STATISTICAL APPROACH TO INCORPORATING THE INFLUENCE OF LAND-GRADING PRECISION ON LEVEL-BASIN PERFORMANCE


ABSTRACT. Monte-Carlo simulation was used to determine the suitability of statistical equations for estimating the influence of soil surface elevations on the low-quarter distribution uniformity, DU\(_{LQ}\), of level-basin irrigation systems. It was shown that these equations give reasonable estimates of DU\(_{LQ}\), provided that the advance curve is known. The Monte-Carlo simulations also provided an estimate of the variation in DU\(_{LQ}\) for different fields with the same standard deviation of elevations. The statistical equations demonstrate that irrigation uniformity is influenced by the ratio of soil surface elevation standard deviation to average depth infiltrated. For conditions that would be typical of laser-leveled basins within the U.S., the influence of elevation variations on DU\(_{LQ}\) is small. However, for poorly leveled fields, as frequently occur in Egypt, these elevation variations can significantly reduce DU\(_{LQ}\), thus limiting potential efficiencies. In some cases, these simple equations can be used to adjust level-basin designs to account for the variation in surface elevations without the need for extensive simulation studies.

Keywords. Surface irrigation, Level-basin irrigation, Irrigation efficiency, Land grading, Design, Developing countries.

Level-basin irrigation has been touted in the United States as a potentially efficient surface irrigation method. Conversions to level-basin irrigation systems has often resulted in little or no improvement in irrigation efficiency. Reasons for this include, inappropriate designs, poor management, and inadequate land grading. These occur even though design aides and guidelines are available (USDA, 1974; Clemmens et al., 1995; Clemmens, 1998). These design aides consider basin dimensions (length and width), soil infiltration, resistance to flow, flow rate, and application time. Accordingly, high efficiency can be achieved by balancing these factors such that uniformity is high with a sufficiently light application of water. High uniformity is assumed to result from relatively rapid advance, uniform recession resulting from ponded water, and relatively uniform soil infiltration properties (i.e., with differences resulting only from differences in infiltration opportunity time). The design aides do not consider practical limitations to uniformity such as variable soils and non-uniform soil elevations, the latter of which influence the amount of ponded water that infiltrates. As a result, the design aides tend to overestimate the uniformity and efficiency that is attainable.

A different set of constraints limit the application of level-basin irrigation systems in developing countries such as Egypt. Land in Egypt is often farmed in long strips (e.g., 100 m), regardless of whether tillage and furrow preparation are done by hand with shovels, with plows pulled by oxen, or by tractor-pulled implements. However, for irrigation, the field is divided into small basins (e.g., 5 m × 5 m). Previous studies suggest that this irrigation practice results from the poor land-grading precision attained with traditional practices; that is, with the small land slopes, poor field grading severely restricts water advance (Osman, 1988). The implication is that land-grading practices have essentially restricted the adoption of modern surface irrigation in Egypt. Tractors are being used more commonly for tillage and furrow preparation, and laser-controlled land grading is growing in popularity. However, even where tractors are used for soil preparation and even where laser-controlled land grading is practiced, traditional irrigation practices are still being used.

Egyptian policies toward privatization have placed a tremendous challenge on Egyptian farmers, since they have to compete with other farmers around the world where modern technology is far easier to adopt. Egyptian farmers have started to use appropriate levels of technology such as certified seeds, compound fertilizers, and combines for harvesting. Improvement of on-farm irrigation is crucial to secure the potential of such inputs. Modern surface irrigation is viewed as having significant potential for the surface irrigated areas of Egypt.

To determine the impact of land-grading precision on irrigation performance, several questions need to be answered. What is the influence of undulations in the soil surface on level-basin performance? How can this be estimated for a specific situation? What level of grading precision should be recommended? Can this be accomplished with existing technology? Can these recommendations be used to drive maintenance grading...
decisions? What level of grading precision is needed for adoption of modern level-basin irrigation in Egypt?

In order to answer the first question, we need to know the relationship between land-grading precision and irrigation uniformity and efficiency. The effects of land-grading precision on level-basin performance could be analyzed with a computer model simulating the hydraulics of level-basin irrigation. Such a model would have to be two-dimensional, in plan, and be able to simulate a non-level field surface. Playan et al. (1994) presented a two-dimensional surface irrigation model, and extended it to consider non-level field surfaces (Playan et al., 1996). This is an explicit model. Strelkoff et al. (1996) questioned the handling of boundary conditions in the model of Playan and developed an implicit two-dimensional model that was developed specifically to handle non-level surfaces. These two-dimensional models are geared toward analysis of a single set of conditions and are computationally intensive. Conducting general studies with these models would be extremely difficult and time consuming. Existing one-dimensional models could be used (e.g., SRFR; Strelkoff, 1990), but such models assume variations in elevation only in one direction. Thus all high elevations are considered as ridges running the full width of the field and the influence of a non-uniform field surface on advance is exaggerated. Such an analysis is presented in a companion article (Fangmeier et al., 1999). Further, it may be difficult to develop general results from these models that can be applied to design.

