Lumber volume and value from elliptical western hemlock logs

Christine L. Todoroki
Robert A. Monserud*
Dean L. Parry

Abstract

The effect of log ovality and orientation on lumber yield and value recovery was examined through sawing simulation. Ovality was modeled on digitized western hemlock (Tsuga heterophylla) logs by altering the ratio of minor to major axes on a given cross section from 1.00 (circular) to 0.80 (oval) in 0.05 unit decrements, while holding cross sectional area and volume constant. The log models were cant-sawn in the AUTOSAW simulator across the full range of rotational settings and “lumber” was tallied and priced according to WWPA grade. Responses (lumber yield and value recovery) were normalized relative to those of the circular logs at their initial orientation to eliminate effects due to the large range in log diameters. Maximum responses were calculated and the orientations attaining those maxima recorded. Log ovality was, contrary to the common assumption, beneficial to lumber yield when the log was sawn at the optimum orientation. A 3.2 percent increase in yield relative to the circular logs was recorded for the optimally rotated logs with an ellipticity ratio of 0.80 (95% CI, 1.1 % to 5.3%). With ellipticity 0.90, approximately equal to the average for western hemlock, the increase was 3.4 percent (95% CI, 1.8% to 4.9%). Ovality was neither beneficial nor detrimental to maximum value recovery; differences were not significant. Optimal lumber yield occurred more frequently at 90° and 270°, equivalent to primary sawing parallel to the major axis. However, value recovery fluctuated considerably, even with circular logs, and there was no unique angle at which maximum value recovery occurred.

Stem cross sections are often assumed to be circular (Kellogg and Barber 1981), though they rarely are (Matérn 1956). The assumption causes errors in cross sectional area and hence errors in estimates of stem volume (Matérn 1956, Williamson 1975, Biging and Wensel 1988), log fiber volume (Mongeau et al. 1993, Skatter 1998) and log value recovery (Maness and Donald 1994). An improvement on the circular shape and more accurate representation of cross sectional shape is obtained with elliptic models (Matérn 1956, Skatter and Hoibø 1998, Saint-André and Leban 2000).

Ellipticity of log cross sections is also described by terms such as noncircularity, ovality, out-of-roundness (OOR), or eccentricity. Although eccentricity is commonly used in the mathematics of the ellipse, the term is avoided here to prevent confusion with existing wood products literature on pith eccentricity (e.g., Pawsey 1966). Pith eccentricity refers to the deviation of the pith from the geometric center of the section, and although the pith may deviate within stems, ellipticity does not necessarily coexist (Pawsey 1966).

Measures of ellipticity have been defined variously as the difference between the maximum and minimum diameters at the small end of the log (Asikainen and Panhelainen 1970), as “the quotient of the difference in length between the longest diameter of a stem cross-section and the diameter at right angles to it, divided by this shorter diameter” (Williamson 1975), and as the ratio of major-to-minor axis (Saint-André 1998, Saint-André and Leban 2000). The definition adopted here, and by Biging and Wensel (1988), is “the diameter perpendicular to the longest diameter divided by the longest diameter,” equivalent to the ratio of minor-to-major axis of an ellipse. Our ellipticity ratio equals one when the cross section is circular, and is less than one when oval.

Ovality tends to increase (and the ellipticity ratio decrease) with increasing log size and to decrease with increasing height.
(Walters and Kozak 1964, Asikainen and Panhelainen 1970, Williamson 1975, Saint-André and Leban 2000, Singleton et al. 2003). The ellipticity ratio typically lies between the range of 0.80 and 1.00. Williamson's (1975) mean ±SD corresponds to our ratios of 0.83 and 0.97, and Monserud's (1979) to 0.945 with a SD of 0.04. Kellogg and Barber (1981) found that the mean ratio (albeit not perpendicular) for western hemlock was 0.92, with a range from 0.85 to 0.96. Biging and Wensel (1988) reported an overall mean of 0.94. They purposefully selected 45 disks that were out-of-round for further analysis; their mean was 0.91 and their range was 0.77 to 0.98. Ellipticity ratios between 0.82 and 0.94 were found for lodgepole pine (Koch et al. 1990), and although with a mean of 0.94 an extreme ellipticity ratio of 0.72 was found for Norway spruce (Saint-André and Leban 2000).

