

## Accounting For Spatially Variable Infiltration in Border Irrigation Models

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In this paper we describe a simple combination of variances technique for incorporating the spatial variability of infiltration rates into the irrigation uniformity predictions of border irrigation models. The mean and variance of the depth of infiltrated water across a border can be calculated directly from the infiltration opportunity time results of a single border irrigation simulation and the distributions of the infiltration parameters for the border. Infiltration depth results are not needed. The single irrigation simulation is run using the expected values for the infiltration parameters to describe uniform infiltration across the border. We also show how these results can be used to calculate the irrigation distribution uniformity coefficient  $DU$ , where care must be used if the depths of infiltration are assumed normally distributed. Although only normal and lognormal distributions are discussed, the techniques described can be applied to any infiltration parameter distribution.

### INTRODUCTION

A major emphasis in agriculture today is the development and fine tuning of irrigation systems that make the most efficient use of water and energy resources. Recently, border irrigation systems, especially level basin systems, have reemerged as viable alternatives with proven water use efficiencies [Erie and Dedrick, 1979]. In addition, the performance of these systems can be accurately simulated by computer models of varying degrees of sophistication [Bassett and Fitzsimmons, 1976; Katapodes and Strelkoff, 1977; Strelkoff and Katapodes, 1977; Jaynes, 1986], which facilitates the rapid testing of irrigation performance for different combinations of system design and field conditions [Clemmens and Dedrick, 1982].

Although these models can use any equation to describe infiltration, the most commonly used equations are the Kostiakov equation

$$I = st^n \tag{1}$$

or the Modified Kostiakov equation

$$I = st^n + at = sx + at \tag{2}$$

where  $I$  is depth of infiltration,  $t$  is infiltration opportunity time,  $x = t^n$ , and  $s$  and  $a$  and  $n$  are empirical coefficients. We will allow  $n$  to vary in (1), but for this paper consider  $n$  to be constant in (2), which then is analogous to Philip's two-term equation. Although empirical in nature, these equations fit observed infiltration data well [Clemmens, 1983] and have been used successfully in irrigation models.

In simulating border irrigation, most models assume uniform infiltration across the border and use a single value for the coefficients in (1) and (2). However, it is well known that infiltration and thus these parameters can vary greatly across a field or border. For example, Sharma *et al.* [1980] using (2) to describe infiltration at 26 sites across a 9.6-ha watershed, found  $s$  and  $a$  to have coefficients of variation  $CV$  equal to 0.5 and 0.6, respectively. In addition, the distribution function of these parameters is also uncertain, mainly because inferences

must be made from small data sets. Sharma *et al.* [1980] found that the measured  $s$  and  $a$  values fit lognormal distributions well but that normal distributions could also be used. The effect of the spatial variation in infiltration rates on calculated irrigation uniformity in model simulations is rarely considered. In this paper we will show how the mean and variance of  $I$  can be found from the distributions of the parameters in (1) or (2). We will then show how these equations can be used to estimate the effect of spatially variable infiltration characteristics on the irrigation uniformity as calculated by border irrigation models using constant coefficient infiltration equations.

### ESTIMATING MEAN INFILTRATION DEPTH AND VARIANCE

Using (1) to represent infiltration we can take the log transform and expand the right-hand side in terms of the mean parameter values to obtain

$$\ln I = \ln s + n \ln t \\ = \mu_{\ln s} + \delta_{\ln s} + (\mu_n + \delta_n)(\mu_{\ln t} + \delta_{\ln t}) \tag{3}$$

where  $\mu_{\ln s}$  and  $\mu_{\ln t}$  are the population means, and  $\delta_{\ln s} = \ln s - \mu_{\ln s}$ ,  $\delta_n = n - \mu_n$ , and  $\delta_{\ln t} = \ln t - \mu_{\ln t}$ . Multiplying, we have

$$\ln I = \mu_{\ln s} + \delta_{\ln s} + \mu_n \mu_{\ln t} + \mu_{\ln t} \delta_n + \mu_n \delta_{\ln t} + \delta_n \delta_{\ln t} \tag{4}$$

Taking the expectation of each term on the right-hand side to find the expected or mean value of  $I$  we obtain

$$\mu_{\ln I} = E(\ln I) = \mu_{\ln s} + \mu_n \mu_{\ln t} + \sigma_{n \ln t}^2 \tag{5}$$

where  $\sigma_{n \ln t}^2$  is the covariance of  $n$  and  $\ln t$ , and  $E(\delta_{\ln s}) = E(\mu_{\ln t} \delta_n) = E(\mu_n \delta_{\ln t}) = 0$  from the definition of the mean.