In a different approach, statistical equations are proposed for determining the effects of a non-level field surface on distribution uniformity (Clemmens, 1991). The purpose of this article is to determine the suitability of these statistical equations for estimating the influence of a non-level surface on level-basin irrigation uniformity and efficiency. In this article, Monte-Carlo simulation is used to test the appropriateness of the statistical equations. The implications of these results for guiding land grading decisions are discussed, along with the limitations of this approach.

**IRRIGATION UNIFORMITY**

Irrigation, particularly non-pressurized surface irrigation, is inherently nonuniform. Because of this, some portion of the field often does not receive the targeted amount of water while most of the field receives more than the target amount. This nonuniformity limits application efficiency and causes subsequent irrigations to be scheduled earlier to avoid crop stress within the underirrigated portion of the field. Irrigation uniformity also has an influence on crop yield. Field experience suggests that more uniform irrigation tends to provide more uniform growing conditions and typically higher, more consistent yields (Solomon, 1983).

The ASCE Task Committee on Irrigation Efficiency and Distribution Uniformity suggests the following definition for the low quarter Distribution Uniformity (Burt et al., 1997):

\[
DU_{1q} = \frac{\text{Average low-quarter depth}}{\text{Average depth of water accumulated}} \tag{1}
\]

where the average low-quarter depth is the volume accumulated in the one-fourth of the total area of elements with the smallest depths divided by one-fourth of the total area of elements. This differs from prior definitions in that water accumulated includes water intercepted by the canopy in addition to the water infiltrated. It also includes a sense of scale related to plant areas (elements). This addition is particularly important for orchards and vineyards. The Task Committee also considers low area fractions other than one quarter. In this article, we deal only with the low quarter and we use the terms infiltrated and accumulated synonymously. The notion of element scale will be discussed in more detail as it pertains to grading precision.

Here, we are interested in the uniformity of level-basin irrigation systems. Clemmens (1991) has identified four components that significantly influence level-basin irrigation uniformity, as follows:

1. Variation in infiltration opportunity time caused by difference in advance and recession curves.
2. Variation in surface retention of water caused by a non-level field surface.
3. Variation in soil infiltration properties.
4. Variation in volumes of water applied to different basins within a field.

Variations in soil infiltration properties are difficult to estimate and even more difficult to adjust for, and will not be further considered in this article. The aim of this work is to study the compound effect of the first two factors, i.e., variation due to surface retention and variation due to advance time, given uniform soil properties. The last item listed is of interest to surface irrigation in developing countries, and is particularly relevant in Egypt. This article is part of a research program aimed at the adoption of long furrow and basins (or border strips) in Egypt as a means for improving surface irrigation. The hypothesis of that research is that the use of long strips could improve efficiency and uniformity by reducing the variability (and gross amount) of water applied to these small blocks. However, this factor is not evaluated in this article.

**MICROTOPOGRAPHY**

Very little research has been done on the spatial structure of land surfaces and its influence on surface irrigation uniformity. The relevant scale has a very significant influence on the nature of soil-surface microtopography, particularly as the prevailing land slope gets smaller. For level basin, where there is no slope, microtopography effects can become dominant. At a small-scale, soil clod size and tillage implement patterns have a big influence on the variability of the soil surface. At a scale of 10 cm, differences in soil-surface elevations have essentially no impact on irrigation uniformity since they are smaller than the scale of most plant root systems. Corrugations, intentional or accidental, represent elevation variations at this scale. For row crops, the furrow spacing is a scale (e.g., 1 m) below which we are likely not concerned with elevation variations. However, measurement of soil surface elevations must be free from the effects of these small scale variations in order to provide realistic estimates of variability. For example, in measuring soil elevations in a furrow irrigated field, the results would be meaningless...
unless one always chose the same part of the furrow to measure (e.g., always on the bottom or always on the top).

