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**Further Development of Stochastic Models of
Water and Solute Flow in Heterogeneous Fields
with Application to Crop Yield**

E. Bresler, G. Dagan, R.J. Hanks

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Title

Further Development of Stochastic Models of Water and Solute Flow in
Heterogeneous Fields with Application to Crop Yield

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A. ABSTRACT

Crop yield depends on a large number of soil-water-atmosphere related parameters some of which are spatially variable and are subject to uncertainty. Consequently, the yield is also spatially variable. The degree of yield variability as a function of variability of various parameters was estimated. The general methodology consists of a numerical deterministic model which predicts relative crop yield for a given irrigation regime and quality of water and for given soil and atmospheric conditions. The model is used to derive the values of expectation and variance of the relative irrigated-crop yield as functions of these values of the various parameters. The data needed to evaluate the impact of the variability of different parameters upon crop yield variability, were collected. The dependence of the relative yield and its derivatives upon each of 19 parameters was investigated. The prediction of yield variability using the probability theory apparatus, have been tested against field data. Among the 19 parameters, six which has a significant impact on yield variability have been singled out. These include two plant parameters, three soil hydraulic parameters and one irrigation boundary condition parameter, related to the nonuniformity of water application. The detection of the parameters of large impact, obtained by both theory and field validation, give an indication of weak links which require further investigation.

Economic optimization of spatially variable crop yield which depends upon variable soil properties and man - controlled water quantity was investigated. The decision maker's attitude to risk was considered. The effects of uncertainty on the level of water quantity was investigated by comparing a deterministic case with risk neutral and risk averse cases. The results suggest that spatially variable soil may substantially affect optimal level of water application. Solutions for unconditional and conditional optimization problems showed that the conditional analysis has more potential of increasing the decision maker's profit substantially as compare with the unconditional one. A methodology to infer the spatial distribution of the statistical moments of a spatially variable component gives estimates of conditional expectations, conditional multivariate covariance and variance and the variance of estimated variance. The uncertainty of the conditional space average of a spatially variable field property is reduced as the number of measured points increases.

B. OBJECTIVES OF THE ORIGINAL RESEARCH PROPOSAL

1. To develop a comprehensive model of water flow and salt transport in heterogeneous irrigated cropped field by adding to our existing models effects of evaporation and water extraction by roots.

2. To suggest a statistical averaging procedure for a heterogeneous field by using conditional multivariate statistics.

3. To apply field results to assess the dependence of the accuracy of estimation of the field average of a quantity of interests, upon the number of sampling points, their distribution in the field plane, and the worth of additional sampling point on improving accuracy.

4. To assess the influence of water application (quantity, quality, and rate) as random space function, upon variability of crop yield and, in turn, on decision making and irrigation policy.

5. To determine the relationship between space average and space variance of the yield and between averages and variances of soil properties, plant parameters, and irrigation management practices.

6. To assess the effects of inclusion of space averages of the field (which come from the random nature of a finite field) on economic optimization of quantity of water applied.

C. BODY OF THE REPORT

1. VARIABILITY OF YIELD OF AN IRRIGATED CROP AND ITS CAUSES:

I. STATEMENT OF THE PROBLEM AND METHODOLOGY

ABSTRACT

Crop yield depends on a large number of soil-water-atmosphere related parameters. Some of these parameters are spatially variable and most of them are subject to uncertainty. Consequently, the yield is also spatially variable and its prediction is uncertain. The aims of this study were to (i) estimate the degree of yield variability as a function of variability of various parameters, and (ii) attempt to single out those parameters, the variability of which are estimated to have the largest impact upon the variability of yield of an irrigated crop. The first part of the study presents the general methodology developed in order to achieve these aims. It consists of a numerical deterministic model which predicts crop yield relative to its potential yield for a given irrigation regime and quality of water and for given soil and atmospheric conditions. Subsequently, the model is used to derive the values of expectation and variance of the relative irrigated-crop yield as functions of these values of the various parameters.

1. INTRODUCTION

The development of models which predict the relative yield of an irrigated crop as a function of the properties of the soil-water-atmosphere-plant complex is one of the central issues of soil physics, agriculture and water resources sciences. Such models are of paramount importance on the qualitative level, helping in the achievement of a basic understanding and a rational approach to the subject. Eventually they may also serve as quantitative tools for the planning and management of water resources and irrigation practices. In spite of the rapid growth of the body of knowledge of water resources and soil physics, this latter task is far from being accomplished.

Among the many existing difficulties, it is worthwhile mentioning two categories. First: the complexity of the various processes which together determine the yield as a function of man-controlled decision variables like the irrigation regime and water quantity and quality. For instance, the mechanism of water transfer to roots on the one hand, and the physiology of plant growth for given water and salt availability and for given atmospheric conditions, on the other, are

far from being understood thoroughly or from being quantified in a general manner. Hence, even under controlled ideal conditions, existing models which predict crop yield are semi-empirical at best and not definitive. The second category involves the uncertainty regarding model parameters under field conditions. As we shall show in Part 2, many parameters (e.g. soil hydraulic properties and irrigation uniformity) influence the yield. It has recently become apparent that these parameters vary in an irregular fashion and to a large degree in the field. Consequently, due to spatial variability, to a limited amount of measured data and to measurement and model errors, the prediction of yield under field conditions is subject to uncertainty.

In other words, even if an accurate model under controlled conditions would be available, prediction of yield on a large scale is still error-prone due to parameter uncertainty.

The aim of the present study is to investigate in detail the impact of the uncertainty of the various parameters that characterize the soil-water-plant complex on crop yields under various irrigation conditions.

In line with recent developments, spatial variability and uncertainty are set in a mathematical framework by regarding the various parameters affecting yield, as well as yield itself, as spatial random functions (or regionalized variables in geostatistical terminology). In the simplest representation, the relative yield of an irrigated crop is characterized by its first two statistical moments (the expected value and the variance) rather than by a well defined, deterministic value.

It should be noted that uncertainty has two opposing effects. Thus, as mentioned before, even if an accurate yield predictive model for controlled conditions is available, uncertainty is still present for large fields. On the other hand, if we accept uncertainty as an inherent feature of the prediction process, approximate models may be quite useful, as some lack of model accuracy becomes part of the overall possible error (Bresler and Dagan, 1983a,b; Dagan and Bresler, 1983).

Under this general approach, the plan of the present study is as follows: (i) In Part 1, we adopt first a conceptual and numerical deterministic model for crop-yield time evolution, depending on one-dimensional vertical water and salt transport through soil and on atmospheric and water application boundary conditions. Although a matter of preference, we believe that this particular model, as described in Sect. 2, is one of the more advanced and realistic models available at present. The model contains a large number of input parameters related to soil hydraulic and salt transport properties, to initial and boundary conditions, to root water extraction in the presence of salt, and to yield dependence on transpiration. We assumed

that these parameters are subject to uncertainty and have to be modeled as random variables. (ii) Our next task was to collect data from the literature about the statistical properties of various parameters, and these are presented in Part 2. (iii) By using the probability theory apparatus, we investigated the variability of yield as influenced by the degree of the parameters' uncertainty through numerical simulations, with the aid of the model mentioned before. This, together with comparison of results with field data will be presented in Part 3.

One of the main specific aims of the study is to single out those parameters which have a significant impact on yield variability of irrigated crops on the one hand, and to establish the magnitude of degree of yield uncertainty, on the other.

There are a few possible benefits from the results obtained here. One of them is that the detection of the parameters of large impact upon yield gives an indication about "weak links" which deserve further investigation and for which the collection of field data is important; another is the advancement of models which are realistic enough to become a useful tool for agricultural management and irrigation practice. The same approach may be considered for other problems in which the water application-soil-climate-plant complex plays an important role.

2. METHODOLOGY; THE DETERMINISTIC MODEL

The deterministic model predicts the crop yield Y as a function of time in the irrigation season for given soil, plant, climate and water application regime. The model presented here is a synthesis of a few models developed in the past and presented in different publications. For the sake of completeness we shall describe it here briefly, referring in each stage to the appropriate literature source.

(a) Theoretical foundations

The deterministic crop-yield model is based essentially on the relationships existing between transpiration and crop yield and on the estimation of transpiration by solute transport and water flow models which consider water uptake by plant roots. The theory of de Wit (1958) suggests that the relationship between transpiration and crop yield can be described by

$$Y = m T_r / E_o$$

[1]

where Y is the total dry matter yield, T_r is seasonal transpiration from a given crop, E_o is seasonal free water evaporation and m is a constant crop factor. For a given crop m is constant, and for a given location and season E_o is constant, so that the relative yield Y_R is obtained (Hanks, 1974) from [1] as

$$Y_R = \frac{Y}{Y_P} = \frac{T_r}{T_P} \quad [2]$$

where T_P is potential - maximum possible transpiration and Y_P is the corresponding potential - maximum possible crop yield. Hence, relative yield Y_R equals relative transpiration ($T_R = T_r/T_P$), and the estimation of relative crop yield is reduced to computation of seasonal transpiration from

$$T_r = \int_0^Z \int_0^T S(z,t) dz dt \quad [3]$$

where S is a macroscopic root-water extraction function, z is the vertical space coordinate positive downward, t is time, Z is rooting depth and T is the duration of the irrigation season.

The sink function S , representing water uptake by the plant root, is expressed (Childs and Hanks, 1975; Bresler et al., 1982) as

$$S(z,t) = b(z,t) \{ [K[\theta(z,t)]] [\Phi(t) - h(z,t) - \Gamma C(z,t)] \} \quad [4]$$

where K is hydraulic conductivity of the soil, $\Phi(t)$ is the equivalent total suction head in the plant root at the root soil interface, h is the soil matric suction head, θ is soil water content, $b(z,t)$ is root distribution coefficient, C is solute concentration of the soil solution, and Γ is a coefficient which transforms salt concentration units into the appropriate water suction (pressure) units. The ΓC represents the osmotic component of the soil water potential and describes the effect of soil salinity on water uptake by plant roots. When salt concentration is high (osmotic potential is low), plants may not be able to extract sufficient water to meet transpiration demand and, as a result, actual transpiration becomes less than the potential and crop yield is reduced. For this to occur, the value of $\Phi(t)$ in [4] must be restricted (see boundary condition [14f]).

The root distribution coefficient b encapsulates the geometry of the flow to the roots and may be written as

$$b(z,t) = \frac{1}{L^2} \frac{w(z,t)}{\bar{w}} \quad [5]$$

where $\bar{L}$ is the average distance between roots when the roots are uniformly distributed over the entire volume of soil and $w(z,t)/\bar{w}$ is the relative distribution of root density with depth at t , such that $1/(\bar{Z}\bar{w}) \int_0^Z w(z,t) dz = 1$. Adopting the approach of Feddes et al. (1974), it is assumed that

$$w(z,t) = w(0,t) \exp[-\alpha(t)z] \quad [6]$$

where $\alpha(t)$ is a root distribution parameter characterizing the rate of root density changes with depth at time t .

Values of $\theta(z,t)$, $h(z,t)$ and $C(z,t)$ to be inserted in [4], in order to calculate T_r from [3], are determined by solving the water flow and solute transport equations [7] and [8], subject to initial and boundary conditions appropriate to irrigation. The saturated-unsaturated one-dimensional water flow in a homogeneous profile, with water uptake represented by a macroscopic extraction term S , is given by

$$\frac{\delta \theta}{\delta t} = - \frac{\delta q}{\delta z} - S(z,t) = \frac{\delta}{\delta z} \{ K[\theta(z,t)] \frac{\delta H(\theta)}{\delta z} \} - S(z,t) \quad [7]$$

where q is the specific (Darcy's) water discharge and H is the hydraulic head, the sum of the suction head $h(\theta)$ and the gravitational head z . In a simple system when effects of adsorption, precipitation, dissolution, and salt uptake by plants may be neglected, the value of $C(z,t)$ in Eq. [4] is determined from the solution of the diffusion-convection equation for transient vertical transport

$$\frac{\delta(C\theta)}{\delta t} = \frac{\delta}{\delta z} [\theta D(\theta,q) \frac{\delta C}{\delta z} - qC] \quad [8]$$

where D is the hydrodynamic dispersion coefficient, which is described approximately by

$$D = \epsilon q / \theta \quad [9]$$

with ϵ being the dispersivity. Note that horizontal flow components were neglected so that only derivatives with respect to t and z were maintained in Eqs. [7] and [8]. This simplifies the problem considerably and is justified because lateral flows are negligibly small compared with vertical ones (e.g. Russo and Bresler, 1981b),

even when active root uptake is involved. While the influence of lateral subsurface flow is small, the lateral overland flow may be important. This effect of surface flow is discussed in Part 3.

To solve Eqs. [7] and [8] simultaneously, the hydraulic functions $\theta(h)$ and $K(h)$ (or $K(\theta)$) must be specified. For this study we adopted the Brooks and Corey (1964) relationships

$$\begin{aligned}\theta(h) &= (\theta_s - \theta_r)(h_w/h)^\beta + \theta_r; & h \leq h_w \\ \theta &= \theta_s; & h \geq h_w\end{aligned}\quad [10]$$

$$\begin{aligned}K(h) &= K_s(h_w/h)^\epsilon; & h \leq h_w \\ K &= K_s; & h \geq h_w\end{aligned}\quad [11]$$

where h_w is air entry value of h , θ_s and θ_r are saturation and residual values of θ , respectively, K_s is saturated hydraulic conductivity, and β and ϵ are constant coefficient characteristics of the soil.

