the United States and Israel as called for by Objectives c and d described earlier in this report.

Theoretical Framework of Input-Output Modeling

Two similiar yet separate approaches were taken in developing input-output models for the U.S. and Israeli economies. One is the development of an aggregative quadratic input model designed to measure the effects of sector capacities on potential GNP. The other is the development of a multi-regional quadratic input-output model. A related effort was also undertaken to project input-output technical coefficients beyond currently available published coefficients.

Framework of the Q10 Model. To meet Objectives c and d in this project, a Quadratic Input-Output (QIO) model was constructed for the U.S. and Israel economies. The essence of this model (for details see Harrington; Penson and Fulton; Penson, Talpaz and Foster) is presented below.

The objective of the QIO model is to maximize the society's welfare by maximizing total consumer surplus plus producer surplus in the economy. Formally stated, the model's objective is as follows:

$$(10) \quad \text{Maximize } Z = C'Q + 1/2Q'DQ$$

subject to

$$(11) \quad SQ \leq 0$$

$$(12) \quad 0 \leq Q \leq B$$

where

$$(13) \quad Q' = [R' : X' : Y']$$

$$(14) \quad C' = [-C_v : 0 : C_u']$$

$$(15) \quad B' = [B_R' : B_X' : 0]$$

$$(16) \quad D = \left| \begin{array}{c|c|c} -G^{-1} & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & F^{-1} \end{array} \right|$$

$$(17) \quad S = \left| \begin{array}{c|c|c} -I_m & A_r & 0 \\ \hline 0 & A_x - I_n & I_n \end{array} \right|$$

and where Q is a $(m+2nx1)$ matrix of quantities; C is a $(m+2nx1)$ matrix of price intercepts for the inverse primary input supply and final demand functions; D is a $(m+2n \times m+2n)$ negative or negative semi-definite matrix of slope terms for the inverse primary input supply and final demand functions (assumed to be linear) accounting for own and cross-price

slopes; S is a $(m+n \times m+2n)$ matrix of coefficients for the fixed proportion production functions; R is a $(m \times 1)$ matrix of the quantity of primary inputs supplied; X is a $(n \times 1)$ matrix of total output of each production sector; Y is a $(n \times 1)$ matrix of the final demand, $-C_v$ is a $(m \times 1)$ negative matrix of price intercepts for the inverse primary input supply functions; C_u is a $(n \times 1)$ matrix of the price intercepts of the inverse final demand functions; $-G^{-1}$ is a $(m \times m)$ matrix of the negative slope terms in the inverse primary input supply functions; F^{-1} is a $(n \times n)$ matrix of the slope and cross price terms in the inverse final demand functions; I_m and I_n are $(m \times m)$ and $(n \times n)$ identity matrices; A_p is a $(m \times n)$ matrix of technical coefficients for the primary inputs and A_r is a $(n \times n)$ matrix of technical coefficients for intermediate products, representing the Leontief relationships.

Solving the above equations under deterministic conditions is equivalent to maximizing producer plus consumer surplus provided the integrability conditions of Slutsky's theorem hold (Samuelson, 1952). Takayama and Judge showed that if D is nonsymmetric, then $\tilde{D} = (D + D')/2$ may substitute for D in equation (10) leaving the solution unchanged while slightly modifying the interpretation. The estimation of D and the details of constructing the model are given in Penson, Talpaz and Foster. Two such models, one for the U.S. and one for Israel, have been constructed and will be discussed later.

Framework of the MRQIO model. The QIO model allows for analysis on the national level. As such, it ignores regional considerations such as regional differences in technology due to environmental effects and geographical effects, transactions and trade between regions, and transfers

of primary inputs from region to another.

A Multiregional Quadratic Input-Output (MRQIO) model provides a systematic framework for describing and analyzing not only the sales and purchases of all industries in the economy but also the shipments among the regions for each industry and the welfare effects of such trades.

To implement the system, a Chenery-Moses fixed coefficient input-output model has been used for Israel only since the U.S. tablets are too old (1963) and we felt that the methodology could be developed for one country and be applied for the other when data becomes available. The MRIO fixed coefficients model is available in details in Polenska; hence, only the basic set of equations are presented here for n_r regions and r_i industries.

$$(18) \quad \bar{X} = \bar{T}(\bar{A}\bar{X} + \bar{Y})$$

$$(\bar{I} - \bar{T}\bar{A})\bar{X} = \bar{T}\bar{Y}$$

$$-\bar{X} = (\bar{I} - \bar{T}\bar{A})^{-1}\bar{T}\bar{Y}$$

$$\bar{X} = (\bar{T}^{-1} - \bar{A})^{-1}\bar{Y}$$

where

$\bar{A} = n_r n_i \times n_r n_i$ matrix of regional technical coefficients with each of the n_r blocks being located along the main diagonal and each block containing technical coefficients of the n_i industries for a specific region. Elements in all blocks off the main diagonal are zero.

$\bar{T} = n_r n_i \times n_r n_i$ matrix with each of the diagonals of the $n_r \times n_i$ blocks containing the trade coefficients and all off-diagonal

elements in the $n_r \times n_i$ blocks being equal to zero.

$\bar{Y}$ = vector of $n_r n_i \times 1$, of final demand, with each element representing the amount of output that industry i must produce and sell directly to final users in region r .

$\bar{X}$ = vector, $n_r n_i$, of total output.

$\bar{I}$ = $n_r n_i \times n_r n_i$ identify matrix.

Then substitute $\bar{A}$, $\bar{X}$, and $\bar{Y}$ of (18) into equation (10) through (17), respectively for A_x , X and Y . Likewise, $\bar{G}^{-1}$ and $\bar{F}^{-1}$ substitute for G^{-1} and F^{-1} in (16), where $\bar{G}^{-1}$ is an $n_r m \times n_r m$ matrix of the slope terms in the inverse primary input supply functions; and $\bar{F}^{-1}$ is an $n_r n_i \times n_r n_i$ matrix of regional slopes of the own and cross-price effects in the inverse final demand functions. $\bar{A}_r$ is $m \times n_r n_i$ matrix of technical coefficients of each industry at each region requirements of the primary inputs. It was assumed that in Israel these primary inputs were available nationally rather than regionally. This constitutes the MRQIO model.

In the process of solving a MRQIO model for the Israeli economy, output multipliers had to be constructed. These are given by the following equations:

$$(19) \quad \Omega_1 = (\bar{I} - \bar{T}\bar{A})^{-1}$$

and

$$(20) \quad \Omega_2 = (\bar{I} - \bar{T}\bar{A})^{-1}\bar{T}$$

where Ω_1 represents the undistributed matrix of multipliers, namely it measures the total increase in output for each industry at each region where the output is produced at the particular region while the inputs are allowed to come from other regions for an increase of final demand by a 1 IS (IS = Israeli shekel, approximately \$1.00 = 50 IS).

Ω_2 is like Ω_1 but the output is redistributed among the industries in the different regions (for a detailed discussion, see Richardson). Ω_2 is a matrix of multipliers which expresses the dependence of each industry on the rest of the economy on its regions. These multipliers were computed for the "open" as well as for the "closed" model. In the "closed" model, the Import and Household sectors are endogenized; hence, they become a part of the output transactions. This allows for the analysis of changes in private consumption, in personal wages, etc.

Data Availability and Updating Methodology. The technical coefficients derived from published input-output tables represent a major data requirement in input-output modeling efforts. Since published tables often lag 5-10 years behind the current year, input-output models relying solely on published technical coefficients are of little or no benefit in current analysis of macroeconomic issues, particularly during times when the economy is undergoing substantial structural changes.

One approach to alleviating this problem is to project these technical coefficients beyond available data using a particular updating technique. A review of the leading updating technique revealed their following features.³

The *RAS* or *Biproportional Method* developed by Professor Stone and his colleagues at Cambridge University during the early sixties is per-

haps the best known and most widely used technique for revising or projecting input-output relationships. Three factors contribute to the method's popularity: (1) its simplicity and preservation of the signs on the technical coefficients, (2) its minimum data requirement (i.e., only those marginal totals associated with intermediate outputs and inputs for the projection year are required), and (3) its flexibility (i.e., one may also incorporate *a priori* information when using this approach to updating coefficients in an input-output model).

The RAS method is based on the hypothesis that all the factors which affect the change in the technical coefficients may be summarized by a biproportional relationship in which each industry is characterized by a "substitution" multiplier and a "fabrication" multiplier which are assumed to operate uniformly over the rows and columns of the A matrix. Stone explained that a value of the substitution multiplier greater than (smaller than) one indicates that the outputs of the i^{th} industry have been substituted for (replaced by) the outputs of other industries as intermediate inputs into industrial processes. Similarly, a value of the fabrication multiplier greater than (smaller than) one suggests that the output of the j^{th} industry absorbs a greater (smaller) ratio of materials fuels, and other current inputs to value added in their production.

The RAS method can be modified to incorporate exogenous information into selected rows, columns, or individual cells of the updated matrix. This simply involves setting specific cells equal to zero and then applying the normal RAS procedure to the modified matrix while the exogenous values of these cells are subtracted from the marginal totals of the forecast year.

Allen and Lecomber examined the performance of this procedure under a variety of assumptions about the origin of the exogenous information, most notably: (1) exogenous estimates of key coefficients derived from actual census data and (2) exogenous pre-census estimates of key coefficients.

The experiment of Malizie and Bond for the state of Washington suggested that the RAS method, used without additional exogenous information on interindustry flows from the projection year, is not powerful enough to generate satisfactory forecasts.

Lecomber has shown that RAS method should be limited to situations where only one input-output transactions table exists. When two or more tables are available, other methods which exploit the additional information are more viable. For example, in estimating a 1971 transactions table, a method which utilizes information from both the 1963 and 1967 tables would be superior to the RAS method that is based upon only one later table.

Two major criticisms cited most often in the literature about the RAS method are that: (1) the substitution multiplier and fabrication multiplier operate uniformly along rows and columns without appealing to economic considerations and (2) any zero elements in the original transactions table cannot be adjusted to be positive, and vice-versa. These criticisms stem mainly from the fact that the RAS method was essentially devised as an operational technique only. Another shortcoming of the RAS method is that the values of the intermediate output (u^*) and input (v^*) for the projection year, which are indispensable in RAS updating, are unlikely to be available. In cases like this, the RAS is useless unless

efforts are made to project u^* and v^* .

Under the RAS method, the updating of the technical coefficients is accomplished through an iterative procedure which minimizes the differences in totals of intermediate input and output between the projected transactions table and the actual transactions table. Under the *Linear Programming Method*, this minimization process through iteration is replaced by the minimization of the objective function, which is the absolute value of the difference between the ratio of the projected and actual technical coefficients and the number one, and the constraints on the totals of intermediate input and output.

The Linear Programming or LP method is very similar to the RAS method in the sense that both methods use the totals of the intermediate inputs and outputs of the projected year and a chosen base-year transactions table to update the technical coefficients. The RAS method provides an easier approach to calculate the updated coefficients. The LP method, however, avoids the fabrication and substitution multipliers in the RAS method and the possibility that 0 elements remain unchanged. Both of these features have been the focus of most criticisms of the RAS method.