There are four components to the spatial structure of field surface elevations; patterns, trends, spatial dependency, and random variations. Patterns tend to be on the scale of the plants and tillage instruments. De Souza et al. (1995) documented patterns in furrow elevations caused by tillage implements. Poor control over the furrow-opening implement, typical in Portugal, caused neighboring furrows to have significantly different elevations (differences as great as 50 mm on a laser-graded field).

Dedrick (1983) attempted to characterize variations in elevation within level basins by measuring elevations on a 30 m spacing, as was the common practice for field staking and grading procedures. However, to remove the small-scale effects, the average of five elevations at each point were used. Elevations were measured at the center and roughly 1 m away from the center in each of the four major compass directions. The distribution of elevations (each an average of five readings) followed a normal distribution for both conventional and laser-controlled grading. The standard deviation of elevations were 13 and 23 mm for the laser and conventionally graded fields, respectively. The conventionally leveled field had a bowl shape, high on the outside, indicating a significant trend. The laser-graded field did not have a discernable trend. No analysis of spatial structure was made. The fields were roughly 4 ha in size.

Playan et al. (1996) analyzed the spatial structure of elevation measurements on level basins in three different fields. The standard deviation of measured elevations were 9, 25, and 40 mm, respectively. These represent part of a very well laser-graded field (16 m × 16 m), a conventionally graded field (0.8 ha), and a poorly leveled field (0.8 ha). The distance over which elevations were correlated (defined as the range of spatial correlation) were 6, 12, and 27 m, respectively. Small-scale variability (estimated with the nugget or intercept of semivariance at zero distance) was estimated as 0, 120, and 300 mm² (0, 11, and 17 mm). Playan (personal communication, 1998) reported studies on a laser graded field. Over the season, the range stayed relatively constant at 4 to 5 mm, while the standard deviation changed from 10 to 20 mm.

Hunsaker et al. (1991) studied the effects of soil water content and surface elevation on irrigation uniformity for level basins on a 4.2 ha field. Analysis was made with elevations computed as the average of five elevations, as described above. Variations in surface elevation explained 60% of the variability in deep percolation. For one basin (0.3 ha) within this same study, Jaynes and Hunsaker (1989) reported a spatial structure for soil moisture content before and after irrigations. The distance over which soil moisture was correlated (defined as the range) was 20 m. Analysis of all measured surface elevations from this study showed no spatial structure, even though the standard deviation of elevations was 20 mm. However, when the analysis was performed with the average of five measurements at each site, the standard deviation was 15 mm and the range was 20 m. The small-scale variability dominated the analysis when all measured elevations were used in the spatial analysis.

These studies do not present consistent results on the range of spatial influence for soil surface elevations, nor is it clear how to properly deal with small scale effects. Other unpublished data collected and analyzed by the senior author and by Playan are equally inconclusive. For all the studies, the range of spatial correlation for laser-graded fields fell within 0 to 20 m, with 5 to 10 m more typical. For conventionally graded fields, the range tends to be slightly higher, say 10 to 30 m, with 20 m typical. Further research is needed to understand the nature of the spatial structure of soil surface elevations and how they influence irrigation uniformity on the scale of plant roots. Under Egyptian conditions, the standard deviations were roughly 15 to 20 mm after laser grading, 30 to 40 mm after traditional grading, and often more than 60 mm for ungraded fields (or fields not graded recently).

METHODS FOR ESTIMATING DISTRIBUTION UNIFORMITY

The combined effect of the four component factors discussed above (as well as other factors not mentioned) determines the actual distribution of infiltrated water. DUₜₗₐᵢ is a suitable performance measure for this distribution. The most straightforward way of determining DUₜₗₐᵢ would be to take a sample of measured infiltrated depths in a field and substitute the results into equation 1. However, for surface irrigation such direct measurement is extremely difficult. Consequently, the usual procedure is to estimate the distribution from other measurements. When only one component is considered, the process is straightforward; for example, the distribution of infiltrated depths stemming from the first component listed above can be generated from measurement of advance and recession and assumption of a single infiltration function for the field.

When several components must be taken into account, a variety of approaches are available. Dedrick (1983) used a brute-force approach to determine the distribution uniformity for an individual irrigation on a specific basin. He took a distribution of infiltrated water based on measured advance and average recession time and added the deviation from average field elevation. This provided a new distribution, essentially superimposing these two distributions. This procedure assumes that the variation in surface elevation translates directly into a variation in infiltrated depth. This is reasonable for soils which exhibit a constant final infiltration rate, or nearly so, and where surface water depths are sufficiently small such that significant lateral movement does not occur. The procedure is demonstrated in figure 1, where the field is broken into 40 increments. In this example, the distribution uniformity based on advance, DUₜₗₐᵢ, was 0.94, the standard deviation of elevations, sᵣ, was 20 mm, and the resulting DUₜₗₐᵢ was 0.85.