In order to solve Eqs. [7] and [8], they must be supplemented with initial and boundary conditions appropriate for irrigation with water of varied salt concentration. During an irrigation season, water and salt intake by the soil profile depends on the rate of water application R , on salinity of the irrigation water C_o , on the duration of irrigation t_i , on the hydraulic functions [10] and [11], and on the transport conditions at the soil surface including irrigation uniformity. The soil surface ($z=0$) boundary conditions during water application (irrigation) are either prescribed maximum possible (potential) flux or maximum θ (*i.e.*, θ_s). Thus, at the soil surface boundary $q(0,t)=R$ as long as $\theta \leq \theta_s$ and $\theta=\theta_s$ when $q(0,t)<R$. The latter conditions may lead either to ponding with creation of water columns of volume $\int [R - q(0,t)]dt$ which is absorbed by the soil after cessation of irrigation, or complete runoff, *i.e.*, evaporation starts ($R < 0$) immediately when irrigation ceases. After cessation of water application and eventual infiltration of excess water and in the absence of runoff, evapotranspiration takes place.

During evapotranspiration there is no solute flux across the soil surface ($z=0$) and the potential rate of evaporation and transpiration depends on atmospheric conditions as well as on crop coefficient

$$F = T_p/E_o \quad [12]$$

where T_p is potential (maximum possible) transpiration and E_o is potential evapotranspiration, the sum of T_p and potential evaporation

E_p . Actual transpiration will be less than potential transpiration when a critical prescribed (limiting) value (h_{cr}) of $\Phi(t)$ in Eq. [4] is achieved. Transpiration (T_p) will equal potential (T_p) if $\Phi(t)$ is above the limiting value (h_{cr}). Similarly, evaporation will equal potential (E_p) if water content (θ) at the soil surface ($z=0$) is above air-dry water content (θ_a), i.e., $-q(0,t) = E_p$ for $\theta > \theta_a$, and it will be less than potential E_p when θ at $z=0$ is at air dry θ_a , i.e., $\theta = \theta_a$ and $E_p < -q(0,t)$, which is a result of $\theta = \theta_a$.

Mathematically, the boundary conditions or the partial differential equations [7] and [8], at $z=0$ and $t>0$, are summarized as follows:

During irrigation ($0 < t \leq t_1$):

$$q(0,t) = -K(\theta)(\delta h / \delta z - 1) \leq R(t) \quad [13a]$$

$$h(0,t) \leq 0 \quad \text{or} \quad \theta(0,t) \leq \theta_a \quad [13b]$$

$$-\theta(0,t)D_p \delta C / \delta z + q(0,t)C(0,t) = q(0,t)C_o(t) \quad [13c]$$

where D_p is molecular diffusivity, C_o is concentration of irrigation water and runoff is given by the difference $R(t) - q(0,t)$. Here t_1 is prescribed and runoff (i.e., $R(t) - q(0,t)$) is allowed. When runoff is not allowed (i.e., in the theoretical case of ponding with creation of a water column), the volume of water V which is eventually taken up by the soil is given by

$$\int_0^{t_p} R(t) dt + \int_{t_p}^{t_1} q(0,t) dt = V \quad \text{for ponding, } (t_1 > t_p) \quad [13d]$$

$$\int_0^{t_1} R(t) dt = V \quad \text{for no ponding} \quad [13e]$$

in which V is prescribed and t_p , the ponding time, and t_1 , the infiltration time, are variables computed according to the application rate R and soil properties which jointly dictate the value of the water intake rate $q(0,t)$.

During subsequent evapotranspiration:

$$-q(0,t) = -K(\theta)(\delta h / \delta z - 1) \geq E_p; E_p < 0 \quad [14a]$$

$$h(0,t) \geq h_a \quad \text{or} \quad \theta(0,t) \geq \theta_a \quad [14b]$$

$$-\theta(0,t)D_p [\theta(0,t), q(0,t)] \delta C / \delta z + q(0,t)C(0,t) = 0 \quad [14c]$$

The boundary conditions [14] are supplemented by

$$T_r(t) = - \int_0^Z S(z,t) dz \quad [14d]$$

$$|T_r(t)| \leq |T_p(t)| \quad [14e]$$

$$\Phi(t) \geq h_{cr} \quad [14f]$$

Here, h_a is air-dry pressure head h , and h_{cr} is the critical limited value of root water potential. The actual critical value of h_{cr} is considered to depend on the irrigated crop to be grown.

The required initial conditions ($t=0$) are:

$$C(z,0) = C_n(z) \quad [15a]$$

$$\theta(z,0) = \theta_n(z) \text{ or } h(z,0) = h_n(z) \quad [15b]$$

where C_n , or θ_n , and h_n are prescribed initial values of salt concentration, water content and suction head, respectively.

Equations [1] through [15] enable one to compute relative crop yield under irrigation conditions as a function of various plant, soil and controllable (decision) parameters. This function can be written in a general form as:

$$Y=f[m,L,\alpha(t),h_{cr},F,K_s,h_w,\theta_s,\theta_r,\epsilon,\beta,b,\acute{e},E_o(t),\theta_a,\theta_n(z),C_n(z),C_o(t),R(t)] \quad [16]$$

Equation [16] contains 19 parameters which can be divided into six categories, as follows:

- (i) Plant parameters which include m - crop factor (Eq. [1]), L - average root distance (Eq. [5]), α - root distribution parameter (Eq. [16]), h_{cr} - limiting root water potential (Eq. [14c]), and F - crop coefficient (Eq. [12]).
- (ii) Soil salinity parameter: $\acute{e}$ - the dispersivity (Eq. [9]).
- (iii) Soil water parameters: β - soil hydraulic parameter (Eq. [10]), ϵ - soil hydraulic parameter (Eq. [11]), h_w - air entry value (Eqs. [10], [11]), θ_s - saturated θ (Eq. [10]), θ_r - residual θ (Eq. [10]), K_s - hydraulic conductivity at soil saturation (Eq. [11]).
- (iv) Non-controllable boundary conditions: θ_a - air-dry water content (Eq. [14b]), E_p - potential evaporation (Eq. [14a]), T_p - potential transpiration (Eq. [14e]). Actually, T_p and E_p are determined by potential evapotranspiration $E_o = E_p + T_p$ and F (Eq. [12]).
- (v) Controllable boundary conditions: R - rate of water application (Eq. [13a]), and C_o - salt concentration of the irrigation water (Eq. [13c]).

(vi) Initial conditions: (C_n - initial salt concentration in soil solution (Eq. [15a]), and θ_n or h_n initial water content or initial suction head (Eq. [15b])).

(b) Numerical Scheme and Computer Program

The aim is to compute Y_R from [2] for the entire irrigation season. For this purpose T_r was computed by a finite difference solution of the partial differential equations [7] and [8] supplemented with the boundary and initial conditions [13], [14] and [15] and with the aid of Eqs. [3] through [6] and [9] through [11]. The numerical scheme has been developed during the last two decades (Hanks and Bowers, 1962; Hanks et al., 1969; Nimah and Hanks, 1973; Bresler, 1973; Feddes et al., 1974) and its essentials are described in Bresler et al. (1982).

In the present finite difference scheme (as adapted by R.J. Hanks) the entire root zone profile from 0 to Z is unevenly divided into 13 depth increments, with prescribed nodal points at $z = 0.5, 2, 3, 8, 16, 32, 48, 64, 80, 96, 112, 128, \text{ and } 144$ cm. The value of the time increment changes with time and adjusts itself to the boundary conditions and balance of the entire flow system, with a minimum value of $t = 0.001$ day and a maximum of 0.25 day. The program starts at $t=0$ with initial conditions of $\theta_n(z) = \theta_r$ and prescribed C_n and the solution is advanced in time until the final time is reached. Irrigation season is arbitrarily selected to be of $T=100$ days, in which irrigation water is applied six times (every 16 days). During any irrigation cycle water application rate $R(t)$, the potential evaporation $E_p(t)$ and the potential transpiration were given as step functions of time t . During the water application period of any cycle in the no ponding case, V (eq. [13]) was prescribed, $R(t)$ was set equal to $0.5 K_s$, and t_1 was calculated by [13e]. Otherwise runoff is allowed ($t_1 > t_p$), so that t_1 and $R(t)$ were set as described in the boundary condition [13]. Therefore, the "runoff" - "no-runoff" switch can be used corresponding to the two cases of prescribed t_1 or prescribed V , respectively.

Input data include: number of depth increments, number of irrigation applications, the 19 parameters of Eq. [16], values of t_1 , V , time for the whole irrigation season ($T = 100$ days) and depth of each nodal point in the z direction. Output data include: water content and salt concentration as functions of time and depth ($\theta(z,t)$, $C(z,t)$), evapotranspiration (ET), transpiration (T_r), evaporation, $Y_R = T_r/T_p$, V , runoff, and balances of water and salt.

3. METHODOLOGY: THE STOCHASTIC APPROACH

The deterministic model presented in the preceding section presumably enables one to predict the crop yield Y as a function of a large number, i.e., 19, of parameters (Eq. [16]). Most of these parameters, denoted here, for simplicity, as $a_1, a_2, \dots, a_n$, are subject to uncertainty for a few possible reasons. First, those parameters which are spatially variable, e.g. soil hydraulic properties and nonuniform rate of water application, change in an irregular fashion over the field. This variability has been documented in a few previous studies, (e.g. Nielsen et al., 1973; Biggar and Nielsen, 1976; Warrick et al., 1977; Russo and Bresler, 1981a; Stern and Bresler, 1983). Since measurements are carried out only at a limited number of points, the actual values at any other point are uncertain. These parameters are regarded, therefore, as random spatial functions (regionalized variables) $a_i(\underline{x})$, where $\underline{x}$ is the coordinate vector in the horizontal plane. We neglect here vertical variability because we consider the upper soil layer with a thickness on the order of 1-2m while field size is on the order of hectares. Also, Russo and Bresler (1981b) showed that due to the length scale characterizing the variability of hydraulic properties in the x, y plane, the lateral flows are negligibly small compared with the vertical flows. The assumption regarding vertical variability may not be adequate if a low permeable layer is present beneath the surface, and the analysis should be modified accordingly.

In the simplest representation, the statistical structure of $a_i(\underline{x})$ is characterized by its expected value $\langle a_i(\underline{x}) \rangle$ and autocovariance $C_i(\underline{x}', \underline{x}'')$ or variogram $G_i(\underline{x}', \underline{x}'')$. Furthermore, we shall assume second order stationarity, i.e.,

$$\langle a_i(\underline{x}) \rangle = \text{const} ; C_i(\underline{x}', \underline{x}'') = C_i(\underline{x}' - \underline{x}'') = C_i(r) \quad [17]$$

The second, additional, source of uncertainty is related to measurement errors of the parameters of interest. If errors are not systematic, this effect is manifested in a degenerate covariance function, i.e.,

$$C_i(r) = C_0, \quad r = 0; \quad C_i = 0, \quad |r| > 0 \quad [18]$$

Generally, the covariance is a sum of the two types of functions [17] and [18] or conversely, the variogram of $a_i(\underline{x})$ has a "nugget" effect C_0 ([18]).

The third aspect is related to estimation errors, i.e. to the inference of values of $\langle a_i \rangle$ and of the parameters characterizing C_i , e.g. $\sigma_i^2 = C_i(0)$. Since these parameters have to be derived from a limited set of measurements or by inference using field data, one

can obtain only their estimates, which in turn are uncertain and can be regarded as random variables.

Since the yield Y of an irrigated crop is a function of a_1 , it is also a spatial random function through the intermediate of $a_1(x)$. The main aim of the present study is to investigate the variability of Y , as expressed by its first two moments, the expected value $\langle Y \rangle$ and the variance σ_Y^2 , as functions of the spatial variability of the various a_i ($i = 1, 2, \dots, n$). The general relationships between the statistical moments of Y and those of a_i were obtained previously (e.g. Feinerman et al., 1985) and are rederived here for the sake of completeness.