In sawing oval logs, Asikainen and Panhelainen (1970) state that "the correct sawing position" is where "in the first frame, the small radius of an oval log is perpendicular to the first frame blades and in the second frame the radius is at right angles to this position. When the log is then sawn in the correct position and its cubic content determined on the basis of the minimum diameter, the sawing yield of an oval log is better than that of a round log of the same size. When sawing is carried out in the wrong position, the opposite applies."

The above statements are applicable to the quantitative analysis of an unswept log. When logs exhibit sweep, rotating to the "horns up" position is generally accepted as being the most beneficial. However, in comparison to the "horns up" position, Maness and Donald (1994) found "the ability to rotate the log into the optimal solution produced significant benefits." They also found, through a series of sawing simulations using SAWSIM with eight log orientations at 45° increments, optimal log rotation to be more highly related to ovality than to sweep. Through regression analysis they demonstrated that although both sweep and ovality had a negative impact on value recovery, on average, the negative impacts due to ovality could be ameliorated through optimal log rotation. Further, they concluded that benefits in value recovery increased with increasing ovality.

Other sawing simulators that model logs with elliptical cross sections include SEESAW (Todoroki 1988), AUTOSAW (Todoroki 1990), and the model of Bindzi et al. (1996). The great advantage of sawing simulators is that the same set of digitized logs can be sawn more than once, allowing for replicated experiments that control sources of variation (Monserud et al. 2004). We will rely on the AUTOSAW simulator of Todoroki (1990) as we want to explore variation due to orientation with a higher level of accuracy (using 5° increments) and examine value recovery associated with knotty log models.

Knobs, if large enough, are defects that lower lumber grade 80 to 90 percent of the time (Walker 1993). Because of this, and the lower prices associated with knotty lumber, the orientation of a log for maximum value might differ from that for maximum volume (Steele et al. 1993, Steele et al. 1994, Todoroki 2001). Variations in value and lumber quality can occur through changes in log orientation (Harless et al. 1991, Samson 1993, Steele et al. 1994, Lemieux et al. 2002), sawing method (Harless et al. 1991, Steele et al. 1994, Todoroki and Lovell 2006) and opening face (Steele et al. 1993, Todoroki 2001, 2003). Rotational variation between ovality and graded and priced knotty lumber has not been explored.

Our objective was to examine the effect of log ovality on lumber yield and value recovery and test the hypothesis that the best sawing position, as described by Asikainen and Panhelainen (1970), holds true for both yield and value. Maximum responses with respect to log rotation were of primary interest because they correspond to a log rotated for optimal lumber yield in one case and optimal value recovery in the other.

Methods

Log sample

The 52 western hemlock logs were obtained from western Oregon. Western hemlock is a commercially important tree species in the Pacific Northwest, second in importance to Douglas-fir (Singleton et al. 2003). These logs were selected to represent the range of growth conditions for western hemlock and were used in a former study (Monserud et al. 2004). Small-end diameters averaged 10.5 inches (geometric mean), with a standard deviation (SD) of 4.1 inches, range of 5.0 to 20.8 inches, and distribution as shown in Figure 1. Log lengths were less variable, ranging from 16.0 ft to 16.9 ft, with a mean of 16.2 ft and SD of 0.20 ft. All logs were straight, with zero sweep, but had taper, calculated as the difference between small- and large-end diameters divided by log length, ranging from 0.05 to 0.52 in/ft with a mean and SD of 0.17 and 0.08 in/ft, respectively. Each log was digitally described in terms of branches and diameters along the stem axis. Longitudinal and radial positions of all externally visible branches were manually measured to provide branch location in a three-dimensional space. The live/dead status of each knot was visually assessed according to whether the wood was continuously ingrown at the stem surface (live) or whether a separation existed (dead). Knot diameters, and the log diameter at the knot location, were measured in addition to diameters at the large and small ends.

| Table 1. — Log ovality multipliers to maintain a constant cross sectional area for a given ratio of minor-to-major axis |
|-----------------|-----------------|-----------------|
| Minor/major     | Semi-major radius | Semi-minor radius |
| Ellipticity     | 1/√Ellipticity   | √Ellipticity    |
| 1.00            | 1.000            | 1.000           |
| 0.95            | 1.026            | 0.974           |
| 0.90            | 1.054            | 0.949           |
| 0.85            | 1.085            | 0.922           |
| 0.80            | 1.118            | 0.894           |

Figure 1. — Diameter distribution of the 52 western hemlock logs.
Lumber was edged and trimmed to remove wane. The primary saw had a 0.14 inch kerf while that at the edger was 0.18 inch.