The Kostiakov-branch function (Clemmens, 1983, sometimes referred to as the Clemmens branch function) is particularly appropriate for this method of combining components. The function is defined as:

\[
    d = c + kτ^a \quad i = a k τ^{a-1} \quad \text{for } τ ≤ τ_c
\]

\[
    d = c_2 + bτ \quad i = b \quad \text{for } τ ≥ τ_c
\]

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where \(d\) is the depth infiltrated, \(i\) is the infiltration rate, \(\tau\) is the infiltration time (current time minus time water arrived), \(c\) is an initial instantaneous infiltration depth, \(b\) is the final infiltration rate, \(k\) and \(a\) are the Kostiakov infiltration parameters, \(\tau_c\) is the branch point for the two parts of the equation, and \(c_2\) is the intercept of the linear part of the function at time zero. The advantage of this equation is that the infiltration rate is exactly constant for long irrigation times and when \(\tau \geq \tau_c\), is the same at all points within the basin. This makes the assumption that water infiltrates vertically much more reasonable.

Clemmens and Dedrick (1981) used this to develop a simple equation for estimating distribution uniformity in level basins. Dedrick (1983) also used this function for computing the combined uniformity. This method for directly combining distributions has the same limitation as simulation models. It is useful for a given situation, but does not provide any way to generalize the results. A large number of trials covering a wide range of input variables would be necessary to establish any general conclusions. One alternative is to develop statistical equations, such as suggested by Clemmens (1991) and Clemmens and Solomon (1997). If these statistical equations prove sufficiently accurate, then they can be used in the design process without the need for cumbersome simulation for each design scenario of interest.

**Statistical Equations**

The Statistical Distribution Uniformity, SDU, has been proposed to provide an estimate of the global DU from measurements of the components that influence DU (Clemmens and Solomon, 1997). They define SDU for the low quarter as:

\[
SDU_{lq} = 1 - K_{lq} CV
\]

in which \(K_{lq}\) is a coefficient which depends on the statistical distribution, and \(CV\) is the coefficient of variation (standard deviation divided by mean). For a normal distribution, \(K_{lq} = 1.27\). This means that the average depth of the low-quarter area is 1.27 standard deviations away from the mean depth accumulated. Clemmens and Solomon also provide a general procedure that can be used to develop statistical equations for combining the influence of several factors in order to arrive at an estimate for the global DU.

For the analysis presented here, the two components under consideration are the influence of advance times (with recession time assumed constant) and the influence of soil surface elevation differences (which accounts for variable recession times). The infiltrated depth at any location after the irrigation, \(d\), can be found from combining the second half of equation 2 with \(r\), the deviation in soil surface elevation from average:

\[
d = c_2 + b\tau_0 - r = d_A - r
\]

where \(\tau_0\) is the infiltration opportunity (recession time minus advance time) at a given point. The variable \(d_A\) is the depth computed from the opportunity time between the advance curve and average recession curve. Since this represents two terms added, the variances of each component are simply added to give the variance of the total, assuming that these two components are independent:

\[
s_0^2 = s_A^2 + s_r^2
\]

where \(s\) is the standard deviation, \(s^2\) is the variance, and the subscripts \(0\), \(A\), and \(r\) refer to the combined value, the advance-recession component, and the surface-elevation component, respectively. Equation 5 gives the correct variance for the combined distribution, but does not give any indication about the characteristics of the final distribution. If any one of the distributions is not normally distributed, then \(K_{lq}\) is not equal to 1.27, and the value of SDU_{lq} predicted from equation 3 with \(s_0\) from equation 5 may not be accurate.

Applying equation 3 for each component and the total gives:

\[
SDU_{lq0} = 1 - K_{lq0} \frac{s_0}{d_{avg}}
\]

\[
SDU_{lqA} = 1 - K_{lqA} \frac{s_A}{d_{avg}}
\]

\[
SDU_{lqr} = 1 - K_{lqr} \frac{s_r}{d_{avg}}
\]

Solving each part of equation 6 for the standard deviation and substituting that into equation 5 gives:

\[
(1 - SDU_{lq0})^2 = (1 - SDU_{lqA})^2 + (1 - SDU_{lqr})^2
\]

assuming that all the \(K_{lq}\) are the same. Clemmens (1986) found that if the measured value of DU_{lq} was used in equation 7 rather than that computed from the standard deviation, then equation 7 gave a good prediction of SDU_{lq}. Further, Dedrick (1983) showed that the distribution of elevations tended to be normally distributed, such that \(K_{lq}\) could be reasonably estimated as 1.27. The resulting equation for the statistical distribution uniformity is:

![Figure 1–Infiltrated depths and surface elevations demonstrating method used for combining the effects of advance (assuming uniform recession) and variations in surface elevations under level-basin irrigation.](image-url)
where DU_lqA and CV_r are assumed independent, and CV_r is s_r divided by the average depth infiltrated. The suitability of equation 8 will be tested in this article.