The expected value $\langle Y \rangle$ can be written in a general fashion as follows

$$\langle Y(x) \rangle = \int \int \dots \int Y(a_1, a_2, \dots, a_n) f(a_1, a_2, \dots, a_n; x) da_1 da_2 \dots da_n \quad [19]$$

where $f(a_1, a_2, \dots, a_n)$ is the joint probability density function (p.d.f.) of the values of $a_i(x)$ ($i = 1, 2, \dots, n$). The simplest assumption, and the one adopted here, is that a_i are statistically independent. This is definitely true for many of the parameters of Eq. [16], while it might not be correct for some others such as soil properties (see reservation in Part 3 of this study about parameters' dependence). For the purposes of the present study, in which second order approximations are considered, this simplification would not affect significantly the main conclusions. Thus, we may write

$$f(a_1, a_2, \dots, a_n) = f(a_1)f(a_2) \dots f(a_n) \quad [20]$$

A further simplification may be achieved if some variables, say $a_{m+1}, \dots, a_n$, have small variances or coefficients of variation. For such variables, we may expand Y in a power series as follows:

$$Y(a_1, a_2, \dots, a_m, a_{m+1}, \dots, a_n) = Y(a_1, a_2, \dots, a_m, \langle a_{m+1} \rangle, \langle a_{m+2} \rangle, \dots, \langle a_n \rangle) + \sum_{i=m+1}^n \frac{\delta Y(a_1, \dots, a_m, \langle a_{m+1} \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_i \rangle} a'_{i1} + \frac{1}{2} \sum_{i=m+1}^n \sum_{j=m+1}^n \frac{\delta^2 Y(a_1, \dots, a_m, \langle a_{m+1} \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_i \rangle \delta \langle a_j \rangle} a'_{i1} a'_{j1} + \dots \quad [21]$$

where $a'_{i1} = a_i - \langle a_i \rangle$ is the residual. Substitution of Y from [21] and f from [20] into [19] yields

$$\begin{aligned} \langle Y \rangle = & \int \dots \int Y(a_1, a_2, \dots, a_m; a_{m+1}, \dots, a_n) f(a_1) f(a_2) \dots \\ & f(a_m) da_1 \dots da_m + \frac{1}{2} \sum_{i=m+1}^n \sigma_{i1}^2 \frac{\delta^2}{\delta \langle a_i \rangle^2} \int \dots \int Y(a_1, a_2, \dots, a_m, \\ & \langle a_{m+1} \rangle, \dots, \langle a_n \rangle) f(a_1) f(a_2) \dots f(a_m) da_1, \dots da_m + O(\sigma_{i1}^4) \end{aligned} \quad [22]$$

The simplification in $\langle Y \rangle$ [22], as compared with [19], stems from the reduction of the number of integrals from n to m . In particular, if the Taylor expansion [21] is appropriate for all a_i ($i=1, \dots, n$), i.e., $m=0$, no quadrature is needed, and Eq. [22] becomes

$$\begin{aligned} \langle Y \rangle = & Y(\langle a_1 \rangle, \dots, \langle a_n \rangle) + \frac{1}{2} \sum_{i=1}^n \sigma_{i1}^2 \frac{\delta^2 Y(\langle a_1 \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_i \rangle^2} + \\ & + O(\sigma_{i1}^4) \end{aligned} \quad [23]$$

If we retain only the first term of [23], the simple result is that the expected value of the yield is the yield for the expected value of the parameters. The sum following the first term is a correction depending on the variances and the second derivatives of the yield, and this term is supposed to be small compared with the first, because of the smallness of the variances σ_{i1}^2 ($i = 1, \dots, n$).

In the absence of the true values of the expected values $\langle a_i \rangle$, with only their estimates $\hat{a}_i$ available, the estimate of $\langle Y \rangle$ denoted by $\hat{Y}$, has the following expression, replacing [23]:

$$\hat{Y}(\underline{x}) = Y(\hat{a}_1, \dots, \hat{a}_n) + \frac{1}{2} \sum_{i=1}^n (\hat{\sigma}_{i1}^2 + \sigma_{i1}^2) \frac{\delta^2 Y(\hat{a}_1, \dots, \hat{a}_n)}{\delta \langle a_i \rangle^2} \quad [24]$$

where $\hat{\sigma}_{i1}^2$ is the estimate of the variance of $a_i(\underline{x})$ and σ_{i1}^2 is the

variance of estimation of a_1 (Mood and Graybill, 1963).

The variance of the yield (σ_Y^2) is also of considerable interest. In a general manner, it has the expression (similar to [19])

$$\sigma_Y^2 = \int \int \dots \int [Y(a_1, \dots, a_n) - \langle Y \rangle]^2 f(a_1, a_2, \dots, a_n) da_1 da_2 \dots da_n \quad [25]$$

which again can be simplified by using [20]. Under the same assumption which leads to [22], namely that $a_{m+1}, \dots, a_n$ have small variances, substitution of the Taylor expansion [21] into [25] yields

$$\begin{aligned} \sigma_Y^2 = & \langle [Y(a_1, \dots, a_m, \langle a_{m+1} \rangle, \dots, \langle a_n \rangle) - \langle Y(a_1, \dots, a_m, \\ & \langle a_{m+1} \rangle, \dots, \langle a_n \rangle) \rangle]^2 \rangle + \sum_{i=m+1}^n \sigma_i^2 \int \int \dots \int \\ & \left[\frac{\delta Y(a_1, \dots, a_m, \langle a_{m+1} \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_1 \rangle} \right]^2 f(a_1) f(a_2) \dots f(a_m) \\ & da_1, da_2, \dots, da_m + O(\sigma_i^4) \end{aligned} \quad [26]$$

Again, if we adopt the Taylor expansion [21] for all parameters a_i , [26] becomes

$$\sigma_Y^2 = \sum_{i=1}^n \sigma_i^2 \left[\frac{\delta Y(\langle a_1 \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_i \rangle} \right]^2 + O(\sigma_i^4) \quad [27]$$

The simple formula [27] renders the variance of the yield in terms of the variances of the various random functions $a_i(x)$, on which Y depends, and on the first derivatives (slopes) of Y with respect to the expected values of a_i . Again, the estimate $\hat{\sigma}_Y^2$ of σ_Y^2 is given by a similar equation, with σ_i^2 replaced by $\hat{\sigma}_i^2 + \sigma_{a_i}^2$ and $\langle a_i \rangle$ by $\hat{a}_i$, respectively.

Summarizing these developments, Eqs. [22] and [26], or their simplified versions [23] and [27], gives the dependence of the expected value and variance of the yield upon those of the various random parameters $a_i(x)$ ($i = 1, \dots, n$). The aim of the remaining parts of the study is to apply these results to the large group of parameters of the list at the end of Sect. 2.

These results can be extended to the statistical moments of the yield averaged over the space (e.g. Feinerman et al., 1985), but this line is not pursued here and we satisfy ourselves with examining the impact of the variability of various a_i upon $\langle Y \rangle$ and σ_Y^2 solely.

4. SUMMARY OF METHODOLOGY: THE COMPREHENSIVE SCHEME

The two models described in Sects. 2 and 3 provide the elements needed in order to achieve our goal, namely, to analyze the impact of various soil water and irrigation parameters upon the yield. We shall enumerate here the various steps followed toward this aim.

(i) First, all flow parameters of Eq. [16] have to be assigned values pertaining to the particular field under consideration. Part 2 of this study is devoted mainly to the collection of some existing data from the literature. For the statistical computations of Sect. 3 we need both the expectation value and the standard deviation of each stochastic parameter of [16]. Furthermore, if the variance of a random parameter a_i is not sufficiently small, we have to evaluate the integral over the p.d.f., $f(a_i)$, of this particular parameter.

(ii) Subsequently, it was assumed that the prerequisites for the Taylor expansion for all parameters are satisfied and approximations [23] and [27] are valid. Then, in order to evaluate the yield expected value [23] and variance [27], we have to compute for $i=1, 2, \dots, n$ the three quantities

$$Y(\langle a_1 \rangle, \langle a_2 \rangle, \dots, \langle a_n \rangle) \quad [28a]$$

$$\frac{\delta Y(\langle a_1 \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_i \rangle} \quad (i=1, 2, \dots, n) \quad [28b]$$

$$\frac{\delta^2 Y(\langle a_1 \rangle, \dots, \langle a_n \rangle)}{\delta \langle a_i \rangle^2} \quad (i=1, 2, \dots, n) \quad [28c]$$

i.e., to evaluate the yield and its derivatives, at the expected value $\langle a_i \rangle$. These quantities have been computed numerically as follows.

First, Y (eq. [28a]), with the parameters of Eq. [16], was evaluated numerically by using the numerical procedure described in Sect. 2. Subsequently, each stochastic parameter a_i was given a set of values around $\langle a_i \rangle$, while all other parameters were kept constant and equal to their expected value. The variation of $Y(\langle a_1 \rangle, \dots, \langle a_{i-1} \rangle, a_i, \langle a_{i+1} \rangle, \dots, \langle a_n \rangle)$ as a function of a wide range of a_i for $i = 1, 2, \dots, n$ has been represented graphically and a few such graphs will be illustrated in Part 2. Furthermore, by using splines, the derivatives in [28] have been evaluated at $a_i = \langle a_i \rangle$. The procedure requires, therefore, solution of the yield problem numerically $n \times N$ times, where n is the number of random parameters and N is the number of values assigned to each parameter in order to obtain its dependence upon a_i .

(iii) To check the validity of the Taylor expansion approximation [21] and its outcomes [23] and [27], the graph $Y(\langle a_1 \rangle, \dots, \langle a_{i-1} \rangle, a_i, \langle a_{i+1} \rangle, \dots, \langle a_n \rangle)$ has been examined in the range $\langle a_i \rangle - 2\sigma_i$ to $\langle a_i \rangle + 2\sigma_i$. If the graph is linear, the Taylor expansion approximation was accepted (as a matter of fact Eqs. [23] and [27] are exact if Y is linear).

(iv) Following Eq. [27], the contribution $\sigma_1^2 (\delta Y / \delta \langle a_1 \rangle)^2$ for each a_1 was subsequently evaluated as well as the sum σ_Y^2 [27]. The ratio

$$\frac{\sigma_1^2 (\delta Y / \delta \langle a_1 \rangle)^2}{\sigma_Y^2} \quad [29]$$

is a measure of the impact of the uncertainty of variable a_1 upon the yield variability. Furthermore,

$$\frac{\sigma_Y}{\langle Y \rangle} = \frac{[\sum_{i=1}^n \sigma_i^2 (\frac{\delta Y}{\delta \langle a_i \rangle})^2]^{1/2}}{Y(\langle a_1 \rangle, \dots, \langle a_m \rangle)} \quad [30]$$

is the yield coefficient of variation, reflecting the overall degree of yield variability.

The above results exhaust the information which serves for the analysis and discussion of yield variability in Part 3.

(v) In those cases in which the requirements of linearity of the Taylor expansion approximation were not met, the yield expected value for the particular parameter a_1 was computed by numerical integration of the first term of [22]. This implied, obviously, knowledge of the p.d.f. $f(a_1)$. The calculation was carried out by using the extended Simpson's rule.

As for the variance in the first term on the r.h.s. of [26], the same numerical quadrature with Y replaced by $(Y - \langle Y \rangle)^2$ was employed. These expressions are not restricted to small variance σ_1^2 , but are subjected to numerical errors. To limit those errors, the value of Y were derived by using splines to represent the relationship $Y(a_1, \dots, a_m)$, and by taking a sufficiently large N .

As for the contribution to σ_Y^2 [26] of the remaining terms, we used approximation [27] instead, with the derivatives of Y evaluated for $Y = \langle Y \rangle$.

Finally, if more than one parameter has large variance, the numerical evaluation of the multiple integral of [26] becomes prohibitive. Indeed, it is then necessary to calculate numerically the yield Y for N^m combinations of values, where m is the number of such parameters and N is the number of values. To circumvent this difficulty, in the few cases of this nature, we carried out the numerical quadratures for each parameter separately.

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VARIABILITY OF YIELD OF AN IRRIGATED CROP AND ITS CAUSES:
II. INPUT DATA AND ILLUSTRATION OF RESULTS

ABSTRACT

The data needed in order to evaluate the impact of the variability of different parameters upon crop yield variability, were collected from various sources in the literature. Two fields (Panoche in the USA, Bet Dagan in Israel), which had been studied extensively in the past, were selected as representative for a wide range of soil conditions. In a similar manner, standard values of the expectations and extreme values of variances of other soil-plant-atmospheric parameters were inferred from the literature. The adopted values are summarized in a table. The dependence of the relative yield and its derivatives upon each of 19 parameters, while the remaining ones are kept fixed at their standard expected value, is illustrated.

1. INTRODUCTION

In Part I (Bresler and Dagan, 1986) which will be denoted hereafter for brevity P1, a model which predicts values of crop yield expectation and variance, as functions of mean and variance of the various factors affecting yield, was presented. The main aims of the model are to: (i) single out from among the 19 soil-plant-atmospheric parameters (P1, Eq. [16]) those stochastic parameters which have a significant impact upon the statistical moments of yield; and (ii) evaluate the yield moments.

The methodology outlined in P1 permits one, in principle, to achieve these aims. However, knowledge of the probability density function (p.d.f.) of various parameters is required. In the absence of information about specific marginal p.d.f., we arbitrarily assumed normal p.d.f. Hence, the expected value and variance of the various stochastic parameters of Eq. [16] (P1), which serve as input, are then needed. The measurement of these data, and the inference of their statistical moments, constitute a formidable task which is not practical, if at all possible. Indeed, the considerable effort to be invested in a particular field cannot be justified by the worth of the information for evaluating yield. Furthermore, some of the parameters cannot be measured directly or known a priori. Hence, one is caught in a vicious circle: in order to focus attention on a small number of parameters, the variability of which has a definite impact upon yield, all parameters must be known first.