The variance of  $I$  in (1) can be found from  $\sigma_{\ln I}^2 = \text{var}(\ln I) = E((\ln I)^2) - E^2(\ln I)$  [Mood *et al.*, 1974, p. 180,]. Thus

$$\sigma_{\ln I}^2 = \mu_n^2 \sigma_{\ln t}^2 + \mu_{\ln t}^2 \sigma_n^2 + \sigma_{\ln s}^2 + \mu_n \mu_{\ln t} \sigma_{n \ln t} \\ + 2\mu_{\ln t} \sigma_n \ln s^2 + 2\mu_n \sigma_{\ln s \ln t}^2 + 2\mu_{\ln t} \sigma_{nn \ln t}^3 \\ + 2\mu_n \sigma_{n \ln t \ln t}^3 + 2\sigma_{n \ln s \ln t} \\ + \sigma_{n \ln t \ln t}^4 - \sigma_{\ln t}^2 \sigma_{n \ln t}^2 \tag{6}$$

where  $\sigma^3$  and  $\sigma^4$  are analogous to the covariance (i.e.,  $\sigma_{n \ln s \ln t} = E(\delta_n \delta_{\ln s} \delta_{\ln t})$ ). Normally, (5) and (6) are not helpful

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for calculating  $\mu_I$  and  $\sigma_I^2$ , since we don't know a priori how or if  $\ln s$  and  $\ln t$  are correlated and thus the magnitude of the covariance terms are unknown. For border irrigation, however, where ponded water is free to move across the surface, we can assume that there will be little correlation between  $\ln s$  and  $\ln t$  and therefore  $\sigma_{\ln s \ln t}^2 = \sigma_{\ln t}^2 = \sigma_{\ln s}^2 = \sigma_{\ln s \ln t}^3 = \sigma_{\ln t \ln t}^3 = 0.0$  and  $\sigma_{\ln s \ln t \ln t}^4 = \sigma_{\ln s}^2 \sigma_{\ln t}^2$  [Mood et al., 1974, p. 180]. With this simplification we have

$$\mu_{\ln I} = \mu_{\ln s} + \mu_n \mu_{\ln t} \quad (7)$$

$$\sigma_{\ln I}^2 = \mu_n^2 \sigma_{\ln t}^2 + \mu_{\ln t}^2 \sigma_n^2 + \sigma_{\ln s}^2 + 2\mu_{\ln t} \sigma_{\ln s} \sigma_n \quad (8)$$

As will be illustrated later, for the case where (2) describes the infiltration, the last two terms in (8) will not contribute significantly to the total and can be ignored. Therefore the variance can be found from

$$\sigma_{\ln I}^2 \approx \mu_n^2 \sigma_{\ln t}^2 + \mu_{\ln t}^2 \sigma_n^2 + \sigma_{\ln s}^2 + 2\mu_{\ln t} \sigma_{\ln s} \sigma_n \quad (9)$$

which can be calculated from the distributions of  $\ln s$ ,  $\ln t$ , and  $n$ .

The mean and variance of the actual infiltration depth for the border can then be calculated from (7) and (9) using the relationships [Miller and Freund, 1965, p. 80]

$$\mu_I = \exp(\mu_{\ln I} + 1/2\sigma_{\ln I}^2) \quad (10)$$

$$\sigma_I^2 = \exp(2\mu_{\ln I} + \sigma_{\ln I}^2)(\exp(\sigma_{\ln I}^2) - 1) \quad (11)$$

However, these equations are valid only for lognormally distributed  $I$  which limits the applicability of (7) and (9) for describing the distribution of  $I$  when (1) is used for infiltration.

If (2) is used instead to describe infiltration, similar equations for the mean and variance of  $I$  can be derived. The resulting equation for the variance is considerably more complicated, however, although the log transform is not required, since  $I$  is the sum of two products. From (2) the mean infiltration depth is

$$\mu_I = \mu_s \mu_x + \sigma_{sx}^2 + \mu_a \mu_t + \sigma_{at}^2 \quad (12)$$

To find the variance of  $I$  we employ the same method used to derive (6). The resulting expression is