Equation 8 provides only the expected value of SDU_lq. For any particular site, the actual value of DU_lq can be different because of different random combinations of components. Clemmens and Solomon (1997) have proposed procedures for estimating how much the real DU_lq can vary. They show that the standard deviation of SDU can be obtained from:

\[ s_{SDU_lq} = \left(1 - DU_{lq} \right)^{2} \left(1.27 CV_r \right)^{2} n \tag{9} \]

where n is the number of elements (or number of surface elevations used in the simulation). This equation applies only to the random components. In the present study, only the variations in elevations is random. Thus, DU_lq is the value only for the measured surface elevations. Equation 9 becomes:

\[ s_{SDU_lq} = \left(1.27 CV_r \right)^{2} n \tag{10} \]

**APPROACH**

Monte-Carlo simulation provides a method for testing these statistical equations by combining these two components with a large number of trials. It is important to establish a reasonable range of field conditions (inflow, application time, infiltration, roughness, and field dimensions) so that the results are representative. We also need to establish a reasonable set of elevation profiles.

**MONTE-CARLO SIMULATION**

For the above components, it is difficult and time consuming to measure each component parameter for each element in a field. If instead we know the statistical properties of a particular parameter—mean, standard deviation and distribution type (e.g., Gaussian, i.e., normal)—it is still possible to estimate the effect of that component on the final distribution of water. In a Monte-Carlo simulation, a set of random numbers, one for each element, is generated, with the set reflecting the given mean, standard deviation and distribution type. The parameter values for each element are then combined, as if the randomly varying component had actually been measured. Of course, two sets of random numbers with the same statistical distribution will produce two different DU_lq values, since the component values for the elements will be different. However, this is the key to the success of Monte-Carlo simulation, the variation in calculated DU_lq with different sets of random numbers is limited. The distribution of calculated DU_lq values itself will have a mean, a standard deviation, etc. Thus by generating a statistically significant number of calculated DU_lq, each with a different set of component values (but sharing a common statistical description), it is possible to determine the expected value of DU_lq (the mean of the sample) as well as its standard deviation and variance.

Here, the field length was divided into 40 intervals of equal length. Advance and recession are determined for the given set of conditions (discussed below) to provide a value for d_A and DU_lqA. Then for a particular value of s_r, the deviations in elevations, r, are simply added to this depth, as in equation 4. The result is a distribution of infiltrated depths, for which DU_lq can be directly calculated.

**GENERATED ADVANCE AND RECESSON CURVES**

In order to provide a meaningful evaluation of equation 8, the component DU_lq should be varied by assuming a wide range of conditions. Dimensional analysis is useful for reducing the number of parameters that have to be varied. Clemmens et al. (1995) developed non-dimensional design curves for level basins. These curves present the distribution uniformity based on the minimum depth, DU_min, which ranges from 0.50 to 0.95. Clemmens (1998) suggests that the practical design range is from cutoff at completion of advance to cutoff based on the limit line—or cutoff when stream advance reaches roughly 85% of the basin length. This actually provides a narrow range of design conditions. The only other variable for the non-dimensional design curve is the infiltration exponent a.

To provide a more understandable basis for the chosen non-dimensional design conditions, we used data from the results of a field evaluation of an irrigation event on wheat in an experimental field at Zagazig University, Mostohor, Egypt. That evaluation gave values for infiltration parameters from equation 2 of c = 35 mm, k = 65 mm/h, a = 0.4, b = 16.4 mm/h, and values for resistance from the Manning equation of n = 0.21. The basin length was 150 m and the depth applied during this irrigation event was 190 mm. Advance, recession, and DU_lq were determined from simulations with the SRFR surface irrigation simulation program (Strelkoff, 1990).