To avoid this vicious circle it was decided to screen data from the literature and to give a wide range of conceivable values to the expected value of the various parameters. Furthermore, the variances of the stochastic parameters have been given the largest values

compatible with data found in the literature. It is hoped that under these conservative conditions, those factors which have a small effect upon yield uncertainty can be eliminated from further investigations.

In the present part we present the input data on the various parameters and subsequently their incorporation in the scheme of P1.

2. DATA ON THE PARAMETERS AFFECTING YIELD AND ON THEIR STATISTICAL MOMENTS

We have selected two fields, the soil properties of which have been investigated extensively in the past, as test cases for our evaluation. The two fields are Panoche in the USA (Nielsen et al., 1973, Warrick et al., 1977) and Bet Dagan in Israel (Russo and Bresler, 1981). These two fields differ markedly in three respects: size (Panoche, 150 ha; Bet Dagan, 0.80 ha), average saturated soil hydraulic conductivity (Bet Dagan, high saturated conductivity of ~ 15 cm/h; Panoche, low conductivity of 1 cm/h), and variability of soil properties (Panoche is much more variable than Bet Dagan, due in part to their different areas). It was postulated, therefore, that the conclusions drawn from the simulation of these two fields and from the validation of the Bet Dagan field (see Part 3, Dagan and Bresler, 1987 of this series) are of some generality, as they encompass a large range of soil parameters.

For the other soil data enumerated in Eq. [16] (P1), we had to rely on other literature sources, since little information is available for crops grown in the above two fields. Again, we have tried to cover a large range of expected values and to adopt high values for the variances of the various stochastic parameters. A detailed list of data follows, including the pertinent literature sources.

(a) Plant parameters ($\bar{L}$, α , h_{cr} , F , m)

Although the physics of the flow process from soil into the roots is poorly understood, the tendency is to describe water uptake by analogy to Darcy's law (Eq. [4] in P1). In this expression, the geometry of the flow path is represented with the aid of two parameters: $\bar{L}$, which is the average distance between individual roots, and α , a single parameter describing the distribution with depth of root density. Measurements of $\bar{L}$ are not available or are very scarce, but α data for various field crops were published by Feddes et al., (1974). As for the expectation and variances of $\bar{L}$, Nimah and Hanks

(1973) and Feddes et al. (1974) used in their numerical models a value of $\bar{L} = 1$ cm, which seems to be a reasonable standard expected value.

For the variance σ_L^2 , we assumed a uniform (rectangular) p.d.f. and decided arbitrarily on the basis of physical meaning of $\bar{L}$ that the range d_L of the p.d.f. would be between 0.01 and 2 cm, which leads to $\sigma_L^2 = (1/3) (d_L/2)^2 = 0.33 \text{ cm}^2$ (i.e., $CV = 0.57$).

Values of expectation and variance for the parameter α , which characterizes the root distribution, were taken from Table 2 of Feddes et al. (1974). In that table, time average values of α were taken and linear regression analyses of the equation $\ln[w(z)] = \ln[w(0)] - \alpha z$ were performed on data of root distribution with depth, $w(z)$, of clover, cotton, corn, grasses, oats, peas, sorghum, soybean, tomatoes, wheat, and orange trees grown on soil with various textures, i.e., sand, clay loam, sandy loam, clay loam, silt loam, and sandy clay. Values of α in this table range between 0.01 and 0.07 cm^{-1} , with an estimated mean of 0.0375 cm^{-1} . The variance σ_α^2 was estimated from the standard deviation of the regression to be $2.25 \times 10^{-4} \text{ cm}^{-2}$. Note that one may get a similar value of σ_α^2 with the assumption of uniform distribution for α , i.e.,

$$\sigma_\alpha^2 = \frac{1}{3} \left(\frac{0.07 - 0.01}{2} \right)^2 = 3 \times 10^{-4} \text{ cm}^{-2}$$

An additional important plant parameter appearing in the root extraction equation [4] of P1 is the limiting value h_{cr} of the total effective root water potential Φ . Explicit information about ranges, expectation, and variance of h_{cr} is scarce because direct experimental determinations of this factor are extremely difficult to achieve. It is customary to assume a standard value of -1.5 MPa as the critical value in soil which causes all plants to wilt and this was indeed the value that was used by Nimah and Hanks (1973), Feddes et al. (1974), Bresler et al. (1979) and others. However, in view of a recent analysis (Bresler and Hoffman, 1986, and Part 3 of this study), we chose two extreme standard deterministic values for h_{cr} , namely, -0.4 MPa and -1.5 MPa. Since data on the variance of h_{cr} are lacking, we assumed h_{cr} to have a uniform p.d.f. and variance of $\sigma_{h_{cr}}^2 = (0.3/2)^2 (1/3) = 0.0075 \text{ (MPa)}^2$, for an interval of variation of 0.3 MPa (i.e., $\ln 1.4$). We also considered h_{cr} as deterministic (see table 1), i.e., $\sigma_{h_{cr}}^2 = 0.0$ as another extreme case.

Crop coefficient $F = t_p/E_o$ is an empirical ratio that may be derived from experimental data. In field crops F is generally a function of time because the area covered by the crop canopy depends on plant growth; hence, F is maximal during that part of the growing season when crop canopy is at effective full cover. Ideally, potential evapotranspiration, E_o , characterizes the evaporative demand

determined by meteorological conditions and considered to be uniform and deterministic over a given field, while F is a measure of the capability of the crop-soil surface to meet this demand. The problem is that E_0 cannot simply be described for all climates and crops because of the effects of leaf area index, and other morphological and physiological characteristics of crop canopy, or exchange of energy and aerodynamic processes over the field. This complexity accounts for large uncertainties in estimating of crop coefficient. The crop coefficient described by Eq. [12] of P1 includes only the effects of potential transpiration (T_p) from plants, while potential evaporation directly from the soil is given by the difference between deterministic E_0 and stochastic T_p . Data are available, however, for the crop coefficient which is defined as the ratio between actual (not potential) evapotranspiration from the crop and standard reference ET, which is generally a little less than evaporation from a standard Class A pan (Wright, 1981). Because data on F based on potential transpiration and potential evapotranspiration are lacking, we use for estimating standard F and its variance, the value of the ratios between actual ET and potential - reference ET as they appear in Table 1 of Wright (1981) for seven crops (barley, peas, sugar beet, potato, corn, beans, winter wheat) and for 70% time from planting to effective cover. Estimated values, which are based on an average of the seven crops are 0.7 for the expectation and 0.022 for σ_F^2 .

Estimates of the fifth crop parameter (m) (Eq. [1] of P1) are necessary when the actual crop yield, rather than the relative yield, is of interest. This may be an important factor when sensitivity analysis and screening of various parameters are done with respect to economic optimization of the profit the farmer gets from the yield. However, in the general framework of the present analysis, when the relative effect of the various parameters on the relative crop yield (Eq. [2] of P1), rather than the absolute yield, is sought, the effects of m need not be considered here, and have been eliminated from our analysis.

(b) Soil water parameters (K_s , h_w , θ_s , θ_r , ϵ , β)

The data on expectation and variances of the soil parameters of the Bet Dagan and Panoche fields were taken from the articles mentioned before.

The saturated conductivity K_s is related to a scaling factor (Peck et al., 1977) by

$$K_s = K_s^* \delta^2 \quad [1]$$

with K_s^* being defined by

$$K_s^* = \left[\int_0^{\infty} K_s^{1/2} f(K_s) dK_s \right]^2 \quad [2]$$

where $f(K_s)$ is the probability density function (p.d.f.) of K_s . It was found that the scaling parameter, δ , is log-normally distributed. Hence, for convenience, we define a new, normally distributed parameter

$$\tau = \ln \delta : N(\mu_\tau, \sigma_\tau^2) \quad [3]$$

with μ_τ being the expectation of τ , and σ_τ^2 its variance. Thus, saturated conductivity K_s (Eq. [1]) depends on the deterministic soil parameter K_s^* and on the stochastic variable τ with p.d.f. characterized by μ_τ and σ_τ^2 . The pertinent data are: Bet Dagan (Russo and Bresler, 1981) $K_s^* = 12$ cm/h, $\mu_\tau = -0.1$, $\sigma_\tau^2 = 0.16$; and Panoche (Warrick et al., 1977) $K_s^* = 0.22$ cm/h, $\mu_\tau = -0.6$ and $\sigma_\tau^2 = 1.4$.

The extremely low values of K_s of Panoche soil caused the prescribed rate of water application (R) to be too low for a reasonable irrigation time t_i . We decided to increase the deterministic value of K_s^* for Panoche soil from 0.22 to 1.5 cm/h, but K_s^* for Bet Dagan remained unchanged (12 cm/h). These two practical values of K_s^* serve our purpose of comparing two extremely different fields.

The above values have also been employed for the appropriate h_w to be used in the model according to

$$h_w = h_w^* \exp(-\tau) \quad [4]$$

because h is inversely proportional to the characteristic pore scale, while K is proportional to the square of characteristic length (Peck et al., 1977). We selected h_w^* of 10 and 4 cm for Bet Dagan and Panoche, respectively, which resulted in expected value of $h_w = 13$ cm for both soils.

When selecting values for θ_s , θ_r , ϵ , and β we faced the fact that the estimated variances (and in some cases of the variances of estimate or expectation values) are available only for Bet Dagan soil (Russo and Bresler, 1981). We therefore decided to adopt the same value of expectation and variance for Panoche and Bet Dagan. Hence, we use as a standard for θ_s a value of 0.4 (the average between 0.43 for Panoche and 0.37 for Bet Dagan), for θ_r a value of 0.085, for ϵ a value of 2.9, and for β a value of 0.45 or of -0.8 for $\ln \beta$ (since the $\ln \beta$ rather than the β was inferred by Russo and Bresler, 1981, to obey a normal p.d.f.). From the same data of Russo and Bresler (1981) we adopted the following estimate of the variances: for θ_s and θ_r a

value of $\sigma_e^2 = 0.0018$; for ϵ a value of $\sigma_\epsilon^2 = 1.4$; and for $\ln \beta$ a value of $\sigma_{\ln \beta}^2 = 0.15$.

(c) Salt transport parameter (ϵ)

Vast amounts of data have been collected on the value of the dispersivity, ϵ , under laboratory conditions. Field data, on the other hand, are rather scarce. Mean values and variances of D and q/θ (Eq. [9] in P1) calculated from intensive sampling have been published by Biggar and Nielsen (1976, Table 1) for Panoche soil. Using their values, we (Bresler and Dagan, 1981) concluded that ϵ is log-normally distributed with estimated values for $\mu_{\ln \epsilon}$ and $\sigma_{\ln \epsilon}^2$ of 0.41 and 1.4, respectively (ϵ is given in centimeters). Note that the estimated mean of field scale ϵ for Panoche is 3 cm (Biggar and Nielsen, 1976), compared with a laboratory scale value on the order of 0.1 cm.

(d) Non-controllable boundary conditions (E_0 , T_p , E_p , θ_u , θ_d)

Maximum possible (potential) evapotranspiration, E_0 , characterizes the evaporative demand as determined by meteorological conditions. To estimate E_0 , measurements of the evaporation from the U.S. Class A pan may be used. Ideally, the pan sites are in the middle of the plot and are surrounded by irrigated fields planted with the same crop. Hence, E_0 is taken here as a deterministic property, although temporal variations of E_0 have been measured. We selected arbitrarily an average (day and night) deterministic value of $E_0 = 0.3$ cm/day to be used for all the computations. As T_p and E_p (Eqs. [2], [14], P1), are obtained from appropriate values of E_0 and F ($E_0 = E_p + T_p$; $F = T_p/E_0$), these two parameters are spatial variables with statistical properties depending on the stochastic parameter F and the deterministic value of E_0 . Air-dry water content θ_d (Eq. [14b] in P1) has been taken to be identical to θ_r , while values for θ_u , the upper limit of θ , have been discussed before.

(e) Controllable boundary conditions - (decision variables) R and C_0

The salt concentration of the irrigation water, C_0 , does not change in a random manner over a particular field and was taken as a deterministic variable. Nevertheless, to test effects of salinity, we have made our computations with two deterministic values of C_0 expressed in terms of electrical conductivity of the irrigation water.

These are: $C_0 = 0.3$ dS/m to test irrigation with nonsaline water, and $C_0 = 5$ dS/m for irrigation with saline brackish water.

Contrary to C_0 , application rate, R , over a particular field is never uniform because any irrigation method exhibits nonuniformity to some degree. The Christiansen coefficient is commonly used to express quantitatively the degree of uniformity of water application for any given irrigation method. A unique relationship between the Christiansen coefficient, C_h , and coefficient of variation, CV , can be calculated for random water application and some common p.d.f. For example, if water application is normally distributed, then $C_h = 1 - .8CV$.