To solve the updating problem, Chavas and Chow proposed the *Disequilibrium Method*, which transforms the basic prediction equation $x = (I-A)^{-1}f$ into a regression problem and uses a dynamic-adjustment disequilibrium theory to explain the error term. Chavas and Chow empirically compared the relative efficiency of RAS method and disequilibrium method in updating technical coefficients matrices. They reported that the RAS method and the Disequilibrium method have the same predictability in terms of mean and variance of prediction error by the ANOVA test. They

concluded that, by providing the interpretation in the context of dynamic adjustment in the disequilibrium situation, the disequilibrium method represents an alternative technique for updating technical coefficient matrices that preserves all desired properties and relaxes the rigid assumptions of the RAS method.

By adding a stochastic $n \times 1$ vector ϵ on the right-hand side of the fundamental input-output accounting identity, the resulting equation, $x = Ax + f + \epsilon$, represents the regression equation used in the *Simultaneous Equation Method*. This stochastic term can be explained as the measurement errors that are usually observed between the actual values of x and $Ax + f$. The value of ϵ has the following properties: $E(\epsilon) = 0$ and $\text{Var}(\epsilon) = \Sigma$ is a positive-definite matrix.

With the exception of the constraints on the estimated coefficients model, the SE method is very similar to the Disequilibrium method proposed by Chavas and Chow in the sense that both are based on the fundamental relationship $x = Ax + f$. The major differences are that: (1) in order to form their regression model, the SE method applies this relationship directly by adding a measurement error term as $x = Ax + f + \epsilon$, while the Disequilibrium method transforms this relationship to $x = (I-A)^{-1}f$ and adds a "dynamic adjustment" error term ϵ to $x = (I-A)^{-1}f + \epsilon$, and (2) the constraint on primary inputs in the Disequilibrium method is included in the regression equation as part of the updating model.

The *INFORUM Macroeconomic Forecasting Model* was developed by Almon and his colleagues at the University of Maryland. Since its introduction, it has been applied to the economies of the United States, Japan, and

many European countries. INFORUM stands for the project of Interindustry Forecast at the University of Maryland.

In the INFORUM model, the technical coefficients are updated using logistic curves. Almon assumed that all coefficients in a row of the A matrix follow the same logistic curve, percentage-wise. This logistic curve reflects the general trends of the input-output technical coefficients. Logistic curves can appear in many forms as Almon found, but all of them eventually approach a floor or ceiling as asymptotes. Coefficients in a row with individual changes are skipped by the across-the-row operation.

Almon found in his American experiment that, for most producers such as steel which flow largely to intermediate uses, the fit is quite good. For products such as cleaning and toilet products which go chiefly to final demand, the estimate had a wide margin of error. Thus, it is not surprising in the later case that the fit was not as good.

Under the INFORUM method, instead of using some relevant economic variables, a simple assumption of a logistic curve with its asymptote and adjustments to its speed are used to capture the future trend of the changing technical coefficients. Compared with other updating techniques, there are three shortcomings. First, no information in the forecast year is used in the updating process except the year itself. In the RAS method, for example, the total intermediate inputs and outputs of the forecast year are used to capture the latest information. Second, all coefficients in the same row of the A matrix are assumed to follow the same logistic curve. This assumption is similar to the substitution multiplier in the RAS method. Third, no specific way is proposed as to how

to choose the base period. The evidence indicates that the later the year chosen, the better the results obtained. Of course, more empirical work is necessary to substantiate such a conclusion.

Considering the above three shortcomings, the INFORUM method is regarded as a useful updating technique only when there is no drastic change in the economic environment during the period for which one is forecasting. This is because changes in economic conditions are not taken into account. In light of these shortcomings, a new updating method was proposed and applied in this study.

According to production theory, the optimal usage of any input in a production process can be expressed as a function of the quantity of output and the prices of all inputs. That is,

$$(21) \quad x_{ij}^* = x_{ij}(p, y_j)$$

where x_{ij}^* is the optimum quantity of the i^{th} input used in the production of the j^{th} output, p is the vector of input prices, i.e., $p' = (p_1, p_2, \dots, p_m)$, m inputs and y_j is the quantity of the j^{th} output, $i = 1, 2, \dots, m$, and $j = 1, 2, \dots, n$. This price vector suggests that a change in input prices will effect the usage of intermediate goods and services.

All the updating methods discussed above do not incorporate input prices in their updating processes. They instead rely upon the various properties of the time-series data for the technical coefficients together with some other minor assumptions.

Recent developments provide a theoretical background for directly incorporating relative input prices in a regression model which explains

changes in technical coefficients over time. This method, which we shall refer to as the *RP Method*, can be stated as follows:

The fundamental principle of duality in production theory suggests that the cost function summarizes all economically relevant aspects of production technology just as all the economically relevant aspects of production. In other words, an optimized cost function $c(p, y_j)$ contains the same information about technology as the production function $y_j = f(x)$, where p is the vector of input prices, y_j is the level of the j^{th} output, and x is the vector of input quantities. If production function f is homothetic (i.e., $y_j = f(x) = h[g(x)]$, where $g(x)$ is linearly homogeneous and $h' > 0$), then the above cost function c can be expressed as follows:

$$(22) \quad c(p, y_j) = h(y_j) \cdot \sum_i \sum_j a_{ij} p_i^{1/2} p_j^{1/2}$$

where $h(y_j)$ is a monotonically increasing function of y_j ($h(0) = 0$), and a_{ij} represent the parameters ($a_{ij} = a_{ji} \geq 0$). This represents the generalized Leontief cost function derived by Diewert in 1971.⁴

Application of Shepherd's lemma shows that the cost-minimizing input demand functions are equal to the partial derivatives of the cost function with respect to input prices. Stated another way,

$$(23) \quad x_{ij}(p, y_j) = \frac{\partial c}{\partial p_i} = h(y_j) \sum_j a_{ij} p_j^{1/2} p_i^{-1/2}$$

$$= h(y_j) \sum_j a_{ij} (p_j/p_i)^{1/2}$$

where $x_{ij}(p, y_j)$ is called the conditional factor demand equation.

To derive the technical coefficients, one must assume that $h(y_j)$ is homogeneous of degree 1 in y_j (i.e., let $h(y_j) = y_j$). Doing so, equation (23) can be rewritten as follows:

$$(24) \quad x_{ij}(p, y_j) = y_j \sum_j a_{ij} (p_j/p_i)^{1/2}$$

which means that

$$(25) \quad a_{ij} = \frac{x_{ij}}{y_j} = \sum_j a_{ij} (p_j/p_i)$$

where a_{ij} is the technical coefficient for the i^{th} input in the production of the j^{th} product. This specification suggests that the ij^{th} technical coefficient is the linear regression function of relative prices P_j/P_i .

An apparent advantage of using the method proposed in this study is its simple data requirements and its ease of calculation. The RAS method is easy to calculate, but it requires additional effort to obtain the necessary data on projected totals of intermediate input and output.

Unlike the Disequilibrium method and the Simultaneous Equation method, the RP method proposed in this study does not require the use of data on projected final demands. The projection of final demands for the products produced in different sectors of the economy, may well take hundreds of estimated equations in disaggregated input-output models.

As mentioned earlier, a shortcoming of the Almon Method is that a simple logistic curve does not contain much economic information and thus

cannot handle the case when drastic changes in economic conditions occur. The RP method, on the other hand, uses the relative input prices to update technical coefficients. Thus, this method incorporates information about changing economic conditions in the economy.

Applications in Israel

The application of the input-output models developed in this study to the Israeli economy includes a multi-regional input-output (MRIO) model, a quadratic input-output (QIO) model, and a multi-regional quadratic input-output (MRQIO) model. Each of these applications are discussed below.

The MRIO Model for Israel. Let us consider the MRIO model first.⁵ The Israeli economy was divided in this model into three regions: Negev-Arava (Prairie), Negev-Har (Mountain), and the Rest (i.e., the rest of Israel). The first region includes the eastern portion of Israel's south in the valley. The second is the western portion of Israel's south in the Negev Heights, and the third is the northern-half of the country. The main reason for this geographic division was that the Negev is the least-developed area. It is expected that a great deal of public and private development will occur in the Negev in the future. A major step in that development is already nearly complete, i.e., the redeployment of Israel's defense forces in the Negev following the Sinai evacuation as part of the 1979 Egypt-Israel peace treaty. This division is supported in terms of data by the Central Bureau of Statistics (CBS). A 4th "Region" was formed to account for the foreign trade, called "World", as Israel is heavily involved in the Imports-Exports Market. Some 50% of its GNP are

exported goods and services and 75% of it are imported.

The analysis was carried in two levels of sector aggregations. A 40-sectors level ($n_i=40$) was used for the MRIO output multiplier analysis. A 16-sector aggregation level was used for the MRQIO model.

The construction of the MRIO table was a multistage process, detailed in Freeman, et al., pp. C-1 through C-17. Hence, only a brief summary of this process follows.

The basis for these input-output tables was the *Input-Output Tables 1975-76, Special Publication No. 625*, CBS, Jerusalem 1980. This included the 40 sectors tables and four primary inputs: Imports, Labor (Housholds), Taxes and Subsidies, and Other Value Added. Using total outputs and final demand data (CBS Abstracts, 1979), which was available from the 1977-78 input-output tables, were updated via the RAS method (Bachrach). The strategy for building the MRIO tables was to obtain sectors outputs from the two Negev region and to subtract it from the national totals to obtain the outputs for the Rest (of Israel) region. It was decided to survey the following industries in the Negev region: Field crops, livestock, citrus, other agriculture, mining and quarrying, processed food, chemical and medicine, glass, ceramics and cement. To complement these data sources, we have used annual financial reports for public utilities and transportation firms. When a region's total sector output has been estimated, it was subtracted from the national output and the final tables were adjusted using the RAS technique.

The trade matrices, $\tilde{T}$, were calculated based on coefficients obtained in the above mentioned survey, regional output and total regional demand. Basically two computational methods were used: (1) coef-

ficients of trade shares obtained from the survey were sufficiently reliable and (2) the trade flows computed by the "go-between" process which basically assumes that producers are marketing their products within their region first. Surpluses are "sent" to a "go-between inventory" which are then forwarded to other regions according to their prespecified demand. If deficits occur, then the region receives it from that "inventory".