A series of different DU_lq values were obtained by changing the flow rate, with the cutoff or application time calculated to give the same volume applied. Table 1 gives the values of flow rate per unit width (q) and cutoff time (t_cot), and the SRFR results: advance time (t_a), distribution uniformities (low quarter and minimum), and R, the relative advance distance at cutoff as a fraction of field length. Note that the relative advance distance at cutoff, R, are all within the design range of 0.85 to 1.0. A more systematic approach to covering the design range is taken in Fangmeier et al. (1999). The advance curves generated for each SRFR simulation were each fit with a power advance function so that advance times could be interpolated for each grid point used by the Monte-Carlo simulation.

The specific conditions for these simulations are not of great significance, except that they cover a reasonable

<table>
<thead>
<tr>
<th>q (L s⁻¹ m⁻¹)</th>
<th>4.90</th>
<th>2.90</th>
<th>2.00</th>
<th>1.57</th>
<th>1.30</th>
<th>1.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_cot (min)</td>
<td>96.9</td>
<td>164.0</td>
<td>237.5</td>
<td>300.0</td>
<td>365.4</td>
<td>386.2</td>
</tr>
<tr>
<td>t_a (min)</td>
<td>112.9</td>
<td>177.6</td>
<td>253.8</td>
<td>329.1</td>
<td>418.2</td>
<td>466.0</td>
</tr>
<tr>
<td>DU_minA</td>
<td>0.94</td>
<td>0.90</td>
<td>0.85</td>
<td>0.81</td>
<td>0.76</td>
<td>0.72</td>
</tr>
<tr>
<td>DU_minA</td>
<td>0.91</td>
<td>0.86</td>
<td>0.79</td>
<td>0.73</td>
<td>0.64</td>
<td>0.51</td>
</tr>
<tr>
<td>R</td>
<td>0.89</td>
<td>0.94</td>
<td>0.96</td>
<td>0.94</td>
<td>0.94</td>
<td>0.90</td>
</tr>
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</table>
range of \( DU_{\text{min}} \) and \( R \). Of importance is the shape of the infiltrated water distribution profile from a specific advance curve. That is, for a \( DU_{lq} \) of say 0.81 resulting from the advance, the shape of the distribution of infiltrated depths should be similar regardless of the specifics of the irrigation. When the infiltration function used here was fit with a Kostiakov equation (eq. 2 with \( c = 0 \) and \( b = 0 \)), the exponent was roughly 0.3. The shape of this distribution may change somewhat from those calculated here if the infiltration exponent \( a \) changes significantly, e.g., very low exponents (typical of low permeability cracking clay soils) or very high exponents (typical of sandy soils with small capillary rise). The infiltration function used here represents a moderately cracking soil, but with a high final infiltration rate that is more typical of a non-cracking soil. We do not anticipate large differences in results for other soils (or values of \( a \)).

**Generated Distributions of Elevations**

Several steps were needed to generate elevation profiles for the Monte-Carlo simulation. Forty grid elements were chosen to provide a sufficiently random sample (i.e., greater than 32), but not too large so that each sample represents an area whose elevation might be expected to be spatially independent. For a 200 m basin length, this would require a range less than 5 m. While this might be reasonable for a laser-graded field under U.S. conditions, it might not represent traditional grading practices in third-world countries. However, since we have insufficient real data on the spatial structure of elevations, we chose to assume that the elevations of the grid elements are spatially independent and thus random. As discussed above, Dedrick (1983) has already established that measured field elevations follow a normal distribution.

First, a random number generator was used to obtain 40 points for a Gaussian (normal) distribution with a specified mean of zero and standard deviation of one. If the mean value of the generated points was different from zero, the difference was added to each point to make the mean equal to zero. If the standard deviation was less than 0.95 or greater than 1.05, the sample was discarded. Otherwise, the individual elevation differences were divided by the sample’s standard deviation, so that the standard deviation was forced to 1.0.

Twenty distributions satisfying this criteria were generated. For these Monte-Carlo simulations, we chose to use the actual distributions from the advance and recession curves—not randomized. This represents a trend. Any unintentional, randomly selected trend in the elevation profiles may result in interaction between the two components and exaggerate the effects of these two parameters. For example, a random selection of elevations could result in a field with an average adverse slope. The uniformity in this case would be significantly reduced from “average”. To reduce the possibility of this occurring, we took each of the 20 elevation distributions and reversed them to provide an additional 20 distributions. This was felt to provide a sufficiently random, unbiased sample.

For each Monte-Carlo simulation, the elevation profiles were multiplied by the standard deviation selected for the given test to generate the distribution of surface elevations, and ultimately depth infiltrated. The simulation was performed for the 40 bottom profiles for each value of the elevation standard deviations given in table 2. These range from a well laser-graded field to an extremely non-level field. Also shown are \( CV_t \) and \( DU_{lqr} \).