The relationship is slightly different if a uniform (rectangular) p.d.f, is assumed, i.e., $C_n=1-0.87CV$. At any rate, the relationships between the variance σ_R^2 and C_n depend on the distribution of R as well as on the average R (since $CV = \sigma_R/R$) or on total application depth V (since $V = \int R dt$ for the no-runoff case). Hence, the value of

σ_R^2 that would be selected for this study depends on a few factors: (i) uniformity of application, i.e., the assumed value for C_n ; (ii) type of distribution either normal or uniform (other p.d.f. may also be considered); and (iii) value of R (or V) selected as a standard. From Eqs. [11]-[4] and [13] of P1, one may infer that crop yield response to water quantity will also depend on the evaporative demand and therefore on E_o . Hence, for the sake of generality, and for the no-runoff case, we express the variable R in terms of seasonal water application relative to seasonal (cumulative) potential evapotranspiration. Such relative depth of irrigation water quantity (V_r) is defined by

$$V_r = \sum_{j=1}^N \int_0^{t_{1j}} R(t) dt / \int_0^T E_o(t) dt \quad [5]$$

In [5] j is the index of irrigation ($j=1, \dots, N$), N is the number of water applications (irrigations) in one season, and T is the length of an irrigation season. For each level of water salinity and soil type (Panoche and Bet Dagan), we considered several levels of V_r but selected $V_r = 0.6$ as the standard.

With this standard value the seasonal depth of applied irrigation water is 17.5 cm, whereas the seasonal cumulative evapotranspiration is 29.2 cm. Assuming a Christiansen coefficient of $C_n = 0.82$ and a normal distribution of water application gives $\sigma_{V_r}^2 = [(1 - 0.82)(0.6/0.80)]^2 = 0.018$. This value will increase as application becomes less uniform, as more water is applied, and as potential evapotranspiration decreases.

(f) Initial conditions (θ_n , C_n)

Initial water content, θ_n , at the beginning of the irrigation season was chosen to be identical to residual water content, θ_r , to simulate arid conditions in which the profile is completely dry at the end of one season and before the next season has started. A uniform profile of $\theta_n(z)$ was assumed. The estimated variance of θ_n was taken to be equal to that of θ_r .

Standard values of initial concentration (C_n) were taken to be equal to the concentration of the irrigation water (C_o). Data collected by Bresler et al. (1984) showed that initial salinity was normally distributed with a coefficient of variation (CV) value

averaged over four soil depths, of 0.68. Similar results and the same CV value for C_n were obtained by Russo (1984). Using this CV value in the two standard values for C_n leads to $\sigma^2_{C_n} = 0.042 \text{ (dS/cm)}^2$ when $C_n = 0.3$ and to $\sigma^2_{C_n} = 11.6$ when the initial concentration profile is 5 dS/m.

(g) Summary

Standard values of 18 stochastic and deterministic parameters and estimated variances of 14 stochastic parameters are summarized in Table 1.

The computation of Y and σ_Y^2 (P1) is carried out by assigning to each parameter its standard (or expected) value. The total number of cases which result from various combinations of the parameters of Table 1 is 16. Indeed, we have the two fields differ in their K_s^* , values, two standard deterministic values of C_o , two extreme values of h_{er} (either deterministic or stochastic), and two types of soil surface boundary conditions (runoff and no-runoff). Recall that in the case of runoff boundary conditions, all excess of ponded water is assumed to disappear instantaneously from the field by evaporation, by very fast runoff out of the field's borders, or by both. For the "no-runoff" case the ponded water infiltrates in the soil subsequent to interruption of water application. Since some of the 16 cases do not differ essentially, we have limited our investigation to 12 combinations of parameters. These various cases are enumerated in Table 1 of Part 3 (P3) of this series (Dagan and Bresler, 1987).

It should be remembered that for each combination of n parameters we have to keep $n-1$ fixed and equal to their standard-expected values and to derive numerically the dependence of Y and its derivatives (Eq. [28], P1) on the value of the remaining random parameter in a range of, say, its standard (or expected) value $\pm$ two standard deviations. In sect. 3 we shall illustrate the results of two such runs rendering relative crop yield $Y_R = Y/Y_p$.

Table 1: Standard expected values and estimated variances of 18 plant, soil, and boundary conditions parameters.

Type	Name	Eq.	Symbol	Units	Standard	Variance
Plant	Distance between roots	[5] ¹⁾	L	cm	1.0	0.33
	Root distribution	[6] ¹⁾	α	cm ⁻¹	0.0375	2.25x10 ⁻⁴
	Critical h: stochastic	[14 _r] ¹⁾	h_{cr}	MPa	-0.4; -1.5	0.0075
	deterministic	[14 _r] ¹⁾	h_{cr}	MPa	-1.5; -0.4	0.0
	crop factor (T_p/E_o)	[12] ¹⁾	F	-	0.7	0.022
Soil water	Square of mean $K^{1/2}$	[2] ²⁾	K_m^*	cm/hr	1.5* 12.0**	0.0 0.0
	ln of scaling factor	[3] ²⁾	τ	-	-0.6* -0.1**	1.4 0.16
	Mean pressure head	[4] ²⁾	h^*	cm	-4.0* -10.0**	0.0 0.0
	Constant in $\theta(h)$	[10] ¹⁾	$\ln \beta$	-	-0.8	0.15
	Saturated θ	[10] ¹⁾	θ_m	-	0.4	0.0018
	Residual θ	[10] ¹⁾	θ_r	-	0.085	0.0018
	Constant in $K(h)$	[11] ¹⁾	ϵ	-	2.9	1.4
Soil salinity	Dispersivity	[9] ¹⁾	$\ln \epsilon$	cm	0.41	1.4
Boundary conditions	Potential evapotranspiration	[12] ¹⁾	E_o	cm/day	0.3	0.0
	Relative irrigation depth	[5] ²⁾	V_r	-	0.6	0.018
	Air dry θ	[14b] ¹⁾	θ_a	-	0.085	0.0018
	Irrigation water	[13c] ¹⁾	C_o	dS/m	0.3	0.0
	concentration	[13c] ¹⁾	C_o	dS/m	5.0	0.0
Initial conditions	Initial salt concentration	[15a] ¹⁾	C_n	dS/m	0.3	0.042
		[15a] ¹⁾	C_n	dS/m	5.0	11.6
	Initial water content	[15b] ¹⁾	θ_n	-	0.085	0.0018

* Panoche soil.

** Bet Dagan soil.

¹⁾ Part 1 of this study

²⁾ Part 2 - the present part - of this study.

3. ILLUSTRATION OF RESULTS

The assigned standard values for expectation and variances of each parameter (Table 1) can now be used for the computations of the statistical moments described in Eqs. [23] and [27] in P1. To illustrate the results, we selected two representative cases which differ in their soil hydraulic properties, surface boundary conditions, and yield response relative to the variations of the relevant parameter. The two cases are:

(i) The runoff case in Panoche soil ($K_B^* = 1.5$ cm/hr, $\langle \tau \rangle = -0.6$, $\sigma_{\tau}^2 = 1.4$) with $C_0 = C_n = 5$ dS/m and $h_{cr} = 1.5$ MPa (case no. 12 in Table 1, P3). Impact of the soil hydraulic parameter, τ , on relative yield is illustrated when τ varies in the range $\langle \tau \rangle \pm 2\sigma_{\tau}$, i.e., between $-0.6 - 2.8 = -3.4$ and $-0.6 + 2.8 = 2.2$. Since runoff was permitted, V is changed with surface flux, $q(0,t)$, (which in turn is changed with τ), so that Eq. [5] is replaced by

$$V_r = \sum_{j=1}^N \int_0^{t_{1,j}} q(0,t) dt / \int_0^T E_0(t) dt.$$

All other parameters are taken at their standard (expected) values as given in Table 1.

(ii) Bet Dagan soil ($K_B^* = 12$ cm/h, $\langle \tau \rangle = -0.1$, $\sigma_{\tau}^2 = 0.16$) with no runoff (i.e., $R = 0.5$ $K_B = 4.9$ cm/h, see P1 sect. 2), $C_0 = C_n = 5$ dS/m and $h_{cr} = 1.5$ MPa (no. 5 in Table 1, P3). The impact of the controllable boundary condition, the decision parameter V_r on the relative yield, is illustrated when V_r varies between 0.2 and 1.1 (note that the range of water quantity relative to T_p , rather than to E_0 , is between 0.3 and 1.6). Again, all other parameters are at their standard values of Table 1.

To evaluate the expected value of the yield and its variance (Eqs. [23] and [27] in P1), we need to compute $Y(\langle a_1 \rangle, \langle a_2 \rangle, \dots, \langle a_n \rangle)$, $Y' = \delta Y / \delta \langle a_1 \rangle$ and $Y'' = \delta^2 Y / \delta \langle a_1 \rangle^2$, in which the random parameter a_1 is τ for case (i), and $a_1 = V_r$ for case (ii). The graphs of the functions $Y(\tau, \langle a_2 \rangle, \dots, \langle a_n \rangle)$ $Y(V_r, \langle a_2 \rangle, \langle a_3 \rangle, \dots, \langle a_n \rangle)$, $\delta Y / \delta \tau$, $\delta Y / \delta V_r$, $Y''(\tau)$, and $Y''(V_r)$ in the range $\langle a_1 \rangle \pm 2\sigma_1$ are illustrated in Figs. 1 and 2.

Figures 1 and 2 were obtained from the numerical scheme described in P1 for a given set (usually 10) of values of τ and V_r , with all the other parameters constant at their standard values. Cubic splines (i.e. piecewise cubic polynomials, with continuous second-order derivatives) were then used to interpolate in the intermediate points and to obtain the first and second derivatives of relative yield (Y_R) with respect to τ and V_r . Figures 1 and 2 are essentially graphs of the spline functions and their derivatives.

An examination of Fig. 1 shows that $Y(\tau)$ looks like the classical sigmoid type function. At low values of τ , the hydraulic

conductivity is very low, so that most of the applied water runs off at the soil surface and does not infiltrate into the soil profile. Consequently, the profile remains dry, transpiration is suppressed, and crop yield is low. As the value of τ increases, runoff is less, more water infiltrates into the soil, water content is higher, soil solution is diluted, and salt is displaced (leached) downward. As a result transpiration increases and crop yield is higher. As τ increases further, saturated K becomes greater than R ($K_s = R = K_s^*$ when $\tau = 0$), so that all applied water infiltrates into the soil. From this value of τ onward, any additional increase in τ would not create changes in $\theta(z,t)$ and $C(z,t)$, and therefore transpiration and crop yield do not change as τ increases further. Note that the plateau in the $Y(\tau)$ function occurs at values of relative yield slightly above 0.6, which is the "standard" yield which is obtained with all standard values of Table 1.

The bell shape of the curve of the first derivative $\delta Y / \delta \tau$ in Fig. 1 is obvious from the sigmoid $Y(\tau)$ curve. It starts at values close to zero at $\tau = -2.8$, increases slightly up to τ close to -1 , from where it rises to a maximum at the inflection point of $Y(\tau)$; then it goes down as τ increases and again reaches a value of zero along the corresponding plateau in the $Y(\tau)$ curve. The behavior of the second derivative $\delta^2 Y / \delta \tau^2$ is illustrated in Fig. 1. The picture of Y , $\delta Y / \delta \tau$, and $\delta Y / \delta V_r$ and $\delta^2 Y / \delta V_r^2$ of case (ii) in Fig. 2 is completely different from case (i). The $Y(V_r)$ curve increases, almost linearly until $V_r = 0.8$, it then rounds off, and approaches asymptotically the value of 1, when V_r approaches 1.2. Here, relative yield climbs beyond the plateau of Y in Fig. 1, because V_r exceeds the standard value of $V_r = 0.6$. Again, the first derivative $\delta Y / \delta V_r$ in Fig. 2 reflects the shape of the $Y(V_r)$ curve. It decreases slightly (*i.e.*, remains almost constant) with V_r in the almost linear $Y(V_r)$ range, decreases steeply in the rounded off range, and reaches a value close to zero at the largest value of V_r . Values of the second derivative in the $\delta^2 Y / \delta V_r^2$ curve of Fig. 2 are negative for the whole range of V_r between 0.2 and 1.2. This means that variations (non-uniformity) in water application would result in yield reduction (see Eq.[23] in P1).

To check the validity of the approximations [23] and [27] of P1, which are the outcome of the second order truncated Taylor expansion in σ^2 , we added horizontal bars to the lower part of Figs. 1 and 2 to denote the range $\langle \tau \rangle \pm \sigma_\tau$ and $\langle V_r \rangle \pm \sigma_{V_r}$, respectively. An examination of the curves at these ranges shows that the graph is approximately linear for most of the range of V_r (case (ii), Fig. 2), but non-linear for the range $\langle \tau \rangle \pm \sigma_\tau$ (case (i) Fig. 1). This means that the truncated Taylor expansion approximation is acceptable for V_r (Fig. 2), but is not valid for the case of the variable τ (Fig. 1).