The output multiplier analysis reported in Freeman, et al. was based on these MRIO tables. Recently, the CBS Input-Output Division published the final national input-output tables for 1977-78. It was concluded that although no substantially marked differences were found between our RAS adjusted national input-output tables and CBS's, the multiplier analysis should be recomputed. This has just been completed but has not been published yet. As an example, the direct and indirect output multipliers for each sector in each region for an increase of IS1 in the demand of all sectors in Region 2 is given in Table 8. This table shows the overall effects for the sectors in all regions. The rest of these results and their interpretation can be found in Freeman, et al. The overall mean of the output multipliers was 3.03 where agricultural industries were above it by about 0.5, excluding the citrus industry which was 2.7. Somewhat surprising was the finding that the regional differences for the total multipliers were small and insignificant. It is contributed to the specialization of the industries on regional basis, e.g., citrus is grown only in the "Rest" region while mining and ceramics is in the Negev-Har. For the nonspecialized sectors, it means that technology is more or less uniformly leveled for all regions. However, the individual sector-

Table 8. Multiregional undistributed output multipliers 1, resulting from an increase demand of 1 for each branch in region ARAVA ISRAEL 1977/8.*

BRANCH	REST	ARAVA	NEGEV M.	WORLD	TOTAL
1. Field Crops	1.479	1.576	0.008	0.445	3.507
2. Livestock	1.779	1.598	0.034	0.409	3.820
3. Citrus	0.850	1.551	0.005	0.316	2.722
4. Other Agriculture	1.265	1.581	0.008	0.310	3.163
5. Mining and Quarrying	1.378	1.855	0.010	0.394	3.636
6. Meat, Fish, Oil & Dairy Products	1.970	1.830	0.023	0.397	4.221
7. Other Food Products	1.752	1.747	0.024	0.456	3.979
8. Textiles	1.316	1.398	0.039	0.486	3.239
9. Clothing	1.220	1.346	0.050	0.437	3.052
10. Leather & Leather Products	1.329	1.433	0.010	0.523	3.296
11. Wood & Carpentry	1.053	1.560	0.007	0.520	3.139
12. Paper & Products	0.775	1.342	0.004	0.628	2.749
13. Printing & Publishing	1.276	1.603	0.007	0.422	3.308
14. Rubber & Plastic	0.835	1.400	0.011	0.467	2.712
15. Chemicals & Medicine	0.856	1.417	0.006	0.489	2.768
16. Petroleum	0.121	1.065	0.001	0.925	2.112
17. Glass, Ceramics, Cement & Their Products	0.988	1.645	0.013	0.442	3.088
18. Basic Metal & Products	0.924	1.413	0.006	0.504	2.847
19. Machinery	1.079	1.533	0.007	0.486	3.105
20. Electrical & Electrical Equipment	0.561	1.502	0.005	0.426	2.495
21. Manufacture & Repair of Transport Equip.	0.951	1.782	0.008	0.648	3.389
22. Diamonds	0.130	1.110	0.001	0.743	1.983
23. Miscellaneous Manufacture	0.894	1.542	0.011	0.525	2.971
24. Precision and Optical Instruments	1.302	1.607	0.009	0.528	3.446
25. Residential Building	1.242	1.708	0.016	0.392	3.358
26. Other Building	1.309	1.745	0.015	0.446	3.515
27. Electricity	1.080	1.378	0.004	0.647	3.108
28. Water	1.041	1.912	0.006	0.458	3.416
29. Buses and Taxicabs	0.736	1.641	0.006	0.281	2.664
30. Trucks and Railway	0.889	1.710	0.006	0.366	2.971
31. Sea Shipping Transport	0.308	1.239	0.002	0.682	2.232
32. Air Transport	0.639	1.602	0.005	0.670	2.916
33. Warehouses and Other Transport	0.917	1.843	0.007	0.300	3.067
34. Communication	0.862	1.804	0.007	0.359	3.032
35. Food Services and Hotels	1.394	1.832	0.014	0.376	3.616
36. Finance, Insurance & Legal Services	0.684	1.609	0.005	0.219	2.517
37. Garages, Other Repairs & Maintenance	0.755	1.591	0.005	0.559	2.910
38. Domestic Service, Barbers & Laundries	1.153	2.121	0.010	0.417	3.700
39. Trade	0.879	1.733	0.007	0.248	2.867
40. General Expenditures	2.862	1.799	0.015	0.465	5.141
41. Import	0.000	1.000	0.000	0.000	1.000
42. Households	1.336	1.502	0.013	0.431	3.282
TOTAL	44.166	66.202	0.451	19.237	130.056
MEAN	1.05	1.58	0.01	0.46	3.10

*Source: Freeman et al P. D-9.

regional multipliers show significant differences among the regions which account for the environmental and geographic differences. As an example, Table 8 exhibits the Ω_1 output MRIO multipliers for an increase in demand of IS1 for each sector in the Negev-Har region. Table 9 exhibits the Ω_2 output MRIO multipliers for an increase demand of IS1 for each sector in the region Negev-Har. A summary of the regional output multipliers is given in Table 10.

It seems that while the arithmetic means of the total multipliers of the two types are nearly identical, the regional components of their effects differ sharply among the regions which means that regional developments will have a substantially different economic effect for the same amount of investment. This will depend on the particular region under development.

Two case studies have been carried out using the Israeli MRIO model.

Case I / Evaluation of alternative development plans for the two Negev regions. It is beyond the scope of this report to discuss this study. Chapter E in Freeman, et al. holds the details. The usefulness of the MRIO model was demonstrated in evaluating the impact of the alternative plans on other industries in all regions including imports and exports.

Case II / Evaluation of alternative scenarios related to the trade coefficient matrix $\bar{T}$. For example, the impact of increasing/decreasing of trade among the regions was evaluated, revealing the high sensitivity of this aspect on the output multipliers. This evaluation which was completed rather recently will be published in the near future by Freeman and Talpaz.

These two studies were beyond the original objectives and have been requested by the Israeli settlement authorities and funded by them. Additional funding for other applications have been offered to the group to further study development projects. To name just one: Impact of the Medi-

Table 9. Multiregional distributed output multipliers 2, resulting from an increase demand of I 1 for each branch in region ARAVA. ISRAEL 1977/8.*

BRANCH	REST	ARAVA	NEGEV M.	WORLD	TOTAL
1. Field Crops	2.897	0.005	0.027	0.351	3.280
2. Livestock	2.363	0.963	0.030	0.417	3.772
3. Citrus	2.382	0.003	0.006	0.316	2.707
4. Other Agriculture	3.116	0.021	0.020	0.368	3.525
5. Mining and Quarrying	1.385	1.838	0.010	0.393	3.626
6. Meat, Fish, Oil & Dairy Products	3.246	0.522	0.019	0.479	4.267
7. Other Food Products	2.328	0.569	0.058	0.525	3.480
8. Textiles	2.587	0.004	0.152	0.487	3.230
9. Clothing	2.550	0.003	0.053	0.438	3.044
10. Leather & Leather Products	2.740	0.004	0.015	0.525	3.284
11. Wood & Carpentry	2.273	0.329	0.008	0.521	3.130
12. Paper & Products	2.104	0.002	0.004	0.628	2.739
13. Printing & Publishing	2.539	0.324	0.008	0.421	3.292
14. Rubber & Plastic	2.217	0.003	0.012	0.467	2.699
15. Chemicals & Medicine	2.223	0.003	0.010	0.489	2.725
16. Petroleum	1.179	0.001	0.001	0.925	2.106
17. Glass, Ceramic, Cement & Their Products	2.484	0.016	0.108	0.449	3.056
18. Basic Metal & Products	2.318	0.006	0.013	0.505	2.841
19. Machinery	2.600	1.006	0.011	0.488	3.105
20. Electrical & Electrical Equipment	1.138	1.072	0.007	0.454	2.671
21. Manufacture & Repair of Transport Equip.	0.951	1.782	0.008	0.648	3.389
22. Diamonds	1.236	0.001	0.001	0.743	1.981
23. Miscellaneous Manufacture	1.015	1.419	0.011	0.525	2.970
24. Precision and Optical Instruments	2.892	0.006	0.010	0.530	3.437
25. Residential Building	1.242	1.708	0.016	0.392	3.358
26. Other Building	1.309	1.745	0.015	0.446	3.515
27. Electricity	1.292	1.165	0.004	0.646	3.107
28. Water	1.426	1.524	0.006	0.457	3.413
29. Buses and Taxicabs	1.249	1.122	0.006	0.281	2.658
30. Trucks and Railway	0.889	1.710	0.006	0.366	2.971
31. Sea Shipping Transport	1.432	0.110	0.003	0.683	2.227
32. Air Transport	1.662	0.568	0.006	0.671	2.906
33. Warehouses and Other Transport	2.288	0.453	0.008	0.300	3.049
34. Communication	1.857	0.803	0.008	0.360	3.029
35. Food Services and Hotels	2.619	0.033	0.011	0.269	2.933
36. Finance, Insurance & Legal Services	1.842	0.436	0.006	0.218	2.502
37. Garages, Other Repairs & Maintenance	0.755	1.591	0.003	0.559	2.910
38. Domestic Service, Barbers & Laundaries	1.738	1.319	0.010	0.366	3.433
39. Trade	1.422	1.181	0.007	0.247	2.858
40. General Expenditures	2.862	1.799	0.015	0.465	5.141
41. Import	0.000	0.000	0.000	1.000	1.000
42. Households	1.336	1.502	0.013	0.431	3.282
TOTAL	79.984	27.668	0.749	20.247	128.648
MEAN	1.90	0.66	0.02	0.48	3.06

*Source: Freeman et al P. D-6.

Table 10. Arithmetic Means of the MRIO Multipliers*

REGION	Arithmetic Means of Output Multipliers									
	Undistributed (I - TA)					Distributed (I - TA) T				
	Rest	Negev Arava	Negev Har	World	Total	Rest	Negev Arava	Negev Har	World	Total
"Rest"	2.55	0.00	0.01	0.46	3.02	2.51	0.01	0.02	0.48	3.03
Negev-Arava	1.05	1.58	0.01	0.46	3.10	1.90	0.66	0.02	0.48	3.06
Negev-Har	1.11	0.00	1.47	0.45	3.03	2.12	0.01	0.41	0.48	3.03
World	0.00	0.00	0.00	1.00	1.00	1.72	0.02	0.07	0.64	2.44

* Source: Freeman et al p. D-17.

terranean Sea-Dead Sea Channel on related industries for each region. This study will start this fall.

The QIO Model for Israel. The major stumbling block in building a QIO model is the estimation of the D matrix as required in equation (1) and defined by equation (7). Time-series data for sector aggregated quantities consumed and their respective prices are needed. Furthermore, ideally D has to be estimated as a set of simultaneous equations with minimum degrees of freedom. This translates into a requirement for a long time series. Faced with data deficiencies as related to many industries in the economy, we were forced to give up the 40-sectors aggregation level as used in the MRIO model and to reaggregate them into a 16-sectors level, where some of the agricultural sectors have been disaggregated to provide greater details in the analysis. Based on these considerations the 16-sectors are now defined in terms of their total outputs X_i , as

- X_1 = fruit (other than citrus)
- X_2 = citrus
- X_3 = field crops
- X_4 = cattle and dairy
- X_5 = fish
- X_6 = poultry
- X_7 = other agriculture branches
- X_8 = food products beverages and tobacco
- X_9 = electricity and fuel
- X_{10} = clothing, shoes and personal belongings
- X_{11} = furniture and house equipment
- X_{12} = cars

X_{13} = other industrial products

X_{14} = housing and construction

X_{15} = fixed services

X_{16} = measured services

From equation (16), we need the elements of G^{-1} which are the slopes of the primary inputs supply curves. These inputs have been aggregated into:

R_1 = labor (or households)

R_2 = tax subsidies and other value added

R_3 = imports

The slopes of R_1 and R_2 have been obtained by assuming a unitary elasticity of supply. This assumption follows previous works (Penson and Fulton; Talpaz, Penson and Foster) and is a result of a lack of time-series data needed to obtain these slopes. Then the slopes S_i , $i=1, 2$ are computed from the definition of elasticity, by

$$(26) \quad S_i = \eta_i q_i / P_i, \quad i=1, 2$$

where

η_i = supply elasticity of input: (=1.0, by assumption)

q_i = quantity of input i (CBS, for 1977-78)

P_i = price index of input i (=100 by convention for the base year).