**Summary of Procedure**

This procedure assumes that the advance curve is not influenced by the variations in field-surface elevation (an assumption tested in a companion article, Fangmeier et al., 1999). For field evaluation of a measured irrigation event, this assumption is reasonable. However, it may not be reasonable when advance is determined from simulation based on a perfectly plane field surface (e.g., for design). This does not negate the statistical equation, but alters the component related to advance (i.e., the two components are not independent).

The following procedure was used:

- Generate an advance curve consisting of 41 distances and times (40 intervals or elements).
- Determine a single recession time based on the average depth infiltrated and the given infiltration function.
- Calculate average depth for each element and determine the resulting distribution of infiltrated water, \( DU_{lq, A} \).
- Generate a random distribution of surface elevations for each element based on a given \( s_r \) (± differences from average).
- For each element, add surface elevation to element-average infiltrated depth generated from advance and recession.
- Compute \( DU_{lq} \) for the resulting combined depth distribution.

The \( DU_{lq} \) computed above represents the results of one irrigation on one field with a particular combination of conditions. Because the random distributions of surface elevations can be paired with different advance times, the above procedure was repeated 40 times with 40 different sets of elevations, all with the same \( s_r \), so that a representative average \( DU_{lq} \) value could be determined, e.g., representing 40 fields with the same advance curve and same \( s_r \), but each with a different set of elevation values. The number 40 was chosen to provide sufficient randomness amongst the random samples.

**Results**

The values of \( SDU_{lq} \) computed from equation 8 are shown in table 3. The average values of \( DU_{lq} \) calculated from the 40 Monte-Carlo simulations for each case are given in table 4. The use of the “average” simulated value is consistent with \( SDU_{lq} \), since equation 8 gives the “expected” value of \( DU_{lq} \). A comparison between the expected value of \( DU_{lq} \) from the statistical equation and the average value of \( DU_{lq} \) from Monte-Carlo simulation is shown in figure 2. In all cases, the differences are small, with the differences tending to increase as \( DU_{lq} \) decreases.

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**Table 2. Values of \( s_r \) used in the Monte-Carlo simulations and resulting values of \( CV_t \) and \( SDU_{lqr} \)**

<table>
<thead>
<tr>
<th>( s_r (\text{mm}) )</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CV_t )</td>
<td>0.053</td>
<td>0.105</td>
<td>0.211</td>
<td>0.316</td>
<td>0.421</td>
<td>0.526</td>
</tr>
<tr>
<td>( DU_{lqr} )</td>
<td>0.933</td>
<td>0.866</td>
<td>0.732</td>
<td>0.599</td>
<td>0.465</td>
<td>0.332</td>
</tr>
</tbody>
</table>
Thus under the assumption that advance is independent of variation in surface elevations, equation 8 gives a good estimate of the expected value of $DU_{Uq}$.

A comparison between the standard deviations of $DU_{Uq}$ from Monte-Carlo simulation, simulated $SDU_{Uq}$, and the standard deviation of $DU_{Uq}$ predicted from equation 10 is shown in figure 3. Clearly, equation 10 provides a conservative estimate of the possible variation in $DU_{Uq}$. It may also suggest that the computer generated field elevations are not truly random, a common problem with computer random number generators, even though these distributions each seem sufficiently random.

Table 3. Statistical low-quarter distribution uniformity, $SDU_{Uq}$ (from eq. 8), as influenced by variation in soil surface elevation ($s_r$) and by different advance curves (as expressed by $DU_{UqA}$)

<table>
<thead>
<tr>
<th>$s_r$ (mm)</th>
<th>0.94</th>
<th>0.90</th>
<th>0.85</th>
<th>0.81</th>
<th>0.76</th>
<th>0.72</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.94</td>
<td>0.90</td>
<td>0.85</td>
<td>0.81</td>
<td>0.76</td>
<td>0.72</td>
</tr>
<tr>
<td>10</td>
<td>0.91</td>
<td>0.88</td>
<td>0.84</td>
<td>0.80</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td>20</td>
<td>0.85</td>
<td>0.83</td>
<td>0.80</td>
<td>0.77</td>
<td>0.73</td>
<td>0.69</td>
</tr>
<tr>
<td>40</td>
<td>0.73</td>
<td>0.72</td>
<td>0.70</td>
<td>0.67</td>
<td>0.64</td>
<td>0.61</td>
</tr>
<tr>
<td>60</td>
<td>0.60</td>
<td>0.59</td>
<td>0.57</td>
<td>0.56</td>
<td>0.53</td>
<td>0.51</td>
</tr>
<tr>
<td>80</td>
<td>0.46</td>
<td>0.46</td>
<td>0.45</td>
<td>0.43</td>
<td>0.41</td>
<td>0.40</td>
</tr>
<tr>
<td>100</td>
<td>0.33</td>
<td>0.32</td>
<td>0.32</td>
<td>0.31</td>
<td>0.29</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 4. Average low-quarter distribution uniformity from Monte-Carlo simulation as influenced by variation in soil surface elevation ($s_r$) and by different advance curves (as expressed by $DU_{UqA}$)