For this latter particular parameter, Eqs. [22] and [26] of P1 become, respectively,

$$\langle Y \rangle = \int_a^b Y(\tau) f(\tau) d\tau \quad [6]$$

$$\sigma_Y^2 = \int_a^b [Y(\tau) - \langle Y \rangle]^2 f(\tau) d\tau. \quad [7]$$

Here, $a = \langle \tau \rangle - 2\sigma_\tau$, $b = \langle \tau \rangle + 2\sigma_\tau$, $Y(\tau)$ is given by the bottom figure in Fig. 1 for the range of $\langle \tau \rangle \pm 2\sigma_\tau$, and $f(\tau)$ is the p.d.f. of τ , which is normally distributed (Warrick *et al.*, 1977), *i.e.*,

$$f(\tau) = \frac{1}{(2\pi)^{1/2} \sigma_\tau} \exp \left[-\frac{(\tau - \langle \tau \rangle)^2}{2\sigma_\tau^2} \right] \quad [8]$$

To calculate $\langle Y \rangle$ and σ_Y^2 from Eqs. [6] and [7] for Panoche soil, we substituted [8] with values of $\langle \tau \rangle$ and σ_τ^2 from Table 1, into [6] and [7], and then performed numerical quadratures by extended Simpson's rule. This resulted in $\langle Y_R \rangle = 0.29$ and $\sigma_Y^2 = 0.07$. These values of $\langle Y \rangle$ and σ_Y are not restricted to small variance of the random parameters, but are subject to numerical errors which were minimized by using the splines as illustrated by the graphs of Figs. 1 and 2.

To illustrate the results further, the relative contribution of τ and V_r to σ_Y^2 is calculated from $\sigma_\tau^2 (\partial Y / \partial \tau)^2 / \sigma_Y^2$ and $\sigma_{V_r}^2 (\partial Y / \partial V_r)^2 / \sigma_Y^2$ (Eq. [29] of P1) by taking the first derivatives from Figs. 1 and 2, the variances from Table 1, and σ_Y^2 from Eq. [27] of P1. Thus, impact of uncertainty of τ on variability of Y was calculated in this case (see Table 1 in P3, Dagan and Bresler, 1987) to be as large as 72%, while the impact of V_r variability upon the yield variability was computed by Taylor approximation for the case of Fig. 2 to be 38%.

The comprehensive results will be reported in Table 1 of P3 of the present study and their analysis, discussion, and partial field validation, constitute Part 3 (Dagan and Bresler, 1987) of this series of papers.

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Fig. 1: Relative yield ($Y_R = Y/Y_p$), its first derivative ($\partial Y_R / \partial \tau$) and second derivative ($\partial^2 Y_R / \partial \tau^2$) as a function of the parameter τ for Panoche soil with runoff; $h_{er} = -1.5$ MPa and $C_n = C_o = 5$ dS/m. The horizontal bar intersecting the curve at the standard value of $\langle \tau \rangle$ denotes $\langle \tau \rangle \pm \sigma_\tau$.

Fig. 2: Relative yield ($Y_R = Y/Y_p$), first derivative $\partial Y/\partial V_r$ and second derivative (Y_R'') as a function of relative water quantity (V_r) for Bet Dagan soil without runoff; $h_{er} = -1.5$ and $C_n = C_o = 5$ dS/m. The horizontal bar intersecting the curve at the standard value of $\langle V_r \rangle$ denotes $V_r \pm \sigma_{V_r}$.

YIELD VARIABILITY OF AN IRRIGATED CROP AND ITS CAUSES:
III. NUMERICAL SIMULATION AND FIELD RESULTS

ABSTRACT

The prediction of yield variability of an irrigated crop, as influenced by the uncertainty of soil, plant, and boundary conditions parameters, and an investigation through numerical simulation, using the probability theory apparatus, are tested against field data. Among the large number (19) of parameters, six which have a significant impact on yield variability have been singled out. These include two plant parameters with an average relative contribution of 23%, three soil hydraulic parameters with a relative impact of 47%, and one irrigation boundary condition parameter, related to the nonuniformity of water application, with 30% average contribution upon yield variability. An additional set of results is related to the yield coefficient of variation. Its range was simulated to be generally between 0.3 and 1, with an average of approximately 0.6. A similar range, but with a somewhat lower average was measured in the field. The detection of the parameters of large impact, obtained by both simulation and field validation, give an indication of weak links which require further investigation, and for which collection of field data is important.

1. INTRODUCTION

In Part 1 (P1) and Part 2 (P2) of the present study (Bresler and Dagan, 1987a and b), we presented the methodology and the input data for evaluating crop yield variability by numerical simulation. In particular, in Table 1 of P2, we elaborated on the various cases resulting from combinations of parameters' values. We selected a set of 12 combinations and computed the expected values of relative crop yield and its coefficient of variation, as influenced by the variability of the factors enumerated in P1. The methodology, *i.e.*, the use of the first-order Taylor expansion or of the full quadratures, was dealt with in P1 (Eqs. [23] or [24], [27]) and in P2 (Eqs. [6] and [7]) and illustrated in P2, Sect. 3.

In this third part, the final results of the simulation are tested against field data and the outcome is presented and discussed. The parameters with significant impact upon crop yield variability, and which may deserve further investigation, are singled out. Simulated and field results of yield coefficient of variation are also compared.

2. RESULTS AND DISCUSSION OF NUMERICAL SIMULATION

(a) Summary of results

The various cases and the final simulated results are presented in Table 1. Recall that in order to obtain the results of Table 1, for

each case given in the table, a set of curves relating relative yield to each parameter (similar to those of Figs. 1 and 2 of P2), was produced. Herewith, we give a few explanations about the structure of Table 1.

In Columns 1 to 5, the pertinent conditions of each case are summarized, the data being explained in detail in P2. In Table 1 we have omitted four of the 16 combinations from table 1 of P2. Those cases, pertaining to irrigation water of low salinity ($C_0 = 0.3$ dS/m), do not differ significantly from the ones presented here.

In Column 6 we present expectations of the relative yield, i.e., of the ratio $\langle Y_R \rangle = \langle Y \rangle / Y_p$ between the expected value of the crop yield and the potential yield, based on the first term of the Taylor expansion of Eq. [23] or [24] of P1. This expression is exact if Y depends linearly on the various parameters a_i in the range $\langle a_i \rangle \pm 2\sigma_i$, as in P2, Sect. 3. Attempts to correct these expressions by accounting for the second derivatives of Y in Eq. [23], P1, did not generally achieve significant results, as in most cases these terms were quite small.

Column 7 of Table 1 presents the expectation $\langle Y^*_R \rangle = \langle Y^* \rangle / Y_p$ as computed by a quadrature (Sect. 4 in P1, in Sect. 3 P2). This has been done solely for those parameters a_i for which Y changes nonlinearly with a_i near the expectation $\langle a_i \rangle$. Values of $\langle Y^* \rangle$ are given only for those cases in which $\langle Y_R \rangle^*$ differs from $\langle Y_R \rangle$. Since variability of a parameter does not influence $\langle Y_R \rangle$ if Y_R depends linearly on a_i , the difference between Columns 6 and 7, reflects the impact of the variability of various parameters upon the expected value of the relative yield. When such a difference is absent, the expected yield is that corresponding to the expected value of a_i .

Column 8 encapsules simulated results of the coefficient of variation of the relative yield. This was calculated by the first-order Taylor expansion, Eq. [30] of P1, for those parameters for which Y was approximately linear in a_i , and by the quadrature (Eq. [26] of P1 and Eq. [6] and [7] of P2), for those which did not satisfy this requirement.

Finally, in Column 10 we present the contribution of various random parameters a_i to the yield variance. Each stochastic parameter a_i contributes either by its share $[(\delta Y / \delta \langle a_i \rangle) \sigma_i]^2$ in the first-order Taylor expansion (Eq. [27], P1) or by the quadrature (Eq. [7], P2). The contribution of the latter type has been marked with an asterisk. The contribution of each parameter is divided by σ^2_Y (computed by Eq. [27] of P1) and is represented as percent of σ^2_Y . Generally, only parameters which contributed to σ^2_Y by more than 5% have been included, the impact of the others being neglected. We shall discuss now some of these results in detail.

Table 1: Simulated results for the relative yield expectations ($\langle Y \rangle / Y_p$), coefficient of variation of yield ($\sigma_Y / \langle Y \rangle$), and the impact $\{[(\delta Y / \delta \langle a_i \rangle) \sigma_i]^2 / \sigma^2_Y\}$ of six parameters (h_{er} , F , τ , ϵ , $\ln \beta$, and V_r) on the yield. The twelve conditions are for two fields (Bet Dagan and Panoche), two values of deterministic and stochastic h_{er} (-0.4 and -1.5 MPa) and two runoff situations (yes and no).

Column No.														
1	2	3	4	5	6	7	8	9	10					
CA- SE	F I E L D	C _o dS/m	h _{er} MPa	RUN- OFF	$\langle Y \rangle$	$\langle Y^* \rangle$	σ_Y	σ_Y	$\{[(\delta Y / \delta \langle a_i \rangle) \sigma_i]^2 / \sigma^2_Y\} 100$					
					$\frac{\langle Y \rangle}{Y_p}$	$\frac{\langle Y^* \rangle}{Y_p}$	$\frac{\sigma_Y}{\langle Y \rangle}$	$\frac{\sigma_Y}{\langle Y \rangle}$	PARAMETERS					
					$\langle Y_R \rangle$	Y^*_R		(CV)	h _{er}	F	τ	ϵ	$\ln \beta$	V _r
1	B	0.3	-0.4	NO	0.52	-	0.29	0.55	-	-	-	65	15	20
2	E		-1.5	YES	0.65	0.58	0.24	0.41	-	-	39*	21	10*	30
3	T	5.0	-0.4	NO	0.22	0.19	0.19	1.0	40	-	-	40	4*	16
3**				NO	0.22	0.19	0.14	0.67	-	-	-	65	6	29
4	D			YES	0.20	-	0.19	0.96	39	11	7*	22	4*	17
4**	A			YES	0.20	-	0.15	0.75	-	17	14	35	6	28
5	G													
5	A		-1.5	NO	0.61	-	0.21	0.35	-	9	-	15	38	38
6	N			YES	0.62	0.56	0.22	0.40	-	8	34*	14	11*	33
7	P	0.3	-0.4	NO	0.57	0.50	0.19	0.37	-	21	-	19*	11*	49
8	A		-1.5	YES	0.64	0.31	0.14	0.46	-	-	30*	-	20*	50
9		5.0	-0.4	NO	0.22	-	0.19	0.88	44	24	-	9*	-	23
9**	N			NO	0.22	-	0.14	0.63	-	43	-	17	-	40
10	D			YES	0.22	0.09	0.19	2.15	49	9	24*	8	-	10
10**				YES	0.22	0.09	0.14	1.56	-	18	47	15	-	20
11	C													
11	H		-1.5	NO	0.60	0.57	0.18	0.31	-	26	-	9*	10*	55
12	E			YES	0.60	0.29	0.30	1.05	-	-	72*	-	10	18
AVERAGE IMPACT									14	9	17	18	12	30

* Obtained by quadrature

** Computed with deterministic h_{er}

(b) Expected Values of Relative Crop Yield ($\langle Y_R \rangle = \langle Y \rangle / Y_P$)

Column 6 of Table 1 presents expected values of $\langle Y_R \rangle$ calculated with negligible influence of the second derivative (the second term on the right hand side of [22] in P1 has been omitted). In this case the soil can be considered as an equivalent homogeneous medium in which the solution of the problem for $\langle Y_R \rangle$ is identical to the solution obtained for the average values of the properties of the heterogeneous field. An examination of $\langle Y_R \rangle$ in Column 6 immediately shows two contrasting groups of results. In one group relative yield ranged between 0.52 and 0.65 and in the second group it was as low as 0.20-0.22. The first group contains $\langle Y_R \rangle$ data calculated for both soils, for low salinity water ($C_0 = 0.3$ dS/m) and/or low values of h_{cr} (-1.5 MPa). The second group (small Y_R) is associated with high values of h_{cr} (-0.4 MPa) and with high water salinity ($C_0=5.0$). The plant parameter h_{cr} is sensitive to salinity (see Fig. 1) and may be considered as the salt tolerance parameter of a crop. The association between high C_0 and low h_{cr} amplifies the salinity effect and causes a drastic reduction of the yield. At low salinity, the effect of high h_{cr} (cases 1, 7) is manifested in a reduction of the yield by 9-16% as compared with low h_{cr} (cases 2 and 8). This reduction, however, is still small compared with the drastic yield decline because of the same change in h_{cr} at high salinity.

Differences between expected values of relative yield calculated with the aid of a quadrature technique (column 7, Table 1) and those of Column 6, reflect the influence of variability upon simulated $\langle Y_R \rangle$, as mentioned in section (a). Scrutiny of Columns 6 and 7 shows that calculation using the quadrature technique always gives lower values of expected yield than calculations using the first term of the Taylor expansion (Eq. [24] of P1). This fact may be explained in terms of the shape of the yield function curves $Y_R(a_1)$ which is generally either concave or linear (cf., e.g. Figs. 1 and 2 in P2). When the curve is linear there are no differences between the two types of calculations (the second and higher derivatives are zero, so that $\langle Y_R \rangle$ by the Taylor expansion is identical to one obtained by the quadrature technique). However, when the curve is concave, quadrature for the actual curve will always give lower values of $\langle Y_R \rangle$ than that performed on a line tangent to the $Y(a_1)$ curve at the standard expectation point. Therefore, the quadrature technique, which is more accurate than the second-order Taylor expansion, gives identical or lower values of $\langle Y_R \rangle$ than those obtained by the latter. In most cases, the difference between the two computations is quite small. However, cases No. 8, 10 and 12 are exceptional, the differences between Columns 6 and 7 being 50% or even more. These three cases are characterized by low K^*_s and high variability of the soil parameter τ (i.e., Panoche soil) in association with the runoff case (in which some of the

irrigation water is prevented from infiltrating in the portion of the field with $K_s < R$). Such a combination between soil properties and boundary conditions causes the $Y_R(\tau)$ curve to be highly concave and far from linear.