Note that price indices are necessary since q_i is an aggregated input.

The slopes of the inverse supply functions are given by

$$(27) \quad \tilde{S}_i = 1/S_i.$$

The import slope, $\tilde{S}_3$, was assumed infinitely elastic, i.e., $\tilde{S}_3 = 0$, and

$$(28) \quad G^{-1} = \text{diag}(\tilde{S}_1, \tilde{S}_2, \tilde{S}_3).$$

which follows from the assumption of no cross-effects.

We have been forced to rely on published elasticity and cross-elasticity estimates (Meital) as related to these sectors due to an incomplete time-series data set for sectors X_8 through X_{16} .

Data was available for the seven (X_i , $i=1, \dots, 7$) agricultural sectors, hence, the strategy was to obtain estimates for elasticity and cross elasticity of demand for these sectors. The sources of these data included *Price Statistics Monthly*, 1968 through 1978 published of CBS and *Monthly Bulletin of Statistics*, 1968 through 1978. The monthly data was combined to quarterly data to reduce the noise and errors of measurements and to correspond to the nonavailability of some of the monthly price indices.

A set of simultaneous demand functions much like the Rotterdam Model (see Theil, Barten or Philips) were formulated and estimated using the Three-Stage Least-Square Method. These estimates were obtained subject to symmetry, homogeneity and negativity constraints. Then the slopes were computed through the definition of elasticity and cross elasticity and prices and quantities (per capita) which existed in 1978. Finally, an iterative process, similar to that employed by Talpaz, Shumway and Alexander, was carried to ensure that F^{-1} is negative definite an assumption needed for welfare maximization. The resulting matrix of slopes, F^{-1} , is given in Table 11. Details of the procedures and measurements of statis-

Table 11. Slope Coefficients of the Demand for Israel Sector S Final Demands

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
D1	-0.00017000	0.00000060	0.00001	0.000150	-0.000120	-0.00015	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D2	0.00000060	-0.00001300	0.00001	-0.000006	0.000003	0.00001	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D3	0.00001000	0.00001000	-0.00150	-0.000000	0.005000	0.01770	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D4	0.00015000	-0.00000600	-0.00800	-0.000000	0.020680	0.10050	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D5	-0.00012000	0.00000500	0.00500	0.020680	-0.017200	-0.06030	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D6	-0.00015000	0.00000700	0.01770	0.100500	-0.060300	-0.21160	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D7	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	0.000000	0.00000	0.00000	0.000	0.00	0.0000	0.0000	0.00000	0.00000
D8	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.000022	-0.00008	-0.00008	-0.016	0.04	-0.0060	-0.0050	-0.00070	0.00000
D9	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.000080	-0.00590	-0.00300	-0.077	-0.15	-0.0300	0.0010	-0.00200	0.00000
D10	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.000600	-0.00300	-0.01960	-0.500	1.53	-0.1829	-0.2190	-0.02100	0.00077
D11	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.016500	-0.07700	-0.50020	-14.660	38.66	-4.8800	-5.5560	-0.57500	0.02990
D12	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	0.036000	-0.15000	1.53000	38.660	-231.83	11.9050	32.6800	1.10000	-0.16300
D13	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.006000	-0.03000	-0.18290	-4.880	11.94	-1.9630	-1.7227	-0.21300	0.01250
D14	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.005000	0.00100	-0.21900	-5.556	32.68	-1.7227	-4.6100	-0.16600	0.02300
D15	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	-0.000700	-0.00200	-0.02100	-0.575	1.14	-0.2134	-0.1660	-0.03320	0.00169
D16	0.00000000	0.00000000	0.00000	0.000000	0.000000	0.00000	0	0.000000	0.00000	0.00077	0.030	-0.16	0.0125	0.0230	0.00169	-0.00093

tical properties will appear in the upcoming M.S. Thesis (Yichia).

The basic solution is obtained if C , the vector of intercepts in equation (10) and defined in equation (14), satisfies the following relationship:

$$(29) \quad \partial Z / \partial Q = C + DQ = P$$

where P is the equilibrium price index vector (with all elements = 100). The basic solution for equation (10) subject to equations (11) through (17)⁶ was found using the QP solver developed by Harpaz and Talpaz with the financial support of this project and supplemented by the Texas Agricultural Experiment Station. This basic solution is given in Table 12.

The QIO model for Israel was run under different additional tax per unit labor. This is equivalent to an upward parallel shift of the labor supply curve. Since the initial intercept of the labor supply curve was then increased, labor-tax will be defined through a new intercept ΔC_L . Table 13 shows the results of the QIO solutions under different labor supply intercepts under *ceteris paribus* conditions. Space limitation prevents from showing the impact on the rest of the economy. That, however will be available in (Yichia). The effects of "taxing" labor is somewhat surprising at first glance. Increased labor costs lead to increased output for a few agricultural industries. If, on the other hand, substitution between agricultural and nonagricultural industries (both in terms of competition on the primary inputs and substitution at the consumption level) is accounted for along with the high technology capital-intensive low-labor agricultural processes, then all agricultural industries

Table 12. Equilibrium quantities of R, X, and Y for Israel, 1978.
All values are in IS 00,000,000's.

Index	R	X	Y	Total "Welfare" Z
1	108.87	1.9576	1.7064	3095.2
2	104.69	3.2382	2.9926	
3	64.161	5.2911	3.1004	
4		2.5738	.17426	
5		3.9361	1.3727	
6		.92488	.56924	
7		1.5826	1.2488	
8		24.175	16.958	
9		17.694	4.4542	
10		19.980	10.228	
11		12.057	4.2940	
12		17.683	8.3280	
13		45.071	29.649	
14		25.043	22.247	
15		54.816	48.060	
16		80.744	45.958	

Table 13. Equilibrium Agricultural branches output and final demands various labor's intercept supply curve shifts, under capacity constraints.

Labor's Supply Intercept	B1	B2	B3	X1	X2	X3	X4	X5	X6	X7	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Z
35	63.483	75.299	59.408	2.9364	4.8573	7.9366	.07855	.10395	.018538	.099431	2.9318	4.8530	7.4289	0	0	0	0	18.643
30	64.324	77.446	61.4	2.9364	4.8573	7.9366	.07857	.10397	.018813	.1008	2.9318	4.8530	7.4255	0	0	0	0	400.51
25	67.004	80.162	63.029	2.9364	4.8573	7.9366	.10287	.12871	.55372	.10647	2.9292	4.8505	7.3830	0	0	.50189	0	793.44
20	76.514	84.808	63.426	2.9364	4.8573	7.9366	.11781	.14393	.84499	.12452	2.9276	4.849	7.3374	0	0	.77286	0	1205.9
15	86.452	89.463	63.819	2.9364	4.8573	7.9366	.13248	.15886	1.1304	.14243	2.9261	4.8474	7.2924	0	0	1.0384	0	1641.5
10	100.75	94.236	64.140	2.9364	4.8573	7.9366	.86540	2.3182	1.0707	.16346	2.9060	4.8297	7.1050	.55944	1.8447	.96099	0	2100.7
5	107.98	99.453	64.161	2.3847	4.8573	7.3344	2.3052	2.4944	1.3873	.25332	2.1448	4.6224	5.3904	0	.11667	1.0357	0	2584.9

(except citrus) do contract at the very high "tax".

A somewhat different picture is depicted when specific labor subsidies or taxes, as translated into levels of labor *flexibilities*, are assumed. Table 14 shows this effect. The general trends are similar to those shown in Table 13. Some industries, like citrus, show extreme increase in output; yet, it should be remembered that no capacity constraints were imposed due to their inavailability, which makes these results unrealistic. Nonetheless, the importance of this analysis is in showing the trends and directions due to these kinds of policy changes.

Next, sensitivity to changes in import price was investigated. Results of equilibrium solutions with various import prices are depicted in Table 15.

The last set of sensitivity analysis was carried to study the effects of individual shifts of the demand curves for each sector on the entire economy. Table 16 shows the effect of shifting the intercept of each sectors' demand curve upward by 1% of the "basic" intercept on the rest of the sectors' outputs and final demand. Results for only the agricultural sectors are shown. The effects of each individual demand shifts should be considered carefully. Here, we can state only the more important and general effects.

The fruit industry is vulnerable - it significantly changed for nearly every sectors' demand shift. All the sectors expand, as expected, in response to their own demand increases. Note that these demand shifts are only the direct ones. The indirect shifts - through cross elasticities - may enforce or inhibit them. The Field Crops, Cattle and Dairy, and Fish are generally increasing their output to complement other sec-

Table 14. Equilibrium Agricultural output and final demands various labor supply curve slopes and under capacity constraints.

Labor Supply	R1	R2	R3	X1	X2	X3	X4	X5	X6	X7	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Z
Shops																		
.76417	135.46	122.56	68.343	0.37692	.36829	3.0387	3.8607	5.9041	1.3873	2.3739	0	0	0	.28774	2.1002	0.85921	1.9239	4360.8
.811939	134.73	120	65.884	.37692	.36826	3.0354	3.8607	5.9041	1.3023	2.3739	0	0	0	.28775	2.1002	.77888	1.9252	4009.6
.859701	128.29	115.75	64.515	.37692	.36826	3.0126	3.8607	5.9041	.74890	2.3739	0	0	0	.28780	2.1003	.25667	1.9369	3675.6
.907462	110.31	110.20	64.161	1.7801	.76830	2.5498	3.8607	5.9041	.73664	2.3739	1.5012	.49942	0	1.2418	3.0047	.35207	1.9879	3370.7
1.00	108.87	104.59	64.161	1.9576	3.2386	5.2911	2.5728	3.9350	.92472	1.5833	1.7063	2.9931	3.1009	1.7335	1.3718	.56909	1.2494	3095.3
1.60298	107.46	99.702	64.161	2.3814	4.8573	7.3678	2.3079	2.5063	1.3873	.25522	2.1411	4.6221	5.388	0	.12516	1.0331	0	2846.0
1.05075	101.63	95.181	64.147	2.9364	4.8573	7.9366	3.5443	5.6748	1.3873	.17618	2.8597	4.7923	6.5944	2.8676	4.6218	1.2204	0	2619.4
1.09851	92.209	91.284	63.951	2.9364	4.8573	7.9366	.13705	.16353	1.2179	.14879	2.9256	4.8420	7.2775	0	0	1.1196	0	2412.1
1.14627	82.597	87.806	63.682	2.9364	4.8573	7.9366	.12745	.15374	1.0329	.13616	2.9266	4.8480	7.308	0	0	.94765	0	2220.7

Table 15. Optimal solution under various import prices and branches capacity constraints.