**Grading criteria**

Lumber was automatically graded according to knot size using WWPA (1998) criteria for Light Framing (WWPA chapter 40.00) and Structural Light Framing (WWPA chapter 42.00) for grading the 2 by 4 boards, and Structural Joists and Planks for grading the 2 by 6, 2 by 8, 2 by 10, and 2 by 12 boards (WWPA chapter 62.00). The 1-inch boards were graded as Commons (WWPA chapter 30.10). Codes used within AUTOSAW to represent these grades are shown in Table 2.

Knot size was determined by its average dimension as in a line across the width perpendicular to the edge of the piece. For the Structural Light Framing and Structural Joists and Planks grades, allowable knot size increased proportionately from the size permitted at the edge to the size permitted at the centerline. Allowable edgeline and centerline sizes are dependent on board width (WWPA 1998). A procedure that determined the size, displacement and morphology

### Ovality of log models

This log and branch data set was used to create five sets of 52 log models with the range in ovality modeled to represent the typical range of 0.80 to 1.00 found in empirical studies. The original set had circular cross sections with the ratio of minor to major axis being 1.00, while the other four sets were progressively more elliptical with minor to major axis ratios of 0.95, 0.90, 0.85 and 0.80.

Although ovality changed, the volume of each log was held constant. To accomplish this, log radii of the circular sets was multiplied by the terms shown in Table 1 to produce the semi-major and semi-minor radii of the elliptical sets. Within the AUTOSAW log-sawing simulator, branches can assume any size and spatial orientation. They emanate from a pith that can deviate from the central log axis. Both live and dead branches are described. A live branch is represented by a right circular cone, and a dead branch by a cylinder concatenated to the end of the cone (Todoroki 1997). With the change in radii of the cross sections from circular to elliptic, branch length (live and dead portions) was adjusted to ensure that the branch neither extended beyond nor lay beneath the log surface. A proportional change in knot radius was also computed, keeping the ratio of knot radius to branch length constant.

### Sawing simulation

These 5 ovality sets comprising 52 logs were subjected to 72 sawing replications, each with the log rotation increased by 5 degrees. This resulted in a set of 18,720 (= 72 by 52 by 5) log sawing simulations. These 18,720 simulations, incorporating branches, and ranging in cross section from circular to progressively more elliptic forms, and ranging in orientation to the saw from 0° to 355°, were then digitally “sawn” using AUTOSAW (Todoroki 1990).

At the initial orientation of 0°, the primary saw was parallel to the minor axis of the elliptical cross sections and the cant processed in a perpendicular direction (Fig. 2a). Lumber, primarily of 2-inch nominal thickness and between 4 and 12 inches wide, was graded (Fig. 2b) and tallied in 1-ft increments. A secondary thickness of 1 inch was also allowed.
of knots on both board faces was developed within AUTOSAW (Todoroki et al. 2005). The displacement of the knot was assumed to be the minimum distance between the knot (on either face) and the closest board edge. Using that displacement, the allowable scaled knot size was calculated. Thus, any knot that touched the edge was an edgeknot and the edgeline criteria apply.

The thickness, width, length, and grade of each individual piece of lumber sawn by AUTOSAW were recorded, as well as ovality and rotational angle, and log identification. Lumber was tallied by grade for each log and priced.

Lumber price

Prices were based on the WWPA Coast FOB Price Summary, year-to-date prices, March 2006 (WWPA 2006; Table 2). Prices were applied to each individual piece of sawn lumber based on grade and dimension. The Hem-Fir Dry / Surfaced prices were applied because almost all West Coast lumber is sold Dry / Surfaced (WWPA 2006) and there was not adequate information to value it otherwise. Furthermore, as there was only one price available for the 1-inch boards sawn, sold under Hem-Fir Green Rough (Table 2, Utility and Btr), this price was applied to all other 1-inch Commons grade boards.