To summarize, variability has a large impact upon expectation of the relative yield of an irrigated crop when runoff is assumed to occur and when variability of saturated soil hydraulic conductivity K_s is relatively large. The effect of the variability in K_s , or its related soil parameter, τ , upon soil water content and leaching or accumulation of salts in a heterogeneous field, has been emphasized by the authors in previous studies (e.g. Dagan and Bresler, 1979, 1983; Bresler and Dagan, 1979, 1981).

(c) Yield standard deviation (σ_Y) and coefficient of variation ($CV = \sigma_Y / \langle Y \rangle$)

Evaluation of the standard deviation and of the coefficient of variation of the crop yield is one of the central topics of this study. Their magnitude reflects the degree of uncertainty of the yield prediction which is caused by the uncertainty of the values of parameters which influence the yield. If this uncertainty is associated with spatial variability, σ_Y also reflects the spatial variability of the yield.

In Column 8 the yield standard deviation is represented in a dimensionless form by referring it to Y_p , which is the same for all cases. It is seen that the values of σ_Y / Y_p are not too different: their range is 0.14-0.30, while in seven cases out of 15 the range is narrowed down to 0.19-0.22. Although it is not clear to us whether this is a finding of wide generality, the figure of $\sigma_Y / Y_p \approx 0.2$ ($\pm 50\%$) may be kept in mind as indicative for yield standard deviation on the basis of present results.

Since the dimensionless expected value of the yield $\langle Y \rangle / Y_p$ (or $\langle Y^* \rangle / Y_p$, when applicable) is smaller than unity, the coefficient of variation $\sigma_Y / \langle Y \rangle = (\sigma_Y / Y_p) / (\langle Y \rangle / Y_p)$ may be quite large (Column 9). Furthermore, due to the large differences in $\langle Y \rangle / Y_p$ discussed in (b), CVs may change quite considerably from case to case, with the range being 0.31-2.15. The relatively large values of the CV suggest that yield prediction may be subjected to large uncertainty in some cases. As we shall show in the next paragraph, a few high values of the CV (cases 3, 4, 9, 10 in Table 1) are associated with the variability of h_{er} . These were precisely obtained for the group of low $\langle Y \rangle / Y_p$ discussed in (b), for which h_{er} has a large impact. Since the assumed variability of h_{er} was of a speculative nature, we have repeated in this case the simulation, with h_{er} deterministic and equal to its expected value (Column 4, Table 1). The results (cases 3**, 4**, 9**, and 10** of Table 1) show indeed a significant reduction of the yield

CV, which is now in the range 0.63-1.56. This range of values will be compared with some field results in Sect. 3.

(d) Parameters Affecting Yield Variability

Once it has been established that simulated yield may have a large coefficient of variation, the next major aim of the study was to determine the relative contribution of the various parameters upon σ_Y . A first screening has already reduced the number of significant parameters to the six in Column 10 of Table 1. We shall discuss now in detail the results concerning the impact of these parameters.

A first observation refers to cases 3, 4, 9 and 10 of Table 1, for which the combination of high h_{er} and low C_0 causes a definitely low $\langle Y \rangle / Y_p$ (see paragraph b above). In the same cases the assumed variability of h_{er} has a large contribution, of 39-44% of σ_Y^2 . Since the values of both expected value and variance of h_{er} were selected quite arbitrarily (see P2 of this series), we have also simulated the cases 3**, 4**, 9** and 10** in which h_{er} is taken as deterministic. These two sets should be considered as extremes for the possible variability of h_{er} .

We consider now in Table 1 the cases with h_{er} either of small impact or deterministic, and examine the impact of other parameters. It is seen that the plant parameter F has a rather small impact upon σ_Y^2 , of less than 17% for Bet Dagan soil and as much as 43% but generally less, for Panoche soil. The soil parameters τ , ϵ , and β contribute together quite substantially to the yield variance, in the range 17-82%, with most values higher than 50%. Among these parameters, τ , which is associated with the hydraulic conductivity at saturation, has a particularly large impact for the more variable Panoche soil and for runoff conditions (a similar conclusion was arrived at by Bresler and Dagan, 1981). Finally, the spatial variability of water application, represented by V_r , has a significant impact, contributing 16 - 55% of σ_Y^2 .

We shall discuss now in detail the significance of these findings for each of the aforementioned parameters which have a significant impact.

(i) Impact of plant parameters (h_{er}).

h_{er} : Remember that this parameter specifies in the model adapted here for root extraction the limiting value for the water head. Low values are associated with the ability of plants to extract increased quantities of water from soil and vice versa. The combined effect of salinity and h_{er} values is illustrated in Fig. 1, which represents dependence of simulated relative yield Y_R and its derivative $\delta Y_R / \delta h_{er}$ upon h_{er} for the two values of C_0 of Table 1. Two findings are apparent in Fig. 1: first, h_{er} has but little influence upon yield for $h_{er} < 0.8$ MPa, irrespective of water quality. In this range, the knowledge of h_{er} is immaterial for predicting yield and its

variability, as illustrated in Table 1. In contrast, for high salinity and for $h_{cr} > -0.8$, Y_R and its slope drop markedly with h_{cr} , explaining the results of cases 3, 4, 9, and 10 of Table 1.

(ii) Impact of soil parameters (τ, ϵ, β).

τ : The log of the scaling parameter (Eq. [3] in P2) has a large impact upon yield variability only when runoff occurs (cases 2, 6, 8, 10, and 12, Table 1). This effect, discussed previously in relation to transport (Dagan and Bresler, 1979), is related to ponding which occurs whenever the rate of water application is larger than K_s . In the runoff model the ponded volume of water does not infiltrate into the soil, resulting in a smaller yield in those parts of the field.

ϵ : The impact of ϵ , which is generally moderate, stems from the combined role it plays in the $K(h)$ function which affects water uptake and transpiration (Eq. [4] P1) and in $h(z,t)$. Because of the transient flow conditions it is difficult to pin down a clear tendency of Y_R variability as related to ϵ .

β : Again, its effect is moderate and not influenced by C_0 , h_{cr} or runoff. The generally small impact probably results from the fact that θ and β are directly related for a given h , so that uptake of water is in turn affected by β through Eq. [4] of P1.

(iii) Impact of uniformity of water application (V_r).

V_r : This is the only stochastic boundary condition parameter. It is not surprising that uniformity of water application has a large impact upon yield variability even for the relatively small variance of V_r equivalent to Christiansen coefficient of 80%. The large effect of V_r results from the steep slope of the Y_R vs the V_r curve (Fig. 2, P2), which is caused by the high response to water at $V_r = 0.6$, similar to the response to τ for the runoff cases. This may be the reason why the relative impact of V_r on the yield variability is generally comparable to that of τ for those cases in which runoff occurs.

3. COMPARISON BETWEEN SIMULATED RESULTS AND FIELD DATA

The results summarized in Table 1 and discussed in the preceding sections are based on idealized conceptual models (see Bresler and Hoffman, 1986), on disparate and sometimes scarce data and on computer simulations. Under these circumstances, their validation by field measurements is highly desirable. Until such a validation is carried out, the approach should be regarded as an investigatory one to serve as a conceptual framework and to provide guidelines for execution and interpretation of experiments. It should be realized, however, that carrying out a complete set of measurements to validate the suggested

model constitutes a formidable task. Indeed, the minimal effort would be to carry out a controlled experiment on a spatially variable field in which all, or the most important, parameters influencing yield and yield itself are measured and characterized statistically.

At this stage we shall attempt to compare the present results with field data obtained and published in the past, even if measurements were carried out for only a few of the parameters considered here. Thus, even if such a comparison does not constitute a definitive validation, it still gives some indication about the soundness of the model.

(a) The Deterministic Model

A first indication about the suitability of the present model is obtained by comparing measured and predicted relative yield for a few small plots (Bresler and Hoffman, 1986). To judge objectively the degree of agreement between computed results and measured data, a linear regression with no intercept has been carried out between computed results and measured crop yield data. For the data given in Fig. 2 the least square estimator of the regression coefficient "a" (which is the best linear unbiased estimator of the ratio between calculated and measured data) was estimated to be 0.97 (illustrated by the middle dashed line of Fig. 2), with value of $r^2 = 0.99$ and Student's t value for 53 degrees of freedom of 63.2 (significant at 0.0001 level). To illustrate the statistical results, the limits within which "a" will lie with a probability of 0.95 are plotted in Fig. 2 by the dashed-dotted line emanating from the origin. Also given in Fig. 2 for orientation, is the 1:1 line (solid line with the slope "a"=1.). Examination of Fig. 2 suggests that the agreement between computed and measured relative yield of the nine irrigated crops is as good as could be expected considering the approximation of the model and the uncertainty in the measured data.

(b) The Assumption of Independence of the Random Parameters

One of the assumptions we adopted in P1, to simplify Eq. [19] in order to obtain Eqs. [23] and [27], was to consider the stochastic spatial parameters $a_i(x)$ to be statistically independent. This is probably true for a system containing only one parameter of each of the categories: plant, soil salinity, soil water, water application, and initial conditions, but may not be correct for some of the soil parameters. Estimations of the correlation coefficient (r^2) between the soil parameters τ , ϵ , and $\ln \beta$ were performed for the measurements made at Bet Dagan by Russo and Bresler (1981). Values of r^2 between τ and $\ln \beta$, τ and ϵ , and ϵ and $\ln \beta$, were calculated to be 0.81, 0.77, and 0.99, respectively, all very highly significant at the 0.0001 level. From the relatively high correlation coefficients we may infer that these three soil parameters are not independent in the probability sense, and simplifying [19] into [23] and [27] of P1.

is not rigorous. However, for the purpose of the present study, this simplification would not affect significantly the main conclusions. This is so because statistical dependence adds negligibly small mixed derivatives in the form

$$\sum_{j=1}^n \sum_{i=1}^n \frac{\partial^2 Y}{\partial a_i \partial a_j} \quad \text{to Eq. [23] of P1, and Eq. [27] of P1 remains}$$

essentially unchanged.

(c) Coefficient of Variation (CV) of Crop Yield

To compare field data with values of CV of Table 1, Column 8, we used CV data which were measured in a few fields and published previously together with new unpublished data. Bresler *et al.* (1981) reported CV values of 0.18 - 0.24 for various yield components of a peanut crop. Using data on winter wheat and corn collected by Hanks (personal communication), Bresler *et al.* (1982) calculated CV values for winter wheat between 0.12 and 0.26 and for corn between 0.11 and 0.20. Higher CV values (CV = 0.22 - 0.36 for kernel yield, fresh and dry matter yield of sweet corn) were reported by Stern and Bresler (1983). The range of CV of sorghum yield from the line source data of Morkoc *et al.* (1985) was calculated to be between 0.05 close to the line source and 0.5-1.0 close to the field's margins, with an average CV of 0.18. We found a similar tendency for sweet corn irrigated by line source at Bet Dagan. For uniformity of application equivalent to the Christiansen coefficient of 0.8 (*i.e.*, CV of V_r of 0.25) we found a CV for corn yield of 0.31 close to the line source and CV of 0.94 close to the margins of the irrigated area with average CV values of 0.45.

In general, simulated values of crop yield CV (Table 1, column 9) are consistent with the above field measured data. The simulated results for CV are generally larger than measured CV due to the extreme values we selected for the various σ^2_i (Table 1 of P2). As mentioned before, a thorough validation would require independent measurements of the various factors as well as of yield.

(d) Impact of Various Parameters upon Yield Variability

Five random parameters were found by simulation to have a significant influence on yield variability (Table 1, Column 10), when h_{cr} was regarded as a deterministic parameter. We were unable to collect enough field data to check the relative impact of F on σ^2_v , as obtained by simulation (in any case this factor has been found to have a relatively small impact). On the other hand, we collected sufficient soil and boundary conditions data to examine the impact of variability in soil hydraulic properties and in water application upon yield variability. With this aim in mind it is worthwhile mentioning first that there is a simple relationship for the correlation

coefficient r_{Y,a_i} between yield and a parameter a_i on the one hand, and the results of Column 10, Table 1, on the other. Indeed, if the first-order Taylor expansion is adopted, i.e., the yield is linear in a_i , we immediately have

$$r_{Y,a_i} = \frac{\langle Y'a_i \rangle}{\sigma_Y \sigma_{a_i}} = \frac{\sum_{j=1}^n \frac{\delta \langle Y \rangle}{\delta \langle a_j \rangle} \sigma_{a_j}}{\sigma_Y \sigma_{a_i}} ; \quad (i=1,2,\dots,n) \quad [1]$$

Furthermore, if it is assumed, as in Table 1, that a_i are statistically independent, i.e., $\sigma_{a_i} = 0$ for $i \neq j$, then Eq. [1] degenerates into

$$r_{Y,a_i} = \frac{\delta \langle Y \rangle}{\delta \langle a_i \rangle} \frac{\sigma_{a_i}}{\sigma_Y} \quad [2]$$

Thus, it is seen that the factors appearing in Column 10 of Table 1 are equal to r_{Y,a_i} square. For comparison with field data, we make use of the results reported by Bresler et al. (1981) for peanut crops in the Bet Dagan field. There, the correlation coefficient r^2_{Y,a_i} between the yield and various soil parameters has been computed by statistical inference.