Import Price	R1	R2	R3	X1	X2	X3	X4	X5	X6	X7	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Z
97	130.35	106.23	63.963	.37592	4.8573	2.9638	3.5780	3.6529	.47892	.26936	0	4.4890	0	0	0	0	0	3180.5
97.5	130.02	105.97	64.002	.37592	4.8573	2.9635	3.5780	3.6529	.47892	.26703	0	4.4890	0	0	0	0	0	3115.4
98	129.68	105.71	64.041	.37592	4.8573	2.9631	3.5779	3.6529	.47892	.26470	0	4.4890	0	0	0	0	0	3350.4
98.5	129.35	105.45	64.079	.37592	4.8573	2.9628	3.5779	3.6529	.47892	.26237	0	4.4890	0	0	0	0	0	3285.4
99	129.07	105.3	64.052	.37592	4.8573	2.9681	3.5780	3.6529	.61634	.26137	0	4.4890	0	0	0	.12981	0	3221.1
99.5	127.90	104.96	64.141	.37592	4.8573	2.9681	3.5780	3.6529	.87002	.25967	0	4.4890	0	0	0	.36952	0	3156.7
100.5	91.980	104.03	64.406	2.9364	.053263	.96373	3.8607	3.4305	1.3873	2.3739	2.8702	0	0	3.3202	2.6814	1.2409	2.0813	3047.9
101	88.432	103.09	64.839	2.9364	.067006	.99489	3.8607	5.9041	.90651	2.3739	2.8566	0	0	3.1878	4.8544	.77307	2.0025	3002.9
101.5	85.065	102.13	65.29	2.9364	.067231	.98991	3.8607	5.9041	.98512	2.3739	2.8564	0	0	3.1856	4.8524	.84795	2.0075	2959.5
102	81.684	101.17	65.734	1.0166	.067472	.98567	3.8607	5.9041	1.0660	2.3739	.93639	0	0	3.1803	4.8579	.92585	2.0924	2917.8
102.5	78.311	100.22	66.182	.080481	.067705	.98105	3.8607	5.9041	1.1457	2.3739	0	0	0	3.1884	4.8596	1.0022	2.0473	2877.0
103	74.943	99.252	66.633	.080701	.06793	.97607	3.8607	5.9041	1.2242	2.3739	0	0	0	3.1863	4.8576	1.0770	2.1024	2839.5

Table 16. Optimal solution with the intercept shift of each branch's demand curve by 1% up off the "basic" intercept, under capacity constraints, leaving other intercepts at their "basic" level. Only Agricultural branches are analyzed.

	R1	R2	R3	X1	X2	X3	X4	X5	X6	X7	Y1	Y2	Y3	Y4	Y5	Y6	Y7	Z
Branch 1	104.23	104.69	64.161	2.9364	2.202	.25719	.096298	.10616	.65445	.65445	2.9276	2.1935	0	0	0	.59354	2.0998	3098.1
Branch 2	103.19	104.69	64.161	1.8283	4.8573	.50297	.10066	.11113	.67377	2.3739	1.8193	4.8485	0	0	0	.59473	2.0956	3100.1
Branch 3	104.72	104.69	64.162	.010641	.010324	7.9366	.11989	.13814	.9923	.5802	0	0	7.2786	0	0	.90858	.32964	3102.5
Branch 4	104.95	104.69	64.158	.046828	1.7651	.90107	3.8607	.34100	1.3873	.1767	0	1.7312	0	3.5158	0	1.2606	0	3098.6
Branch 5	105.23	104.69	64.163	.046201	.045291	7.1791	.45558	5.9041	.072428	.2333	0	0	6.3610	0	5.0431	0	0	3100.2
Branch 6	104.29	104.69	64.161	.012424	.012104	7.9366	.13833	.15673	1.3873	1.7945	0	0	7.2524	0	0	1.2796	1.4864	3096.6
Branch 7	104.42	104.68	64.163	2.9364	1.4702	.25827	.095394	.10473	.67496	2.3739	2.9275	1.4616	0	0	0	.61310	2.1013	3097.3

tors direct increases. Citrus and Poultry are complementing some sectors and substituting for others. A thorough discussion of the sensitivity analyses will be available in (Yichia).

The Israeli MRQIO model. The decision to build a MRQIO model for Israel was reached on year into the start of the project. Obviously, this effort was beyond the objectives stated in the proposal. We made this decision when a group of men, D. Freeman, I. Weksler, and G. Alperovitch, from the Center for Settlement Studies at Rehovot, Israel offered to integrate their efforts into building both the MRIO and MRQIO models for Israel. In view of the situation where data needs for the MRIO model had to be partially primary (specially collected) and no meaningful study on the MRQIO model may start ahead of near completion of the MRIO modeling, we had doubted the possibility of finishing that modeling by the end of this project. It should be noted here that this is the first attempt anywhere to build a MRQIO model; hence, we were not aware of some of the theoretical methodological problems that lay ahead. We hoped to expose the problems and to demonstrate the usefulness of such methodology. These attempts were carried out with partial success.

The D-matrix for MRQIO, $\tilde{D}$, is similar to D as in equation (16), but F^{-1} is given by $\tilde{F}^{-1}$, where $\tilde{F}^{-1} = n_r n_i \times n_r n_i$ matrix of regional inverse demand slopes, with each of the n_r blocks being located along the main diagonal, and each block contains the inverse regional demand slopes of the n_i industries for the specific region. Elements in all blocks off the main diagonal are zero. With no information on differences among the per-capita demand curves for each region, it was assumed the same for each region. The blocks on the main diagonal of $\tilde{F}^{-1}$ therefore reflect

only the differences in regional population densities.

No problems should be encountered in solving the regional model aside from the need to solve a bigger model requiring additional computer time. When interregional output trade does occur the matrix $\tilde{T}$ is that of the above definition. Then, equation (29) does not represent the optimal (equilibrium) conditions for $\tilde{C}$. The intuitive reasoning for this is that $\tilde{T}$ in equation (18) transfers total outputs (X's) from one region to another when the corresponding Y's are dictated by the regional demand curves. Since the sectors' supply curves are derived supply, then transferring outputs interregionally break this relationship.

An obvious cure to this problem is to eliminate $\tilde{T}$ entirely and instead redesign $\tilde{A}$ as a regular input-output matrix with $m=n_r n_i$ *seemingly different industries*. Namely, fruit in one region is different from fruit in another. Hence, interregional trade is treated exactly the same as inter-industry flows - as given by the Leontief relations. Unfortunately, this data was unavailable in Israel and may not be available in the near future.

Other avenues for solving this problem appear feasible but not straightforward. Further research into this problem is clearly needed. The severity of this problem was unexpected; thus, no time was budgeted to work on it. To run sensitivity analysis on the MRQIO model without trade appeared senseless to us.

Applications with the U.S. QIO Model

There were two applications made of the QIO model developed in this study for the U.S. economy. The first application involved an examination

of stabilization policies for the agricultural sector. The second application focused on the measurement of the U.S. economy's potential gross national product.

Effects of Agricultural Stabilization Policies. With the growing interdependency between the agricultural sector and the rest of the economy's sectors, it becomes increasingly important to evaluate government policies toward agriculture on other sectors in the economy. In stressing the need for some public intervention even in the most market-oriented industrial countries, Tsukui and Murakami advocate "indicative" or "guided" planning (p. 1-6). They advocate a formation of a future course which the government intends to take. This, they argue, should be based on alternative future policies based on different national goals and forecasts. The existence of uncertainty and instability is a major contribution for the failure of market forces to obtain economic efficiency. One of the most important criticisms of the classical (Marshall) and new welfare economics is that it fails to account for the risk and uncertainty which is differentially distributed among sectors of the economy and among population groups. To avoid such criticism, Willig adopted a value judgement that changes should be made under *pareto* optimal conditions if, and only if, the gainers are able to bribe the losers. He showed that the change in consumer surplus reasonably tracks the consumers' compensation in many cases. Hueth, Just and Schmitz have likewise shown that under deterministic profit maximization the change in producer surplus exactly measures the producer's compensating and equivalent variations. Just (1974) showed that risk, measured in price variation, can be included in the firm's utility function to measure welfare changes under risk aversion or neu-

trality. In light of the major efforts by recent Food and Agricultural Acts to stabilize farm prices using target prices and loan rates in order to raise and stabilize income levels, one wonders what is the full impact of this instability reduction on the other sectors of the economy. This was Objective (D) of this project, namely, to offer a methodology for measuring the effects of variable instability of agriculture on the economic well-being of all producers and consumers. We assume that the government can supply a new set of policies *capable* of dampening the oscillations of prices in the agricultural sector. Our interest, then, is to assess the *potential* gain for society and the impact on the other sectors. Since the specific policies are not formulated, it is impossible to estimate the costs of stabilization. Therefore, only the potential gross gains from stabilizing policies have been pursued.

Most macroeconomic models explicitly ignore the impact of risk on market behavior. It is a serious omission when such models are used in evaluating demand-supply response to government policies aimed at reducing price and quantity fluctuations. In fact, Just (1975) has argued that ignoring a positive response to reduced price risk led policy makers to grossly underestimate agricultural product supplies during the 1950s. Most of the theoretical inroads in risk response have been made at the firm level, or single decision maker levels. Young, et al. provide an excellent state-of-the-art review of this issue.

Explicit inclusion of risk variables in econometric models of supply response (Behrman; Just (1974); Trail; Ryan) have confirmed the hypothesis of *negative* aggregate risk aversion although different levels of aversion factors are related to socio-economic and education characteris-

tics as well as financial position (Patrick, Whitaker and Blake).

Two recent studies are of interest to us. First, Heifner and Plato have developed a stochastic simulation model for measuring the effects of wheat price stabilization on aggregate economic well-being. They postulate that "instability can impose losses upon society in at least three ways: by fostering long run disequilibrium between supply and demand; by shifting the supply function permanently to the left; and by accelerating inflation when price and wage levels are flexible upward but not downward" (p. 119). In simulating alternative price bounding government stabilization programs for wheat, they tentatively concluded that such programs can be expected to reduce farmers' gross incomes but increase their economic well-being when the costs of instability are taken into account. More specifically, the reduction in variability of producers' returns and the consequent shift in the supply curve provides the primary source of gain from stabilization.

Second, Robison and Carman developed a simple aggregative risk model which can account for the influence of risk on the collective actions of individuals who trade in an exchange economy. Avoiding the difficulties of measuring aggregate interpersonal utility, they focused on the efficient set of choices given by $\bar{W} + \lambda \sigma^2$, where $\bar{W}$ is the expected wealth, λ is a constant and σ^2 is the variance determined by the vector of choice variables X . This function possesses some important properties: (1) it is a member of an E-V set, (2) under normal distribution it is the expected utility for a constant absolute risk averter with risk aversion parameter equals to 2λ , and (3) it provides the certainty equivalent ($\bar{W}$) or an approximation for it under certain conditions (pp. 141-142).

In formulating our model presented below, we admittedly take sides in a very long dispute among welfare economists regarding the effects of price instability. While doing so, we make no attempt to add new empirical evidence in resolving this issue. It was Waugh who in 1944 logically showed that price fluctuations (with constant mean) do increase consumers' welfare, only to be joined by Oi (1961) who had obtained similar results concerning results concerning producers. In 1972, however, Samuelson destroyed this paradox by showing that Waugh's result is possible only in a partial equilibrium framework. He showed that "exercise" of replacing stable prices by unstable prices must necessarily have those unstable prices average out to higher than the stable prices' under general equilibrium. Although this dispute is not over, we side with the later claim leading to the formation of an aggregate, concave objective function under general equilibrium conditions.