$$\begin{aligned} \sigma_I^2 = & \mu_s^2 \sigma_x^2 + \mu_x^2 \sigma_s^2 + 2\mu_s \mu_x \sigma_{sx}^2 + 2\mu_x \sigma_{sxx}^3 + 2\mu_s \sigma_{sxx}^3 \\ & + \sigma_{sxxx}^4 - \sigma_{sx}^2 \sigma_{sx}^2 + \mu_a^2 \sigma_t^2 + \mu_t^2 \sigma_a^2 + 2\mu_a \mu_t \sigma_{at}^2 \\ & + 2\mu_t \sigma_{aat}^3 + 2\mu_a \sigma_{att}^3 + \sigma_{aat}^4 - \sigma_{at}^2 \sigma_{at}^2 + 2\mu_a \mu_s \sigma_{ts}^2 \\ & + 2\mu_s \mu_x \sigma_{ts}^2 + 2\mu_t \mu_s \sigma_{ax}^2 + 2\mu_t \mu_x \sigma_{as}^2 + 2\mu_a \sigma_{tsx}^3 + 2\mu_t \sigma_{asx}^3 \\ & + 2\mu_s \sigma_{ats}^3 + 2\mu_x \sigma_{ats}^3 + 2\sigma_{atsx}^4 - 2\sigma_{at}^2 \sigma_{sx}^2 \end{aligned} \quad (13)$$

If we again assume no correlation between opportunity times and infiltration coefficients, these equations reduce to

$$\mu_I = \mu_s \mu_x + \mu_a \mu_t \quad (14)$$

$$\begin{aligned} \sigma_I^2 = & \mu_s^2 \sigma_x^2 + \mu_x^2 \sigma_s^2 + \sigma_s^2 \sigma_x^2 + \mu_a^2 \sigma_t^2 + \mu_t^2 \sigma_a^2 + \sigma_a^2 \sigma_t^2 \\ & + 2\mu_s \mu_x \sigma_{sx}^2 + 2\mu_t \mu_a \sigma_{at}^2 + 2\mu_s \sigma_{tsx}^3 + 2\mu_t \sigma_{ats}^3 \\ & + 2\mu_s \sigma_{ats}^3 + 2\mu_x \sigma_{ats}^3 + 2\sigma_{atsx}^4 \end{aligned} \quad (15)$$

Equation (15) can be simplified further if  $a$  and  $s$  are not correlated, as was found by Talsma [1969]. However, other field experiments have found strong correlations between these parameters [Sharma et al., 1980] and thus this simplification may not always be possible.

We should note at this point that if we set  $a = 0$  in (2) (where  $n$  is constant), the resulting expression is analogous to the equation used by Bralts et al. [1981] to describe drip irrigation emitter flow. Removing the  $a$  and  $t$  terms from (14) and (15) gives

$$\mu_I = \mu_s \mu_x \quad (16)$$

$$\sigma_I^2 = \mu_s^2 \sigma_x^2 + \mu_x^2 \sigma_s^2 + \sigma_s^2 \sigma_x^2 \quad (17)$$

where now (17) is the correct expression for the variance of  $I$  and not the expression given by Bralts et al. [1981, their (14)] which is in error. However, this correction does not affect their subsequent results.

#### INCORPORATING VARIABLE INFILTRATION INTO BORDER IRRIGATION MODELS

In this section we illustrate how the relationships described above can be incorporated into border irrigation models and how infiltration variability affects the calculated irrigation uniformity. These effects will be examined within the context of a simple border irrigation model described by Jaynes [1986], which was modified to describe a border having variable infiltration across its width as well as along its length but where the water still advances uniformly across the border. The model was applied to a border, 100 m long by 12 m wide, having a uniform slope of 0.0011 and a Manning roughness factor of 0.089. Water was applied at a rate of  $12 \text{ m}^2 \text{ min}^{-1}$  across the width of this border for 75 min. Equation (2) with  $n = 0.5$  was used to describe infiltration where  $s$  and  $a$  were assumed to be either constant, normally, or lognormally distributed across the border and  $s$  was considered either independent or linearly dependent on  $a$ .

The border was divided into blocks of approximately  $0.5 \times 0.5 \text{ m}$  for a total of about 4800 blocks ( $200 \times 24$ ). Infiltration parameters were either constant for each block or were assigned randomly. The exact length and number of blocks were calculated during each simulation and varied slightly for each irrigation [Jaynes, 1986]. This size of block was used because it corresponded to the sample area for which infiltration was measured by Sharma et al. [1980], whose data were used to characterize the  $s$  and  $a$  distributions for the border (Table 1). For  $a$  normally distributed and independent of  $s$ , values for each block were found from

$$a = \mu_a + \beta_1 \sigma_a \quad (18)$$

where  $\beta_1$  is a standard normal variate ( $\mu_{\beta_1} = 0$  and  $\sigma_{\beta_1} = 1$ ). For independent lognormally distributed  $a$ , the block values were generated from

$$a = \exp(\mu_{\ln a} + \beta_1 \sigma_{\ln a}) \quad (19)$$

Values for  $s$  were generated in the same way.