Analysis

Both Microsoft Excel® and SAS® (SAS Institute Inc. 1999) were used to process the AUTOSAW output files and summarize results by log (1, 2, . . . , 52), rotation angle (0°, 5°, . . . , 355°), and ovality (0.80, 0.85, . . . , 1.00), ultimately combining all into one dataset. Summaries within the dataset included lumber yield, normalized lumber yield, value recovery, and normalized value recovery.

Lumber yield, \(Y_{ijk}\) for each log \(i\) (\(i = 1\) to 52), rotation \(j\) (\(j = 1\) to 72), corresponding to the angular settings of \(0°, 5°, . . . , 355°\), and ovality \(k\) (\(k = 1\) to 5), corresponding to ellipticity ratios of 0.80 to 1.00, were defined as the ratio of lumber volume, \(V_{ijk}\), to log volume, \(V_{ij}\), and expressed as a percentage.

\[Y_{ijk} = \frac{L_{ijk}}{V_{ij}} \times 100\%\]

where:

\[L_{ijk} = \text{green lumber volume} \ (\text{ft}^3)\]
\[V_{ijk} = \text{gross log volume} \ (\text{ft}^3)\]

with \(V_{ij1} = V_{ij2} = V_{ij3} = V_{ij4} = V_{ij5}\) and

\[0.005454 \times \text{len} \times (0.75 \times \text{sed}^2 + 0.25 \times \text{led}^2)\]

if upper log

\[0.002727 \times \text{len} \times (\text{sed}^2 + \text{led}^2)\]

if lower log

\[\text{sed} = \text{small-end diameter (in)};\]
\[\text{led} = \text{large-end diameter (in)};\]
\[\text{len} = \text{log length (ft)};\]

Lumber yield was normalized relative to the circular log counterparts (\(k = 5\)) at their initial orientation (\(j = 1\)) to eliminate effects due to differing log sizes and shapes.

\[NY_{ijk} = \frac{Y_{ijk}}{y_{1.1.5}}\]

Value recovery, \(P_{ijk}\), was defined as the ratio of lumber value to log volume and expressed in units of dollars per hundred cubic feet. Normalized value recovery, \(NP_{ijk}\), was computed relative to the value recovery of the circular log counterpart at its initial orientation, \(NP_{1.1.5}\).

<table>
<thead>
<tr>
<th>Ellipticity</th>
<th>(y = a + b \ln(x))</th>
<th>(r^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>(y = 3.29 + 21.9 \ln(x))</td>
<td>0.835</td>
</tr>
<tr>
<td>0.85</td>
<td>(y = -0.24 + 23.2 \ln(x))</td>
<td>0.849</td>
</tr>
<tr>
<td>0.90</td>
<td>(y = 0.81 + 22.9 \ln(x))</td>
<td>0.822</td>
</tr>
<tr>
<td>0.95</td>
<td>(y = 0.22 + 22.8 \ln(x))</td>
<td>0.814</td>
</tr>
<tr>
<td>1.00</td>
<td>(y = 1.10 + 21.9 \ln(x))</td>
<td>0.830</td>
</tr>
</tbody>
</table>
$P_{ijk} = \frac{X_{ijk}}{V_{ijk}} \times 100$ value recovery ($\$/100 \text{ ft}^3$)

where: $X_{ijk} = \text{green lumber value ($S$)}$.

$NP_{ijk} = \frac{P_{ijk}}{P_{i1k}}$ normalized value recovery

Maximum lumber yield, maximum normalized lumber yield, maximum value recovery, and maximum normalized value recovery were then calculated for each log $i$ and ovality $k$ as follows:

$Y_{ijk} = \text{Max}(Y_{ijk})$ maximum lumber yield (%)

$NY_{ijk} = \text{Max}(NY_{ijk})$ maximum normalized lumber yield

$P_{ijk} = \text{Max}(P_{ijk})$ maximum value recovery ($\$/100 \text{ ft}^3$)

$NP_{ijk} = \text{Max}(NPY_{ijk})$ maximum normalized value recovery

The rotational angle(s) that delivered maximum yield ($Y_{ijk}$) and maximum value recovery ($P_{ijk}$) over all rotations for each log $i$ and ovality $k$ was determined. The frequency distribution of rotational angles attaining maximum lumber yield and value recovery was then calculated and expressed as a percentage of the 52 logs.