Now that we have outlined the motivation for using a general-equilibrium-under-risk model, let us outline the properties of the model used. The foundations for our model were already laid out in equations (10) through (17), which represents an aggregate QIO model.

Expanding the analysis to focus on uncertainty and instability, an additional term can be added to form a utility function to be maximized, given by

$$(30) \quad \text{Maximize } Z^* = C'Q + Q'DQ - \phi Q'VQ$$

where ϕ is an instability aversion factor ($\phi \geq 0$) and V is a $(m + 2n) \times (m + 2n)$ variance-covariance matrix of the sectors' time series real prices. V can be interpreted as the instability matrix. The implicit

assumption here is that the stochastic forces affecting the economy are reflected by the sectors' price changes. Hence, solving equation (17) subject to equation (11 through (16) while parametrically changing ϕ ($\phi \geq 0$) provides an efficiency frontier similar to Markowitz's E-V curve in portfolio analysis. In this case, however, the interpretation of the efficiency locus is the trade-off between total consumer plus producer surplus and total instability. Modeling stabilization policies which are aimed at the variance minimization of the target variables have been carried out for econometric models by Chow and by Turnovsky.

It is important to note that through the above set of constraints, the instability effects on the final demand "forces" the corresponding changes in the total production of each sector through equation (11) and (30).

All the properties discussed by Robison and Carman do hold here under proper specifications, as well as the proper interpretation of "welfare" (see the discussion in Robison and Carman, pp. 149-152).

The direct and immediate effect of stabilization is the relative stability of the agricultural sector's prices, while the indirect effects are given by the respective equilibrium solutions. Let P_{at} be the observed real price of the agricultural sector, let $\Delta = P_{at} - \bar{P}_{at}$; $t=1, \dots, L$ be the difference vector of that sectors' prices for the considered time span. Then, the stabilized vector is:

$$(31) \quad \Delta^S = \psi \Delta$$

where $0 < \psi < 1$, is a stabilizing parameter which reduces the amplitudes

of the price fluctuations through a set of stabilizing policies like target and support prices, buffer stocks, subsidies, taxes, etc. Obviously, such a set does not exist and very likely is quite impossible, and yet by this assumption we we can measure the potential gross increase in total welfare and sector's output combination. Recall that the variance-covariance matrix in portfolio analysis reflects the decision maker's subjective judgement allowing him to consider different scenarios. The same approach is taken here, by making a *ceteris paribus* assumption with respect to the nonagricultural sectors. Note that there is no loss of generality as far as modeling is concerned. It is possible to account for the cross-price effects of agriculture on the sector. In other words, our assumption is taken for simplification of exposition. With this disclaimer in mind, let us now consider an application for the U.S. economy.

The eight-sector QIO model of the U.S. economy developed by Penson, Talpaz and Foster is the basis for this model. It represents an aggregated version of the 1971 85-sector input-output table published by the U.S. Department of Commerce. The variance-covariance V matrix was computed on the sector's prices from 1961-1971 deflated by CPI (source: U.S. Department of Commerce). The demand functions were estimated according to equation (11) in Penson, Talpaz and Foster.

Table 17 shows the results of the "basic" run, namely the optimal solution for the 8-sectors U.S. economy. The last two columns in Table 17 provide information about the sector's producer and consumer surplus. Values for the consumer surplus were approximated since the Trade and Transportation Sectors' output is computed proportional to the other combined sectors' final demand.

Table -17: The Base Solution for 1971. Values are in \$000,000.

SEC #	Title	Total Output	Final Demand	Producers' Surplus	Consumers' ^a Surplus	Producer and Consumer Surplus
1	Agriculture	78,652	12,717	15,021	15,687	
2	Mining	32,726	1,688	8,680	2,082	
3	Construction	146,566	119,583	35,972	147,509	
4	Manufacturing	597,712	233,499	112,093	288,027	
5	Processed Food and Tobacco	118,798	85,676	16,013	105,684	
6	Energy	86,841	38,776	15,252	47,833	
7	Services	647,729	418,554	227,196	516,297	
8	Trade & Transportation	296,455	194,417	101,534	239,818	
	TOTAL	2,005,479	,104,910	531,761	1,362,939	1,894,700

^a Approximated by distributing the total consumers' surplus according to the final demand.

To obtain the Welfare-Instability (W-I) efficient frontier, equation (30) was solved repeatedly with increasing values of ϕ . The values were: $\phi = 0$ (the basic run); 25; 50; 150; 250; 300; and 500. The efficient frontier is shown in Figure 1 where each w-mark on the curve (going from left to right) corresponds to ϕ values above (in descending order). The relative, not the absolute, values of instability ($Y'VY$) are of interest; hence, these have been scaled down by the maximum element of the matrix. The shape of the W-I curve is typical and often appears in E-V Analysis of Portfolio selection. It means that yielding a relatively small quantity of welfare near its potential maximum decreases rather substantially the level of instability. This effect is decreasing as additional welfare is yielded. Corresponding values to Table 17 were obtained for each ϕ , but unfortunately, space limitation prohibits their presentation. However, Figure 2 illustrates the sectors' output level compared to their levels at the basic run. For each sector, points on the histogram corresponds to the higher values of ϕ (from left to right). The agriculture (1) sector's output increases and sharply decreases as ϕ increases. The large sectors: construction (3), manufacturing (4), and services (7) decrease their value which is the prime cause for the initial increase in Agriculture and food & tobacco (6) and the monotonic increase in mining. The reason lies in the relatively large instability or variance (and covariance) in sectors (3), (4), and (7), and the relatively small instability in sectors (1), and (6), and even more so for mining (2). The reduction in one sector output leads to lower cost of production for other sectors through the supply function for the primary inputs. This is being solved for simultaneously. In conclusion, this figure depicts the sensitivity of

INSTABILITY-WELFARE FRONTIER.

FIG. 1. INSTABILITY - WELFARE FRONTIER.

SECTORS OUTPUT AS FUNCTION OF ϕ

FIG. 2. SECTOR OUTPUT AS FUNCTION OF ϕ
IN RELATIVE TERMS.

each sector to the overall stabilization.

Figure 3 shows the level of producer and consumer surplus as a result of changing ϕ (decreasing going to the right). The consumer surplus decreases while the producer surplus increases. It is primarily due to the increasing equilibrium prices with the decrease of ϕ .

Stabilization of the agricultural sector. As discussed above, we attempted to examine the effect of stabilizing prices of the agricultural sector on the rest of the economy. The value of $\phi = 150$ was arbitrarily picked for the analysis. Considering equation (22), ψ was varied repeatedly for each solution using values: 1.0 (basic run); 0.95; 0.90; 0.80; 0.75; 0.70; 0.60; and 0.50, which corresponds to a set of decrease instabilities. The new set of general equilibrium solutions was obtained and summarized in Figures 4 and 5. The first shows the efficient locus Instability-Welfare as ψ decreases (moving from left to right). The negative slope points to potential benefits since, while instability is decreasing, welfare is increasing. If the costs of stabilization are not zero it is possible for the optimal solution to be somewhere on the curve. Figure 5 shows the relative size of each sectors' output to the respective standard ($\psi = 1.0$) solution. Agriculture is increasing with the decrease of ψ (from left to right). This is obvious in view of the lower variance-covariance for agriculture. The last point ($\psi = 0.5$) is unique since the agriculture sector reaches full capacity of production, and therefore any further decrease in ψ will not change this sectors' output. Four more sectors very modestly increase their output (#2, 5, 6 and 8) while the rest are mildly declining. This combination of effects, namely the consumer plus producer surplus of the solutions results in the increase in welfare.

PRODUCER AND CONSUMER SURPLUSES.

FIG. 3. PRODUCER AND CONSUMER SURPLUSES.
(CONSUMER SURPLUS IS SCALED DOWN BY 0.4)

STABILIZING AGRICULTURE ONLY

FIG. 4. INSTABILITY-WELFARE FRONTIER
AS A FUNCTION OF $\Psi(\Phi=150)$.

SECTORS OUTPUT AS FUNCTION OF Ψ

FIG. 5. SECTORS OUTPUT AS FUNCTION OF Ψ
IN RELATIVE TERMS.

Measurement of potential GNP. An important application of the U.S. QIO model was to derive a series of estimates for annual potential GNP during the 1958-79 period using the annual quadratic input-output models developed in this study and to compare these estimates with the potential GNP estimates published by the President's Council of Economic Advisors (CEA) based upon Okun's Law and by Peter Clark.

While there have been other studies whose purpose was to measure potential GNP, the CEA estimates based upon Okun's Law and Clark's estimates, which were the first to explicitly include capital, are the most widely referenced works in this area. Furthermore, they are representatives of the general approaches previously taken in the literature. This section will begin with an explanation of how the annual quadratic input-output models were modified to estimate annual potential GNP during the time period covered by this study. This will be followed by a discussion of the CEA and Clark's approaches of estimating the potential GNP.

The CEA estimates of potential GNP published in the Economic Report of the President have traditionally been based largely on a relationship between the unemployment rate and GNP gap developed by Arthur Okun:

$$(32) \quad \text{GNP gap} = a (u - \bar{u}),$$

where a is the gap/unemployment parameter, u is the actual unemployment rate and $\bar{u}$ is the full-employment unemployment rate. Okun used a value of the parameter a of about 3, and $\bar{u}$ was taken to be 4% in the 1960s and around 5% in the latter stages of the 1970s. The GNP gap is defined in percentage terms as:

$$(33) \quad \text{GNP gap} = (y_p - y)/y_p$$

where y_p is the potential GNP and y is the actual GNP. Okun's Law stated that for every 1% unemployment over the full-employment rate, there is a corresponding 3% GNP gap. The value of potential GNP is then obtained by adding the real GNP gap to actual real GNP.

Only labor is used in the determination of potential GNP under Okun's approach. The possibility that capital can represent a limiting primary input is ignored. As Perry argues, "... it is hard to argue that capital should not be included in estimating potential output because everyone knows it belongs in the calculation" (p. 11). Okun later recognized this when he stated, "I shall feel much more satisfied with the estimation of potential GNP when our data and our analysis has advanced to the point where the capital factor can be explicitly taken into account," (p. 104).

Clark's explicit accounting of capital when estimating potential GNP was accomplished through the use of the following aggregate Cobb-Douglas production function for the private nonresidential sector of the economy:

$$(34) \quad y_n = cK^{1/3}E^{2/3}e^a$$

where y_n is the private nonresidential output, K is the fixed nonresidential capital stock, E is the labor force used in the private nonresidential sector, and a represents the trend or business cycle. Equation (34) shows that input shares of 1/3 and 2/3 were assigned (arbitrarily) to capital and labor. Clark used this single production function for the

entire private nonresidential sector when deriving the rate of growth of potential GNP because the other components of real GNP, as he explained, do not respond to growth of domestic labor productivity (which he argues is the source of the growth of potential GNP). They are also not related to the domestic business cycle which affects the growth of potential GNP. This implies that the actual outputs for the other components of real GNP are usually the same as their potential outputs.