Table 2 summarizes the estimated correlation coefficient (r^2) values between crop yield components and individual soil and water application parameters in field tests. Unlike Table 1, these relative contributions (of the variance of each parameter to yield variance) in Table 2 should not add up to unity because in reality a_i are not necessarily independent (see preceding section). Hence, about 40% of the yield variance of corn, for example, is influenced by water application uniformity when the Christiansen coefficient is approximately 75% (Stern and Bresler, 1983), and the remaining 60% is influenced by the other soil, plant and boundary condition parameters in addition to all types of errors. A comparison between the results given in the last line of Table 1 and those of Table 2 shows that on the average the impact is somewhat higher in Table 2, but in some particular cases values of Table 1 are higher, due probably to extreme variance values taken for the simulation. In general, however, simulated results of column 9 Table 1 are consistent with correlation field data of Table 2.

Table 2: Correlation coefficients (r^2) between various components of crop yield and individual soil parameters for 4 different crops, and between yield and water application V_r (for $C_n=75$) in one crop. All r^2 are significant at 0.01 level or higher.

Crop	Yield Component	Parameter a_i			
		τ	ϵ	$\ln \beta$	V_r
Peanuts*	Total	.36	.35	.41	-
	Hay	.32	.35	.42	-
	Pod	.27	.19	.23	-
Corn**	Kernel	.16	.25	.26	0.38 - 0.49
	Total	.12	.27	.24	0.40 - 0.45
Wheat	Total	.11	.27	.24	-
	Grain	.07	.18	.13	-
Hay	Total	.18	.29	.25	-
AVERAGE IMPACT		.20	.27	.27	.43

* After Bresler et al. (1981)

** After Stern and Bresler (1983)

The simulated impact of nonuniformity of water application (V_r) on crop yield variability (last column of Table 1) was compared with field data for sweet corn using Eq. [29] of P1 with $a_1 = V_r$, ($\langle a_1 \rangle = 300/500$, i.e., $\langle V_r \rangle = 0.6$) and $\delta Y / \delta V_r$, $\sigma^2 V_r$ and $\sigma^2 Y$ taken from Stern and Bresler (1983). In addition, a line source irrigation experiment with sweet corn, conducted in the Bet Dagan field in 1985, was used for this comparison.

By using the results of Stern and Bresler (1983) we have:

$$\sigma^2 V_r = [(1.-0.8)0.6/0.8]^2 = 0.0225 \quad [3]$$

so that the contribution of V_r variability to the variance of the yield, at $V_r=0.6$, is

$$[(\delta Y / \delta V_r)^2 (\sigma^2 V_r) / \sigma^2 Y]_{V_r=0.6} = (.825)(.0225) / .04 = 0.46 \quad [4]$$

This relative contribution of 0.46 of the variance of V_r , to the variance of the yield is similar to the simulated value given in the last column of Table 1.

From data collected in the irrigation experiments conducted with sweet corn at Bet Dagan during 1985 we estimated, using the maximum likelihood method:

$$(\delta Y_r / \delta V_r)^2 = \{(3.53)(.392) \exp[-.392(0.6 - 0.04)]\}^2 = 1.23.$$

For variance of relative yield $\sigma^2_y = 0.068$ and variance of V_r of $\sigma^2_{V_r} = 0.0225$, as before, the relative impact of V_r variability is: $(1.23)(0.0225)/0.068 = 0.41$, which is again similar to the simulated data.

4. SUMMARY AND CONCLUSIONS

Irrigated crop yield has been modeled as a spatial random variable. Its expected value and variance have been evaluated as functions of 19 different soil-water-plant parameters which influence it. This aim has been achieved by simulating the yield over a season for each case considered.

Various cases resulting from many combination of values of 19 parameters were elaborated. We have selected the 12 combinations presented in Table 1 and have computed expected values of the crop yield and their coefficients of variation as influenced by the variability of the different enumerated factors. To obtain each case for which the final results are given in Table 1, we produced a set of curves relating yield, and its first and second derivatives, to each parameter, and singled out subsequently six random parameters with the greatest impact upon yield variability. Among them there are two plant parameters (h_{cr} and F , or one plant parameter, F , when h_{cr} was considered to be deterministic), three soil water parameters (ϵ , τ , and $\ln \beta$), and one controllable boundary condition parameter (V_r). Simulated yield results as affected by V_r and the three soil parameters were compared with field data and the agreement was reasonable.

The critical suction (h_{cr}) for water uptake by plant roots has been found to have a large impact upon yield only for the case of high concentration of the irrigation water ($C_o = 5$ dS/m) and at high values of critical h ($h_{cr} = -0.4$ MPa). Such a large impact of h_{cr} on yield expected value and variability may be related to the choice of very high values for h_{cr} and its variance. Unfortunately, experimental data regarding h_{cr} and its variability are not available and should be the subject of future investigations. Testing directly the validity of these results at present is not feasible, and furthermore, more research is needed for physical understanding of water uptake by

plant roots and water transfer through the plant into the atmosphere. Although we are uncertain about the parameter h_{cr} and its expected values and variances, this parameter may be interpreted, nevertheless, as reflecting plant genetic and photosynthetic capability properties which affect salt or drought tolerance of a plant. The expected value of h_{cr} may represent the tolerance of a given crop, while the variance of h_{cr} may be interpreted as the genetic and physiological activities' variabilities within that given crop.

The impact of the soil parameters τ , ϵ , and $\ln \beta$, as well as of the boundary condition parameter V_r , on crop yield variability as determined by simulation, was found to be of the same order as the field results. The simulated effect of τ and V_r under runoff conditions was similar, because both determine water quantities that penetrate into the soil profile through the surface. Their effect is more pronounced when h_{cr} is low (-1.5 MPa), because larger quantities of water are potentially absorbed by plant roots. For the cases without runoff, where all applied water V_r eventually penetrates the soil and τ does have a small impact on crop yield variability, V_r has a greater impact on variability and again this is more pronounced for cases of $h_{cr} = -1.5$ MPa. The variability of V_r is directly related to the uniformity of water application (Eqs. [5] and [6]). The smaller the uniformity of application, the larger the impact of V_r on yield (see also Warrick and Gardner, 1983).

The results in Table 1 are based on the selected standard value of $V_r = 0.6$. With the data of Fig. 2 from P2 and Eq. [29] from P1, it is possible to simulate the relative impact of V_r for any other V_r . Since $Y(V_r)$ is almost linear up to $V_r = 0.9$, the relative impact depends upon the variance of V_r only. However, V_r varies with the Christiansen coefficient (C_h) as well as with the total quantity of water applied (Eq. [5]). Hence, the variance ($\sigma^2_{V_r}$) increases as C_h (0.8 in our case) decreases and as relative water quantity ($V_r = 0.6$ in our case) increases. The assessment of the relative impact of V_r in Tables 1 and 2 and the other field data are restricted, therefore, to values of C_h and V_r which were assumed as standards in this study.

5. RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

The present study raises some questions which need further consideration. Thus, two factors which are not well understood, but were formed by simulation to have a large impact on yield, deserve further experimental investigation. We refer to: (i) h_{cr} , a measure of plant tolerance to salinity and drought, and (ii) the degree of runoff.

The assumption of a critical water potential, h_{cr} , in the range of -0.4 to -1.5 MPa at which water extraction practically ceases, is a problematic one. Formally this means that if soil salinity and/or matric suction are at these values, there is no uptake, but in most plants water uptake goes beyond these limits. Although at first glance the h_{cr} limit appears to be a fudge factor necessary to make the model fit the data, it nevertheless may be a deterministic or stochastic macroscopic quantity representing differences in the tolerance genome of a given plant population. Assessment of values of this parameter and estimation of its statistics require intensive and controlled laboratory and field investigations before it can be used with full confidence in practice.

The problem of runoff also deserves further investigation, because part of the water which is running off can infiltrate into the soil in regions of high conductivity to water. Hence, the entire complex (see Smith and Hebbert, 1979) which consists of field slope, average water application intensity, uniformity of application and field variability, should be considered simultaneously in any further investigation.

Another direction to follow is related to the variability of the average yield over a field. As already shown by Feinerman et al. (1985), besides the expectation value and variance of the factors influencing yield, one also has to know the correlation scale (integral scale) of those factors which have a spatial structure. The variability of average yield in a field depends to a large extent on the ratio between the integral scale and the field size. Hence, more information about correlation scales of the factors affecting the point value Y has to be collected in order to estimate the average yield over the field $\bar{Y}$ and its variance $\sigma^2_{\bar{Y}}$.

Another topic of considerable interest is the investigation of the impact of variability of various factors upon economic optimization of applied water quantity and quality (see Feinerman et al., 1985). Such a study may further emphasize the usefulness of the results concerning the relative importance of various factors.

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Fig. 1: Simulated relative yield (Y_R) and its first derivative ($\partial Y_R / \partial h_{cr}$), with respect to critical value of h , as a function of h_{cr} . Upper curves (dashed-dotted line) correspond to $C_0 = 0.3$ dS/m for both Bet Dagan and Panoche soils, lower solid lines are for $C_0 = 5$ dS/m for Bet Dagan and Panoche soils.

Fig. 2: Relationship between computed and measured relative yield for nine different crops (cowpea, oat, barley, celery, cauliflower, tomato, lettuce, sorghum, and wheat). The straight solid line represents the 1:1 ratio, the dashed middle line is the best-fitted line with $a=0.968$, the dashed-dotted lines represent the 95% confidence limits on "a", and the two extreme dashed lines are the predicted 95% confidence limits.

D. DESCRIPTION OF COOPERATION

The model developed by R. John Hanks and his colleagues during the past years has been an essential part of the project. In the beginning stages of the project John visited Israel for two months and helped initiate the research program. During this visit a simplified model was developed to include salinity as well as water stress. The simplified model was tested extensively against the more complete model. The results indicated some instances where the simplified model gave good comparisons and some instances where the comparisons were not good. Therefore the simplified model was not used in this report. The simplified model probably needs further modification. The more complete model (see pages 6 through 12) was adapted for this specific BARD project and the model run well in the VAX 11/730 computer of the Institute of Soils and Water (the computer was purchased before with the help of two previous BARD projects). R. J. Hanks visited Israel twice again for short times on other BARD missions (e.g., TAC) and helped in updating the project. He initiated a project, with a MSc. student, to further develop a simplified soil water and salinity model.

Gedeon Dagan visited various universities and research institutes in the USA during the second year of the project. Eshel Bresler visited Logan Utah close to the termination of the project to summarize and finalize the project in cooperation with John Hanks.

E. EVALUATION OF RESEARCH ACHIVEMENTS

The predictive power of the deterministic model which include water flow, salt transport and relative crop yield, has been tested and gave promising results (Chapt. C section 5). This model served as the basis for the stochastic comperhensive model (Chapt. C Sect. 1 I) for heterogeneous irrigated cropped fields (Chapt. B, Objective 1).

Paper 3 in Capt.C describes the research achievements with respect to objective 2 (Chapt. A). A statistical averaging procedure for a heterogeneous field has been developed using conditional multivariate normal analysis. This part of the report together with Sect. 4 (Chapt. C) suggest economical ways to apply field observation results for the assesment of the dependence of accuracy of field estimations upon the number of sampling points, their distribution in the field and the worth (economically) of additional sampling point on improving accuracy (Objective 3, Chapt. B).

The influence of water application uniformity on crop yield variability and on irrigation policy (Objective 4, Chapt. B) and the benefit to agriculture is given by the research results of Sections 1 and 2 in Chapt. C. The research achievements with respect to objectives 5 and 6, i.e. effects of average and variance of soil, plant, and irrigation management variables on field average of yield and on economic optimization of irrigation water, are described in detail in Sections 1 (I through III), 3, and 4.

F. LIST OF PUBLICATIONS

1. Feinerman, E., E. Bresler, and G. Dagan (1985) Optimization of — spatially variable resource: An illustration for irrigated crops. Water Resour. Res. 21:793-800.
2. Feinerman, E., G. Dagan, and E. Bresler. (1986). Statistical inference of spatial random functions. Water Resour. Res. 22: 935 - 942.
3. Bresler, E., and G. J. Hoffman (1986) Irrigation management for soil salinity control: Theories and tests. Soil Sci. Soc. Am. J. 50: 1552-1560.