For simulations where  $s$  was assumed to be a linear function of  $a$ , (18) or (19) was used to generate an  $a$  value for each block, and the corresponding  $s$  value was calculated from

$$s = C_1 a + C_2 + \beta_2 \sigma_{sa} \quad (20)$$

for normally distributed parameters and

$$s = \exp(C_3 \ln a + C_4 + \beta_2 \sigma_{\ln s \ln a}) \quad (21)$$

for lognormal distributions where  $\beta_2$  was randomly drawn from a standard normal variate ( $\beta_2 \neq \beta_1$ ), and the  $C$  and  $\sigma$  values are the regression coefficients and standard deviations from regression respectively found from linear regression using the measured data or log of the measured data.

TABLE 1. Contribution to Total Variance by Each Term in (13) and the Variance of  $I$  Calculated Both Directly From Simulated  $I$  Values and (13).

Simulation	$\mu_s^2\sigma_x^2$	$\mu_x^2\sigma_s^2$	$2\mu_s\mu_x\sigma_{sx}^2$	$2\mu_x\sigma_{sx}^3$	$2\mu_s\sigma_{sx}^3$	$\sigma_{sxx}^4$	$\sigma_{xx}^2\sigma_{sx}^2$	$\mu_a^2\sigma_t^2$	$\mu_t^2\sigma_a^2$	$2\mu_a\mu_t\sigma_{at}^2$	$2\mu_t\sigma_{aat}^3$	$2\mu_a\sigma_{at}^3$	$\sigma_{aat}^4$
1	0.1631	10.6355	0.0046	0.0092	-0.0012	0.0490	0.0	0.0728	2.5268	-0.0095	0.0139	0.0002	0.0416
2	0.1721	11.1679	-0.0152	-0.0772	-0.0001	0.0574	0.0	0.0787	2.2064	-0.0024	-0.0126	-0.0004	0.0392

Simulation	$\sigma_{at}^2\sigma_{at}^2$	$2\mu_a\mu_t\sigma_{tx}^2$	$2\mu_t\mu_s\sigma_{tx}^2$	$2\mu_t\mu_s\sigma_{ax}^2$	$2\mu_t\mu_s\sigma_{as}^2$	$2\mu_a\sigma_{tx}^3$	$2\mu_t\sigma_{ax}^3$	$2\mu_s\sigma_{ax}^3$	$2\mu_x\sigma_{ats}^3$	$2\sigma_{atxx}^4$	$2\sigma_{at}^2\sigma_{xx}^2$	$\sigma_I^{2*}$	$\sigma_I^{2\dagger}$
1	0.0	0.2176	0.0021	-0.0133	0.0773	-0.0008	-0.0171	0.0003	-0.0328	0.0028	0.0	13.7432	13.7537
2	0.0	0.2335	-0.0107	-0.0033	8.5947	-0.0000	-0.0214	-0.0006	-0.0418	0.0820	0.0	22.4478	22.4507

Infiltration parameters are lognormally distributed, with  $\ln s$  independent of  $\ln a$  (1), or  $\ln s$  linearly dependent on  $\ln a$  (2).  
 \*Calculated from (13) and  $s, x, a,$  and  $t$  data.  
 †Calculated directly from simulated  $I$  data.

Five different infiltration parameter distributions were simulated. The first two used lognormally distributed  $s$  and  $a$  parameters that were either independent or linearly correlated. The next two cases also used either independent or linearly correlated parameters but the parameters were normally distributed. In the last simulation, normally distributed independent  $s$  and  $a$  distributions were used but the variances were double those used in the first four simulations.

RESULTS

To illustrate the applicability of (13), (14), and (15) the results from lognormally distributed parameters with independent and dependent  $s$  and  $a$  are shown in Table 2. Listed are the variances of  $I$  calculated by (13) and the contributions to the variances by each term in (13) as calculated from the simulated  $s, x, a,$  and  $t$  results. Also, the variances of  $I$  as calculated directly from the simulated  $I$  results are shown. The two values of  $\sigma_I^2$  are identical within the errors of numerical round-off for both simulations, verifying (13) and illustrating that  $\sigma_I^2$  can be calculated indirectly without using the actual  $I$  values.