Many different approaches have been proposed in the literature to estimate the current productive capacity of a sector. The time-series data on capacity used in this study were provided by Wharton Econometrics Forecasting Associates. The method they use to derive these estimates is called the 'trend-through-peaks' method and can be explained as follows. First, a time series of the sector's total output is identified. Peaks in this time series of actual outputs are then identified and are assumed equal to the sector's maximum output in that year. The productive capacities between these relative peaks are approximated by points along linear segments which connect these peaks. This method is illustrated in Figure 6, where x_1 , x_4 , x_8 and x_{10} are the total outputs for periods 1, 4, 8 and 10, which represent the peaks in this series, and x_2 , x_3 , x_5 , x_6 , x_7 and x_9 are the productive capacities obtained from this interpolation method. The productive capacities of the four selected sectors in this study for the 1958-79 period are listed in Table 18.

The value of GNP in any year can be derived from the solution values for sector final demands and imports provided by the quadratic input-output model since GNP is equal to the sum of the sector final demands minus imports. However, in order to derive potential GNP from a quadratic

Figure 6. Illustration of the Wharton Trend-Through-Peaks method

Table 18. Production Capacities of Selected Sectors
(Billions of Dollars)

Year	Mining and Quarrying	Manufacturing	Processed Food and Tobacco	Energy
1958	33.33	554.00	115.97	72.03
1961	35.13	618.55	119.13	78.02
1963	34.83	660.50	120.06	86.15
1966	34.99	723.53	125.86	92.21
1967	36.46	726.14	124.19	92.28
1968	36.05	743.99	124.95	92.26
1969	36.54	748.61	126.29	91.66
1970	37.88	718.54	127.70	95.26
1971	39.64	755.42	126.93	97.72
1972	39.90	776.24	151.19	97.78
1973	37.89	886.45	167.94	111.88
1974	42.21	1138.56	215.12	155.59
1975	38.17	1401.13	229.65	177.53

Table 18 cont.

Year	Mining and Quarrying	Manufacturing	Processed Food and Tobacco	Energy
1976	42.05	1269.66	210.00	161.68
1977	49.18	1306.33	224.01	167.62
1978	55.03	1300.45	234.91	174.79
1979	64.37	1537.54	267.66	198.19

input-output model, an extra labor activity needs to be included in the quadratic input-output models to reflect the difference between the size of the *full-employment* labor force and the use of labor. The incorporation of this new employment activity in the annual quadratic input-output models is accomplished in the following two steps. First, a new labor constraint is introduced

$$(35) \quad w_d \leq (w_f - w_1) \text{ if } w_f \geq w_1$$

or

$$(36) \quad w_d = 0 \text{ and } w_f = w_1 \text{ if } w_f < w_1,$$

where w_d is the value of the new employment variable for labor, w_f is the employment level corresponding to the annual full-employment unemployment rates reported by Clark, and w_1 is the actual level of employment. Table 19 reports the actual and full-employment unemployment rates for the years covered by this study.

In the second step, a new term $\beta_{11}w_d$ is added to the original inverse labor supply equation. This supply equation now takes the following form

$$(37) \quad r_1 = \beta_{10} + \beta_{11}w_d + \beta_{11}w_1 = \beta_{10} + \beta_{11}(w_1 + w_d)$$

where r_1 is the wage rate, β_{10} is the original intercept, and β_{11} is the slope coefficient associated with the wage rate.

Table 19. Calculation of Full Employment Labor Force

Year	Clark's Full Employment Rate Percent	Actual Unemployment Rate Percent	Actual Employment Compensations (ALBR) 1972 Bil. \$	Maximum Employment a/ Compensations- (MXLBR) 1972 Bil. \$
1958	4.1	6.8	390.88	402.21
1961	4.2	6.7	437.76	449.45
1963	4.3	5.7	478.09	485.03
1966	4.6	3.8	573.06	568.24
1967	4.5	3.8	596.73	592.67
1968	4.5	3.6	629.84	623.79
1969	4.6	3.5	658.40	650.96
1970	4.7	4.9	666.11	667.84
1971	4.7	5.9	678.01	686.96
1972	4.9	5.6	666.60	671.49
1973	5.0	4.9	706.85	705.83
1974	5.0	5.6	682.03	686.39
1975	5.1	8.5	651.35	675.25
1976	5.1	7.7	675.38	694.42
1977	5.1	7.1	704.90	719.78
1978	5.2	6.1	727.42	734.11
1979	5.2	5.8	731.65	736.70

$\frac{a}{MXLBR}_t = ALBR_t \times \frac{LM_t}{LE_t}$ where LM is the maximum employed labor force corresponding to

Clark's full-employment unemployment rate and LE is the actually employed labor force.

It can be demonstrated that in a maximization programming problem, the requirement that the full employment of labor be achieved will be satisfied if $w_d = w_f - w_1$ when actual employment level of labor is less than the full-employment level of labor (i.e., $w_f > w_1$). The value of w_d is set equal to zero when the actual employment level is greater than full-employment labor force, leaving this variable inoperative. To determine potential GNP when this case occurs (i.e., $w_f < w_1$), the constraint that labor not exceed the full-employment labor force in equation (27) becomes operative.

The three estimation approaches mentioned above can be thought of as representing three evolving stages of sophistication in measuring potential GNP. For example, Clark's approach has evolved from the perceived need to measure the potential output of the economy which includes all factors of production rather than the rule of thumb (Okun's Law) used by the CEA. However, Clark's estimates still do not reflect the functioning of the demand forces which contribute to a new general equilibrium as the economy moves from the current actual labor force to the full employment labor force. That is, these models totally ignore final demand and the effects of changes in prices and wages. As Penson and Webb point out, the estimates of potential GNP supplied by a quadratic input-output model reflect the effects of changes in final demand as the economy moves to the new general equilibrium. The nature of this programming approach also permits us to consider the effects of the sector manufacturing capacities for the various production sectors and their potential bottlenecking effects on GNP. The estimation of potential GNP using the quadratic input-output model thus represents a more comprehensive estimate of

potential GNP since it accounts for both supply and demand forces throughout the economy. The results from these three approaches to estimating potential GNP are listed along with the actual real GNP levels in Table 20 for the 1958-79 period.

The estimates of potential GNP derived from the CEA, Clark, and quadratic input-output models are compared in this section. Two criteria will be used to judge the performance of these three estimation approaches: (1) the degree to which the annual growth rates of potential GNP given by the three different approaches correspond to the annual growth rates of actual GNP, and (2) the ability of alternative estimates of the GNP gaps to explain the inflationary pressures which existed during the time period covered by this study. The alternative growth rates are presented in Table 21 while the alternative GNP gaps and the annual inflation rate are reported in Table 22. Both of these two comparisons are made for the 1958-79 period.

The rate of growth in actual GNP necessarily reflects the changes in the current economic conditions in the economy from one year to the next. That is, these rates of growth reflect the effects of such current economic conditions as corporate profits in different industries, the current level of interest rates, consumer preferences, current technology, etc. These factors, which influence the final demands observed during the year, must necessarily continue to play a role as the economy moves from actual to full employment GNP. Therefore, it is expected that if the growth rate of the actual GNP goes up (that is, if the economic conditions in the economy improve), then the growth rates of potential GNP should also go up. After all, the only difference between the actual GNP

Table 20... Estimates of Potential GNP, Selected Years 1958-79
(Billions of 1972 Dollars)

Year	Actual GNP	CEA Potential GNP	Clark Potential GNP	Quadratic input-output potential GNP
1958	680.9	729.50	722.99	692.32
1961	756.6	808.49	802.48	768.36
1963	832.5	867.78	860.46	839.49
1966	984.8	970.75	961.51	976.94
1967	1011.4	1006.98	978.55	1007.30
1968	1058.1	1047.13	1037.98	1052.00
1969	1087.6	1090.42	1077.92	1080.10
1970	1085.6	1135.68	1116.59	1087.40
1971	1122.4	1185.64	1160.81	1131.50
1972	1185.9	1232.08	1203.92	1190.90
1973	1255.0	1285.89	1252.36	1253.90
1974	1248.0	1348.53	1309.18	1252.60
1975	1233.9	1404.57	1353.26	1257.50
1976	1300.4	1451.79	1391.52	1319.70
1977	1371.7	1508.82	1440.87	1387.00
1978	1436.9	1571.02	1496.36	1443.20
1979	1483.0	1644.76	1564.14	1487.8

Table 21. Growth Rates of Various Measures of Potential GNP, Selected Years 1958-79
(Percent)

Year	Actual GNP	+ change	CEA Potential GNP	+ change	Clark Potential GNP	+ change	Quadratic Input-output Potential GNP	+ change
1958-61*	3.7		3.6		3.7		3.8	
1961-63*	5.0	+	3.6	0	3.7	0	4.4	+
1963-66*	6.0	+	3.9	+	3.9	+	5.2	+
1966-67	2.7	-	3.7	-	3.9	0	2.8	-
1967-68	4.6	+	4.0	+	3.9	0	4.4	+
1968-69	2.8	-	4.1	+	3.8	-	2.7	-
1969-70	-0.2	-	4.1	0	3.6	-	0.7	-
1970-71	3.4	+	4.4	+	4.0	+	4.1	+
1971-72	5.7	+	3.9	-	3.7	-	5.2	+
1972-73	5.8	+	4.4	+	4.0	+	5.3	+
1973-74	-0.6	-	4.9	+	4.5	+	-0.1	-
1974-75	-1.1	-	4.2	-	3.4	-	0.4	+
1975-76	5.4	+	3.4	-	2.8	-	4.9	+
1976-77	5.5	+	3.9	+	3.5	+	5.1	+
1977-78	4.8	-	4.1	+	3.9	+	4.1	-
1978-79	3.2	-	4.7	+	4.5	+	3.1	-
range	-1.1-6.0		3.4-4.9		2.8-4.5		-0.1-5.3	

*Normal average growth rate during the period.

Table 22. Gaps Between Actual GNP and Various Measures of Potential GNP and the Rate of Inflation, Selected Years 1958-79^{a/}

Year	CEA	+ change	Clark	+ change	Quadratic input-output	+ change	Rate of Inflation (percent)	+ change
1958	48.6		42.1		11.4		1.8	
1961	51.9	+	45.9	+	11.8	+	1.5	-
1963	35.3	-	28.0	-	7.0	-	1.6	+
1966	14.1	-	-23.3	-	-4.9	-	3.4	+
1967	-4.4	-	-12.8	+	-4.1	+	3.0	-
1968	-11.0	-	-20.1	-	-6.1	-	4.7	+
1969	2.8	+	-9.7	+	-7.5	-	6.1	+
1970	50.1	+	31.0	+	1.8	+	5.5	-
1971	63.2	+	38.4	+	9.1	+	3.4	-
1972	46.2	-	10.0	-	5.0	-	3.4	0
1973	30.9	-	-2.6	-	-1.1	-	8.8	+
1974	100.5	+	61.2	+	4.6	+	12.2	+
1975	170.7	+	119.4	+	23.6	+	7.0	-
1976	151.4	-	91.1	-	19.3	-	4.8	-
1977	137.1	-	69.2	-	15.3	-	6.8	+
1978	134.1	-	59.5	-	6.3	-	9.0	+
1979	161.8	+	81.1	+	4.8	-	13.3	+

^{a/}Values expressed in 1972 billion dollars.

and the potential GNP is the change in the size of the labor force and the effect this has upon current wages and prices and final demand.