Confidence intervals, CI, for the difference between two means were calculated for the normalized responses of the oval and circular logs. The intervals were calculated using 95 percent confidence levels.

Results

Lumber yield

Due to symmetry, normalized lumber yield for circular logs was constant and equal to one over all rotations. For oval logs, normalized lumber yield fluctuated with orientation (Ellipticity $< 1.00$). This is demonstrated by the four log sets in Figure 3. In the majority of cases, normalized lumber yield of the elliptical logs at their maximum value exceeded that of the circular log counterparts (e.g., Logs 1, 3, and 4). Log 1, a small log with a 6.1” diameter at the small end, reached a maximum (actual and normalized) yield of 33.0, 41.6, and 42.4 percent for the 1.00, 0.90, and 0.80 ovality classes respectively. Log 1 also demonstrated local, yet lesser, peaks at 90° and 270° of 1.17 with the 0.80 ovality class. Log 2’s maximum (actual and normalized) yield was identical for the circular and elliptical counterparts, and was attained at many orientations (0° to 10°, 60° to 120°, 170° to 190°, 240° to 300°, and 300° to 355° with Ellipticity equal to 0.90 and at 75° to 105° and 255° to 285° when Ellipticity was 0.80). Similarly to Log 1, maximum yield for the elliptical Log 3 set exceeded that of their circular counterpart. For Log 4, not only did the maximum yield of the elliptical logs exceed that of the circular counterpart, but yield across all orientations exceeded that of the circular log.

The trend of increased maximum lumber yield with increasing ovality can be observed with the logarithmic regressions used to fit the small-end diameter to lumber yield data (Fig. 4, and Table 3). The difference in regression equations between that for the circular (Ellipticity = 1.00) and most oval (Ellipticity = 0.80) logs equates to about 2.2 percent, i.e., the absolute maximum yield due to those elliptical logs on average exceeded that of their circular counterparts by 2.2 percent, and in relative terms by 3.2 percent.

Normalized maximum yields of the oval logs (Ellipticity < 1.00) were greater than those of their circular counterparts (Ellipticity = 1.00), Table 4. While there is an increased tendency toward greater variation, as indicated by the SDs, as logs become more oval, at the 95 percent level of confidence none of the intervals for oval log sets includes 0, indicating significant differences. For the least oval set (Ellipticity = 0.95), maximum lumber yield was greater than their circular counterparts by an average of 2.1 percent (95% CI: 1.4 to 2.9%). For the most oval set (Ellipticity = 0.80) lumber yield exceeded that of their circular counterparts by an average of 3.2 percent (95% CI: 1.1 to 5.3%), (Table 4).

Log orientation for maximum lumber yield

Maximum lumber yield occurred more frequently at 90° and 270° with oval logs (Fig. 5). At least 50 percent of all oval logs recorded maximum yield at the 90° and 270° orientations. Lesser peaks in maximum yield (25 to 37% of all logs, depending on ovality) were observed at 0° and 180°. For log models with circular cross sections (top of Figure 5), there was no preferred orientation.

Lumber value recovery

Normalized lumber value recovery, unlike normalized yield, was not constant for circular logs (Fig. 6) nor were there

<table>
<thead>
<tr>
<th>Ellipticity</th>
<th>Mean</th>
<th>SD</th>
<th>95% CI for difference in means between oval (Ellipticity &lt; 1.00) and circular (Ellipticity = 1.00) logs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>1.032</td>
<td>0.078</td>
<td>(0.011, 0.053)</td>
</tr>
<tr>
<td>0.85</td>
<td>1.031</td>
<td>0.063</td>
<td>(0.014, 0.048)</td>
</tr>
<tr>
<td>0.90</td>
<td>1.034</td>
<td>0.057</td>
<td>(0.018, 0.049)</td>
</tr>
<tr>
<td>0.95</td>
<td>1.021</td>
<td>0.028</td>
<td>(0.014, 0.029)</td>
</tr>
<tr>
<td>1.00</td>
<td>1.000</td>
<td>0.000</td>
<td>(0.000, 0.000)</td>
</tr>
</tbody>
</table>

Figure 5. Angular frequency distribution of occurrence of maximum yield.
any identifiable unique angles relative to log shape that increased value recovery. Furthermore, differences between means of the oval logs (Ellipticity < 1.00) and their circular counterparts were not significantly different (Table 5). However, similar to maximum lumber yield but with more scatter, maximum value recovery increased approximately logarithmically with increasing small-end diameter (Fig. 7, Table 6).