It is also illuminating to examine Table 1 for terms that significantly contribute to  $\sigma_I^2$ . None of the covariances between the infiltration parameters and opportunity times contribute more than 1% to  $\sigma_I^2$ . Therefore the assumption we made in simplifying (6) and (13) appears valid. In addition, none of the higher degree moments contribute more than 0.5% to  $\sigma_I^2$  and thus can also be ignored. Eliminating these terms from (15), we have

$$\sigma_I^2 \approx \mu_s^2\sigma_x^2 + \mu_x^2\sigma_s^2 + \mu_a^2\sigma_t^2 + \mu_t^2\sigma_a^2 + 2\mu_a\mu_t\sigma_{tx}^2 + 2\mu_t\mu_s\sigma_{as}^2 \quad (22)$$

Table 1 also illustrates that for the conditions modeled the variance in the infiltration terms is the largest contributing factor to the variance of the infiltration depth. The variance in infiltration caused by the variance in opportunity time across the border is virtually insignificant for these well-irrigated examples ( $CV_t < 0.15$  for both simulations). Also, the variance in  $s$  dominates over that of  $a$  in contributing to  $\sigma_I^2$ . This is as expected for a short-duration irrigation and we would expect the contribution to  $\sigma_I^2$  by  $\sigma_a^2$  to increase for longer irrigations.

Equation (22) suggests that the variance of  $I$ , where variable infiltration is assumed across a border, can be calculated from a single simulation using a uniform infiltration equation and knowledge of the variance of  $s$  and  $a$  for the border. This is possible because the interaction between infiltration opportunity time and infiltration rate is not greatly affected by a spatially variable rate. The values for  $\mu_s, \mu_a, \sigma_s^2, \sigma_a^2,$  and  $\sigma_{as}^2$  in (22) can come directly from the known or assumed parameter distributions for the border, while  $\mu_x, \mu_t, \sigma_x^2, \sigma_t^2,$  and  $\sigma_{tx}^2$  can be estimated from the results of a single simulation where a uniform infiltration rate is assumed.

To test this hypothesis we ran five simulations analogous to the variable infiltration simulations but where spatially uniform infiltration (constant coefficients) was assumed. In each case, care was taken to use the correct value for the expected  $s$

TABLE 2. Mean, Variance, Their 95% Confidence Limits, and  $DU$  of  $I$  (cm) for Variable Infiltration Simulations

Distribution	Calculated Directly From Simulated $I$ Results			Estimated From (14) and (22) and Correct Estimates of $s$ and $a$		
	$\mu_I$	$\sigma_I^2$	$DU$	$\mu_I$	$\sigma_I^2$	$DU$
Lognormal, independent	7.95	13.75				
	7.86-8.04	13.31-14.22	0.53	7.94	13.94	0.53
Lognormal, dependent	8.03	22.45				
	7.92-8.14	21.72-23.22	0.43	7.94	22.89	0.44
Normal, independent	8.03	8.66				
	7.96-8.10	8.38-8.95	0.55	8.00	8.85	0.55
Normal, dependent	8.13	12.57				
	8.05-8.21	12.17-13.00	0.47	8.10	13.07	0.48
Normal, independent, double $\sigma_s^2$ and $\sigma_a^2$	9.08	23.73				
	8.97-9.19	22.97-24.54	0.39	9.00	22.89	0.42

$I$  statistics are calculated directly from  $I$  results and from (14) and (22) using the correct parameter statistics.

and  $a$  coefficient for the given probability distribution function (see the appendix). Results are shown in Table 2 where the mean and variance of infiltration calculated from the simulated  $I$  data are listed for the five distributions. Since each simulation is but one realization of border irrigation under variable infiltration, the approximate 95% confidence limits for  $\mu_I$  and  $\sigma_I^2$  were calculated for each simulation [Snedecor and Cochran, 1967, pp. 67 and 75] and are listed in Table 2. Also shown are  $\mu_I$  and  $\sigma_I^2$  calculated from (14) and (22) using the known infiltration coefficient statistics and the opportunity time results from the corresponding uniform infiltration runs.

When the infiltration parameters are lognormally distributed the statistical approach accurately calculates the mean and variance of  $I$  whether the parameters are independent or correlated. When the infiltration parameters are normally distributed, the calculated  $\mu_I$  values also agree very well with the results of the simulations. The predicted variances, however, are outside the 95% confidence limits for the calculated variances in the last two examples. There is no consistent bias in the predicted variance, it being greater than simulated in the normal dependent case and less in the simulation where the variances of the infiltration parameters were doubled. The predicted values are still close estimates of the simulated variances, being within 4% of these values.