The results presented in Table 22 show that the growth rates of the potential GNP derived from the annual quadratic input-output models change direction consistently with the growth rates for actual GNP. More specifically, the only inconsistency in the entire 1958-79 period is that the growth rate suggested by the quadratic input-output model for potential GNP in 1973-74 is lower than that suggested for 1974-75, while the growth rate of actual GNP in 1973-74 is higher than it was in 1974-75. As for the rest of this time period, the direction of changes in growth rates in actual GNP and the quadratic input-output model's estimates of potential GNP are identical.

This consistency in year-to-year changes in the growth rates between actual GNP and quadratic input-output model's estimates of potential GNP is not found for the CEA's estimates and Clark's estimates of potential GNP. For example, a comparison of the growth rates in the CEA's estimates of potential GNP and actual GNP shows that this consistency in the direction of change in growth rates is violated eight times (i.e., 1961-63, 1968-69, 1969-70, 1971-72, 1973-74, 1975-76, 1977-78 and 1978-79). The same comparison between Clark's estimates of potential GNP and actual GNP shows this consistency is also violated eight times (i.e., 1961-63, 1966-67, 1967-68, 1971-72, 1973-74, 1975-76, 1977-78 and 1978-79).

This comparison of the consistency in the direction of changes in growth rates for actual GNP and the different methods of estimating potential GNP clearly demonstrates that the quadratic input-output model estimates more closely reflect the changes in current economic conditions

as the economy moves from actual to potential GNP than either the CEA's estimates or Clark's estimates.

A fundamental macroeconomic relationship sometimes, referred to as the price equation, accounts for the relationship between the inflationary GNP gap ($y - y_p$) measured in real terms and the rate of inflation ($\dot{p}$). This equation takes the form:

$$(38) \quad \dot{p} = \dot{p}^* + a(y - y_p)$$

where $a > 0$ and where $\dot{p}^*$ represents the expected rate of inflation. This expected rate in turn is thought to be affected by the current rate of growth in the nominal money supply as well as past inflationary experiences and other factors. Of interest in this study is the direct relationship between the inflation rate ($\dot{p}$) and the size of the real inflationary GNP gap ($y - y_p$).

In the following examination of how well the different measures of annual real GNP gap explain the annual inflation rates over the time period covered by this study, the overall 1958-79 time period will be subdivided into two periods. The first period will cover the years 1958-71, which were characterized primarily by demand-pull inflationary forces. The second period examined will cover the years 1972-79, which were characterized more by cost-push inflationary forces than demand-pull forces (i.e., rising factor costs (energy) and increasing unemployment of both labor and capital). We would thus expect the real inflationary GNP gap ($y - y_p$) in equation (38) to play a more significant role in explaining the inflationary pressures which existed during the 1958-71 period than the

inflationary pressures which existed during the 1972-79 period.

Two specifications of the partial relationship between the inflation rate and the GNP gap will be examined in the remainder of this section. The first equation takes the form:

$$(39) \quad \dot{p} = a + b(y - y_p)$$

while the second equation takes the form:

$$(40) \quad \log \dot{p} = c + d(y_p - y)/y_p$$

where equation (40) represents the form estimated by Penson and Webb for the 1958-71 period. It is hypothesized that $a > 0$ and $b > 0$ in equation (39) and that $c > 0$ and $d < 0$ in equation (40). The explanatory variable in equation (39) represents the real *inflationary* GNP gap, while the explanatory variable in equation (40) represents the real *deflationary* GNP gap expressed in percentage terms.

The empirical results of all twelve sets of regression estimates (two different estimating equations, three different measures of potential GNP, and two different time periods), are presented in Table 23. A discussion of these different estimates is presented below.

1958-71 Period. No matter whether equation (39) or equation (40) was used, the quadratic input-output model approach to estimating potential GNP always led to better results than the other two methods examined (i.e., the CEA approach based upon Okun's law and the Clark approach). This is based upon a review of the R^2 and t-statistic values achieved by

Table 23. Regression Estimates of the Relationship Between the Inflation Rate and the GNP Gap

1958-71 period:

(A) Model:		$\dot{p} = a + b(y - y_p) \quad (a > 0, b > 0)$
1. Quadratic I-O	$\dot{p} = 3.760620 + 0.153732 (y - y_p)$	(8.621) (2.724)
	$R^2 = 0.5145$	
2. CEA-Okun	$\dot{p} = 3.927211 + 0.019531 (y - y_p)$	(5.314) (1.010)
	$R^2 = 0.1272$	
3. Clark	$\dot{p} = 3.824109 + 0.028619 (y - y_p)$	(6.538) (1.486)
	$R^2 = 0.2398$	
(B) Model:		$\log \dot{p} = c + d(y_p - y)/y_p \quad (c > 0, d < 0)$
1. Quadratic I-O	$\log \dot{p} = 1.261176 - 45.648568 (y_p - y)/y_p$	(10.388) (-3.468)
	$R^2 = 0.6322$	
2. CEA-Okun	$\log \dot{p} = 1.363428 - 9.089217 (y_p - y)/y_p$	(6.687) (-1.841)
	$R^2 = 0.3261$	
3. Clark	$\log \dot{p} = 1.287019 - 10.561199 (y_p - y)/y_p$	(7.983) (-2.271)
	$R^2 = 0.4242$	

1972-78 period:

(A) Model:		$\dot{p} = a + b(y - y_p) \quad (a > 0, b > 0)$
1. Quadratic I-O	$\dot{p} = 9.839949 + 0.172488 (y - y_p)$	(5.362) (1.186)
	$R^2 = 0.1890$	
2. CEA-Okun	$\dot{p} = 6.636558 - 0.013089 (y - y_p)$	(2.033) (-0.508)
	$R^2 = 0.0412$	
3. Clark	$\dot{p} = 7.657839 - 0.008126368 (y - y_p)$	(3.002) (-0.230)
	$R^2 = 0.0087$	
(B) Model:		$\log \dot{p} = c + d(y_p - y)/y_p \quad (c > 0, d > 0)$
1. Quadratic I-O	$\log \dot{p} = 2.200491 - 25.208886 (y_p - y)/y_p$	(8.779) (-0.973)
	$R^2 = 0.1363$	
2. CEA-Okun	$\log \dot{p} = 1.822318 + 2.416343 (y_p - y)/y_p$	(3.867) (0.438)
	$R^2 = 0.0310$	
3. Clark	$\log \dot{p} = 1.953228 + 1.390144 (y_p - y)/y_p$	(5.732) (0.209)
	$R^2 = 0.0072$	

all three measurement approaches for both models as well as the signs of the coefficient corresponding to the explanatory variable. All coefficients in each equation reported the hypothesized sign. The Clark approach, which explicitly accounted for the role of capital, did outperform the CEA-Okun approach, which ignored capital.

1972-79 Period. Once again the results showed that no matter which model (i.e., equation (39) or equation (40)) was adopted in the regression analysis, the quadratic input-output approach to measuring potential GNP always lead to the best fit. However, the coefficient of determination, R^2 , during this sample period is much lower than that obtained for the 1958-71 period. This was expected since the economy was plagued more by cost-push inflationary influences than by demand-pull influences during this period.

Summary and Conclusions

Dual models have much appeal for analysis of economic relationships using real world data. There exist conceptual, computational and econometric advantages in estimating dual models as opposed to estimating production functions and deriving economic implications (primal models). Further, when the dual estimates satisfy certain conditions, inferences about the nature of the production function can be drawn.

There are also certain limitations in dual modeling and situations for which it is not appropriate. For example, it was documented in this

study that a constraining allocatable input generally causes jointness in production; yet, it is still possible to identify input demand allocations among products with a primal approach but not with a dual approach. In such cases, and depending on the research objective, there may be important information loss in dual models.

A procedure for geographically aggregating product supply and input demand equations was presented. It accounts for differences in the nature of input supplies (or product demands) at the aggregate as opposed to the subaggregate level.

Dual tests of production structure were summarized and extended for several common functional forms and for some approximations. When theoretical properties of the profit functions are satisfied, these tests permit unambiguous inferences to be drawn about the structure of production.

Many important and strong interrelationships in agricultural commodity supplies have been documented at state, regional, and national levels in the U.S. Although the particular relationships differ by geographic entity and time period, it is likely that many commodity supplies are highly interrelated in other locations and time periods as well. It appears that production relationships for few commodities can be accurately examined in isolation from others.

Profit function estimates obtained without maintaining the convexity hypothesis did not support that hypothesis implied by a concave production function. Imposing convexity on moderate-sized systems is feasible but it remains very costly computationally.

A dual multiple-product model explained spatial and temporal differences in land prices for one state remarkably well. Parameter estimates

all had the expected sign and were highly significant.

Several quadratic input-output (QIO) models were developed for the United States and Israel in this study. A national QIO model developed for the United States was used to measure the U.S. economy's potential GNP over the 1958-1979 period and to assess the effects of agricultural stabilization policies. The time series for potential GNP developed in this study compared favorably with measures presented elsewhere in the literature on this subject, particularly when used to help explain the inflationary pressures which existed during the 1958-1979 period. The stabilization policies for agriculture were shown to promote the growth of the sector, increase producer surplus, and decrease consumer surplus. The overall change in producer plus consumer surplus for the economy as a whole was positive.

National and multiregional QIO models were developed for Israel. The national QIO model was used to examine the effects of subsidizing labor and changes in import prices. With respect to labor subsidies, some industries such as citrus showed an extreme increase in production. The fruit industry in general was also shown to be vulnerable to changes in the demand for *other products*. A multiregional QIO (or MRQIO) model was also initiated for Israel. Not called for at the time the project statement was approved, this model is nearing completion.

Several methodological advances in input-output analysis were realized under this study. One advance was the endogenization of final demand and primary input supply relationships. Previously, input-output analysis assumed these values prior to solution of the model. Another advance was the development of an updating technique for the technical coefficients

in input-output models that allowed us to project these coefficients beyond available data based upon estimated economic relationships.

Footnotes

- ¹ In some cases, much of the labor used on the farm also can be distinguished by commodity to which it is allocated.
- ² Contemporaneous correlation among the system of equations was not accounted for in this estimation.
- ³ A review of the mathematical properties of these techniques is presented by Alan Lu in, "Estimation of Potential GNP in the U.S. Economy: An Application of Input-Output Analysis and Alternative Updating Techniques," unpublished Ph.D. dissertation, Texas A&M University, May 1983.
- ⁴ If $h(y_j) = y_j$, and $a_{ij} = 0$ for all $i \neq j$, then equation (3) becomes $x_{ij}(p, y_j) = a_{ij}y_j$. This is the Leontief cost function.
- ⁵ The details of this modeling effort are compiled in Freeman, et al.
- ⁶ Actually, the right-hand-side of equation (3) was dropped since no capacity estimates are available for Israel.

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