**Discussion and conclusion**

Maximum lumber yield of the oval logs was significantly greater than that of the circular logs (Table 4). Reports of loss in yield due to ovality (e.g., Saint-André and Leban 2000) do however remain valid in some cases where oval logs are not first rotated to their optimum rotation, as demonstrated by Logs 1, 2, and 3 (Fig. 3) when their normalized yields are less than 1.00. Other logs, such as Log 4, do not support the statement that ovality causes yield losses.

Ovality did not negatively impact maximum value recovery. This was demonstrated by the responses between the oval logs and their circular counterparts that were not significantly different (Table 5). Although Maness and Donald (1994) found that ovality caused significant reductions in value recovery (under conditions different to this study), they also found that these could be partly ameliorated by careful log rotation.

The angular log placement proposed by Asikainen and Panhelainen (1970) whereby the first sawcut is parallel to the major axis, equivalent to a 90° or 270° orientation with our sawing practices, was identified as being the most probable orientation for obtaining increased lumber yield with oval logs. However, it was not identified as being the most probable orientation for maximizing lumber value recovery, nor was any other unique angle identified. This may be due to the fact that log rotation was performed relative to log shape. In a future study we will investigate lumber value recovery relative to branch orientation. The impetus for this future study comes from the large variability demonstrated at the individual log level (Fig. 6), which suggests that there is potential to increase value recovery, even for circular logs, by appropriate rotation.

Through simulation we have dispelled the myth that ovality causes loss in yield. Furthermore, we can confirm the earlier findings of Asikainen and Panhelainen (1970) with their rule-of-thumb for determining the optimal orientation of oval logs. However, we have also found that this rule is appropriate only to the volumetric analysis of logs and does not extend to knotty logs and their qualitative analysis.

**Literature cited**


Figure 6. — Normalized lumber value due to rotation within four log sets with ellipticity ratios of 0.80, 0.90, and 1.00.

![Figure 6](image)

**Table 5. — Statistics for the normalized maximum value recoveries by ovality.**

<table>
<thead>
<tr>
<th>Ellipticity</th>
<th>Mean</th>
<th>SD</th>
<th>95% CI for difference in means between oval (Ellipticity &lt; 1.00) and circular (Ellipticity = 1.00) logs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>1.210</td>
<td>0.222</td>
<td>(-0.040, 0.081)</td>
</tr>
<tr>
<td>0.85</td>
<td>1.209</td>
<td>0.210</td>
<td>(-0.037, 0.077)</td>
</tr>
<tr>
<td>0.90</td>
<td>1.212</td>
<td>0.219</td>
<td>(-0.036, 0.083)</td>
</tr>
<tr>
<td>0.95</td>
<td>1.205</td>
<td>0.211</td>
<td>(-0.041, 0.074)</td>
</tr>
<tr>
<td>1.00</td>
<td>1.189</td>
<td>0.207</td>
<td>(-0.056, 0.056)</td>
</tr>
</tbody>
</table>

**Table 6. — Logarithmic regressions by Ellipticity ratio estimating maximum lumber value recovery, y ($/100 ft²), given small-end diameter, x (inch).**

<table>
<thead>
<tr>
<th>Ellipticity</th>
<th>y = a + b ln(x)</th>
<th>r²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>y = 56.0 + 99.5 ln(x)</td>
<td>0.667</td>
</tr>
<tr>
<td>0.85</td>
<td>y = 44.4 + 105.0 ln(x)</td>
<td>0.707</td>
</tr>
<tr>
<td>0.90</td>
<td>y = 56.6 + 99.6 ln(x)</td>
<td>0.667</td>
</tr>
<tr>
<td>0.95</td>
<td>y = 49.9 + 102.0 ln(x)</td>
<td>0.720</td>
</tr>
<tr>
<td>1.00</td>
<td>y = 49.2 + 100.0 ln(x)</td>
<td>0.717</td>
</tr>
</tbody>
</table>

Figure 7. — Maximum value recovery, P₉₅₅₀, for each of the 52 logs i and ovality sets k, regressed against small-end diameter.

![Figure 7](image)