In irrigation engineering, the infiltration mean and variance is seldom used directly. Instead, a more common measure of irrigation uniformity is the distribution uniformity of the lowest quarter,  $DU$ , which is defined as the average depth over the least irrigated quarter of the field,  $\mu_{I_{0.25}}$ , divided by the average depth for the entire field [Kruse, 1978]:

$$DU = \mu_{I_{0.25}}/\mu_I \quad (23)$$

Table 2 lists the  $DU$  values for the five simulations.  $DU$  was calculated directly for the simulated results by sorting the  $I$  results and dividing the mean of the lowest quarter by the mean of all  $I$  values. For the estimated  $\mu_I$  and  $\sigma_I^2$  values,  $DU$  was calculated by assuming that  $I$  was distributed in the same manner as the  $s$  and  $a$  parameters, either normally or lognormally. This is a good assumption provided the borders are irrigated properly such that the opportunity times across the border do not vary significantly. For lognormally distributed  $I$ ,  $DU$  was calculated as described by Warrick [1983].  $DU$  for normally distributed  $I$  was calculated as described in the appendix. For all simulations except the last the properly estimated  $DU$  values closely approximated the measured values. When the standard deviations for  $s$  and  $a$  are doubled, the calculated  $DU$  overestimates the calculated value by about 10% despite close agreement between estimated and calculated  $\mu_I$  and  $\sigma_I^2$  values. This is because the opportunity times are much more variable for this irrigation, and therefore the  $I$  distribution is more distorted from the assumed normal distribution for which  $DU$  is calculated.

#### CONCLUSIONS

One of the biggest shortcomings of border irrigation models is that they normally do not account for the effects of spatially variable infiltration rates across a border when calculating the irrigation uniformity. Most existing models could probably be modified to allow for spatially variable infiltration and used in Monte Carlo simulations but this may not always be practicable nor is it necessary. We have shown instead that infiltration variability can be accounted for directly in these models by a simple combination of variances technique. While not

exact, the method gives values for the mean and variance of infiltration depth that are very close to simulated values. The method requires only the mean, variance, and covariance of the opportunity time, which can be easily calculated by any irrigation model using constant infiltration coefficients, and the infiltration coefficient statistics for the border. This approach is also advantageous in that the infiltration mean and variance is broken down into its component parts and their contribution to the total easily quantified. Those components limiting irrigation uniformity can then be identified, permitting adjustment in irrigation design or management for improved uniformity.

The techniques described above were demonstrated for normally and lognormally distributed infiltration coefficients. However, the combination of variances technique is valid for any distribution provided good estimates of the mean and variance of the distribution are used. The technique will provide accurate estimates of  $\mu_I$  and  $\sigma_I^2$ , but other measures of irrigation uniformity such as  $DU$  will depend on the  $I$  distribution which, in turn, depends on the soil physical properties and border irrigation management. For more complicated distributions of irrigation coefficients and opportunity times, combination techniques such as those described by Mood *et al.* [1974, p. 175,] may be helpful in approximating the distribution type for the resulting infiltration depths, thus permitting accurate  $DU$  calculations. We should also note that the approach used here was based on assuming completely random infiltration coefficients across the border. Numerous recent studies have shown that many soil properties are not completely random but exhibit a spatial structure [Russo and Bressler, 1981; Sisson and Wierenga, 1981; Vieira *et al.*, 1981]. Spatial structure should have little effect on the results shown here unless there is a consistent trend in the infiltration characteristics down the border. In this case, additional terms would then be needed in the defining equations but the technique would still be valid.

#### APPENDIX: CHOOSING SINGLE-VALUED INFILTRATION PARAMETERS

In performing an irrigation simulation for a specific field or border, we seldom have sufficient data to accurately characterize the probability distribution functions for the  $s$ ,  $a$ , or  $n$  infiltration parameters. Usually, we fit an assumed normal or lognormal distribution to the available data and use the expected values from these distributions in the irrigation model. For lognormal parameters, the expected value is found from (10), while (11) gives the variance of the untransformed data.

If the population is assumed to be normally distributed, then  $\mu_s$  and  $\sigma_s^2$  should be used directly in (14) and (22), and  $\mu_s$  is the best estimator to use in a constant coefficient infiltration equation. Problems arise, however, when the assumed normal distribution  $ND$  is truncated. That is, the distribution is restricted to only part of the predicted parameter range due to some physical restriction on the parameter such as not allowing any negative values. For situations where negative values are not allowed, truncation will occur when the coefficient of variation  $CV$  is greater than 0.25 but becomes significant only when  $CV \geq 0.5$ .

The data of Sharma *et al.* [1980] illustrates this problem. For 26 measurements they found  $\mu_s = 0.639 \text{ cm min}^{-1/2}$  and  $\sigma_s^2 = 0.100 \text{ cm}^2 \text{ min}^{-1}$  for a  $CV$  of 0.5. If a normal distri-

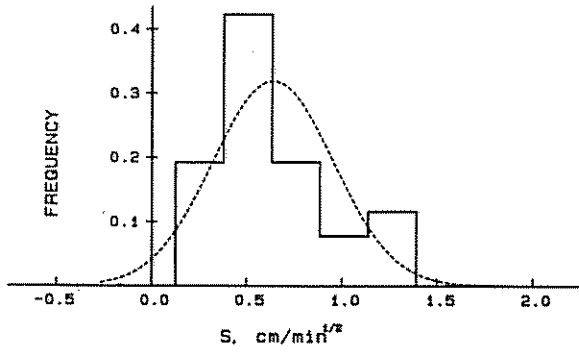


Fig. A1. Histogram of measured  $s$  values and their calculated normal distribution.

bution is assumed to represent these data with this mean and variance, a portion of the population is predicted to have negative values (Figure A1), which is physically meaningless. The expected value of the untruncated portion of the distribution is thus shifted to the right in the figure and the mean of the  $ND$  is no longer the best estimate to use in the irrigation simulation. Instead, the expected value is found by taking the weighted mean of the allowed values [Nakayama et al., 1979]:

$$\mu_{s'} = \mu_s + \sigma_s / (2\pi)^{1/2} \exp(-z_1^2/2) / (1 - P(z_1)) \quad (A1)$$

Where the prime indicates a truncated distribution,  $z$  is the standardized normal variate,

$$z = (s - \mu_s) / \sigma_s \quad (A2)$$

with  $z_1$  corresponding to the point  $s = 0.0$  in (A2), and  $P$  is the cumulative probability operator. The denominator in (A1) is found from cumulative  $ND$  tables and is equal to the cumulative probability under the normal curve for  $z > z_1$ . In this example,  $\mu_{s'} = 0.656$  as compared to  $\mu_s = 0.639 \text{ cm min}^{-1/2}$ .

The effective variance of  $s$  also needs to be adjusted from the calculated value for a truncated  $ND$ . The effective variance is reduced, since it is found by integrating over only the positive portion of the distribution. Defining  $\mu_{s'} = (\mu_{s'} - \mu_s) / \sigma_s$ ,  $\sigma_{s'}^2$  is found from

$$\sigma_{s'}^2 = \frac{\sigma_s / (2\pi)^{1/2} \int_{z_1}^{\infty} (z - \mu_{s'})^2 \exp(-z^2/2) dz}{1 - P(z_1)} \quad (A3)$$

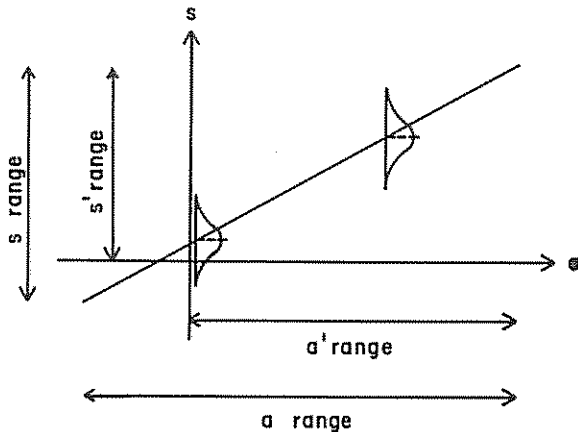


Fig. A2. Calculated ranges for and linear regression between  $s$  and  $a$  values;  $s'$  and  $a'$  ranges are for truncated distributions. Normal distributions about the regression line represent predicted probabilities of  $s$  values given  $a$ .

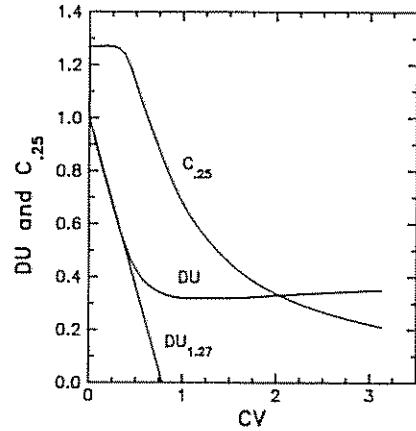


Fig. A3.  $DU$  and  $C_{0.25}$  versus  $CV$ .  $DU_{1.27}$  is calculated using constant value for  $C_{0.25}$  of 1.27.

which is easily integrated numerically. For this example the variance of  $s$  is reduced from  $0.100$  to  $0.089 \text{ cm}^2 \text{ min}^{-1}$ . These corrected values for  $\mu_{s'}$  and  $\sigma_{s'}^2$  were used for the constant coefficient normal independent simulation.

A second problem caused by truncating the  $ND$  is that when one parameter is considered a function of the other, the statistics for the dependent variable will be affected if the independent  $ND$  is truncated. This problem is illustrated in Figure A2 where  $s$  is linearly regressed against  $a$  using the least sum of squares method. As above, the original  $ND$ 's of  $a$  and  $s$  will be restricted to the upper right-hand quadrant of Figure A2 when only positive values are allowed. This limits the range over which the population statistics are calculated with the mean increasing and the variance and covariance being reduced. Assuming that  $s$  is linearly dependent on  $a$ , whose  $ND$  is truncated, we find the new mean of  $s$  is

$$\mu_{s'} = C_1 a' + C_2 \quad (A4)$$

and the effective variance of  $s'$  is

$$\sigma_{s'}^2 = C_1^2 \sigma_{a'}^2 + \sigma_{sa}^2 \quad (A5)$$

where  $\sigma_{sa}$  is the standard deviation from regression of  $s$  on  $a$  and  $C_1$  and  $C_2$  are the regression coefficients [Snedecor and Cochran, 1967, p. 138]. The covariance can be calculated in a similar manner:

$$\sigma_{a's'}^2 = C_1 \sigma_{a'}^2 \quad (A6)$$

For the Sharma et al. [1980] data where  $C_1 = 17.337$ ,  $C_2 = 0.2148$ , and  $\sigma_{sa}^2 = 0.1911$ , the effective mean of  $s$  is increased from  $0.639$  to  $0.666 \text{ cm min}^{-1/2}$ , which is larger than if  $s$  is considered to be independent of  $a$ . The effective variance of  $s$  and covariance decrease to  $0.0896$  from  $0.100$  and to  $0.00306$  from  $0.00376$ , respectively.

A third problem arises when the dependent variable  $ND$  is also truncated because the regression equation is calculated by assuming that  $s$  is normally distributed about it and that the variance of  $s$  about this line is uniform for all values of  $a$  [Snedecor and Cochran, 1967, p. 141]. Figure A2 illustrates that for small positive values of  $a$ , the  $s$  values can no longer be normally distributed about the regression line but instead are limited to positive values only. Therefore the effective population statistics are again slightly modified (mean increased and variance and covariance decreased). The new statistics must be calculated numerically with the expectation of

$s'$  equal to  $C_1 a' + C_2$  except when the expected distribution of  $s'$  about  $a'$  has a nonzero probability of values less than zero. Then (A1) is used to find the expected  $a'$  value. For the data used here, the correct mean, variance, and covariance of  $s$  when expressed as a linear function of  $a$  are  $\mu_s = 0.670$ ,  $\sigma_s = 0.0872$ , and  $\sigma_{sv}^2 = 0.00299$  which are the values used in the normal dependent constant parameter simulation.

The last consequence of having a truncated  $ND$  affects the calculation for uniformity of the lower quarter  $DU$  equation (23). For  $I$  normally distributed across a border, numerous authors have shown that  $DU$  can be calculated from [Warrick, 1983; Hart and Heerman, 1976]

$$DU = 1 - 1.27 CV \quad (A7)$$

where the value 1.27 comes from the average of the bottom quarter of the standard  $ND$ .

In Figure A1, however, it can be seen that for a truncated  $ND$  the area under the distribution that represents the lowest quarter of the allowable values is shifted to the right on the distribution. In effect, this means that the value 1.27 in (A7) is not actually constant but a function of  $CV$  and (A7) cannot be used for large  $CV$ 's [Warrick, 1983]. To find  $DU$  for any  $CV$ , we use (A1) to calculate  $\mu_I$  for the truncated  $ND$  and calculate  $\mu_{I_{0.25}}$  from

$$\mu_{I_{0.25}} = \mu_I + \sigma_I / (8\pi)^{1/2} (\exp(-z_1^2/2) - \exp(-z_2^2/2)) \quad (A8)$$

where, again,  $z_1 = -\mu_I/\sigma_I$  ( $I = 0$ ) and  $z_2$  is defined by

$$P(z_2) - P(z_1) = 0.25 \quad (A9)$$

and can be found from cumulative probability tables.

Figure A3 shows the value  $C_{0.25}$  for various values of  $CV$  where  $DU$  is calculated from

$$DU = 1 - C_{0.25} CV \quad (A10)$$

As can be seen,  $C_{0.25}$  equals 1.27 only for  $0 < CV < 0.25$  and then decreases with increasing  $CV$ . Figure A3 also shows the variation in  $DU$  with  $CV$ .  $DU$  decreases as  $CV$  increases from 0.0 to about 1.0 after which the decrease in  $C_{0.25}$  is balanced by the increase in  $CV$  and  $DU$  remains approximately constant.  $DU$  calculated with a constant  $C_{0.25}$  of 1.27 is also shown in Figure A3. The error in this calculation increases with  $CV$  and becomes especially significant for  $CV > 0.5$ , with  $DU$  becoming negative for  $CV > 0.8$ .

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