<table>
<thead>
<tr>
<th>$s_r$ (mm)</th>
<th>0.94</th>
<th>0.90</th>
<th>0.85</th>
<th>0.81</th>
<th>0.76</th>
<th>0.72</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.94</td>
<td>0.90</td>
<td>0.85</td>
<td>0.81</td>
<td>0.76</td>
<td>0.72</td>
</tr>
<tr>
<td>10</td>
<td>0.91</td>
<td>0.88</td>
<td>0.84</td>
<td>0.80</td>
<td>0.75</td>
<td>0.72</td>
</tr>
<tr>
<td>20</td>
<td>0.85</td>
<td>0.84</td>
<td>0.81</td>
<td>0.77</td>
<td>0.73</td>
<td>0.71</td>
</tr>
<tr>
<td>40</td>
<td>0.73</td>
<td>0.72</td>
<td>0.70</td>
<td>0.68</td>
<td>0.65</td>
<td>0.63</td>
</tr>
<tr>
<td>60</td>
<td>0.61</td>
<td>0.60</td>
<td>0.59</td>
<td>0.57</td>
<td>0.54</td>
<td>0.53</td>
</tr>
<tr>
<td>80</td>
<td>0.48</td>
<td>0.47</td>
<td>0.46</td>
<td>0.45</td>
<td>0.43</td>
<td>0.42</td>
</tr>
<tr>
<td>100</td>
<td>0.35</td>
<td>0.34</td>
<td>0.33</td>
<td>0.32</td>
<td>0.31</td>
<td>0.30</td>
</tr>
</tbody>
</table>

DISCUSSION

The change in the expected $DU_{Uq}$ as a function of $CV_r$ is shown in figure 4. Immediately after laser land-grading, the standard deviation of soil surface elevations is typically 10 to 15 mm. For well managed level basins in the United States, application depths are typically in the range of 100 to 150 mm. This gives $CV_r$ of roughly 0.10 to 0.15. Within this range, the influence of elevation variations on uniformity, as shown in figure 4, can be small. When $DU_{UqA}$ is low, suggesting poor hydraulic design, a $CV_r$ of 0.1 has almost no impact on $DU_{Uq}$ (e.g., from 0.72 to 0.69 in fig. 4). However, when $DU_{Uq}$ is high, soil surface variability can have a large influence on $DU_{Uq}$ (e.g., from 0.94 at $CV_r = 0$ to 0.85 at $CV_r = 0.1$ in fig. 4). Thus as one tries to attain higher uniformity through improved irrigation hydraulics and design, the grading precision is
begins to dominate. As $CV_r$ gets larger, it has a dominant effect on the distribution uniformity, regardless of the irrigation design.

There are two ways to lower the value of $CV_r$. First, one can reduce the standard deviation of elevations. Second, one can increase the depth applied. Where water is scarce or expensive, increasing the depth applied may not be a viable alternative. Surface irrigation has a reputation for poor irrigation efficiency. One reason is that it is often difficult to provide small applications of water when needed. Smaller basins can be designed to provide smaller application depths, but this becomes limited by leveling precision. Many farmers who have been using level basins for decades have their own laser-grading equipment so that they can regrade their field between each crop. The field levelness deteriorates over time and regrading restores a more uniform surface, allowing better uniformity or lighter application depths. These farmers are operating in the high end of $DU_{lq}$, where $CV_r$ has a big impact.

For Egyptian conditions, breaking fields up into small blocks provides less variation in elevations within the block (as least partially due to the spatial structure). It essentially eliminates the non-uniformity due to advance so that hydraulic design of the block is not a significant issue. However, managing water between blocks becomes a serious operational problem, which is beyond the scope of this discussion. However, this method takes land out of production, increases labor, and results is relatively low field irrigation efficiencies. It is also easy to understand why long furrows and basins have not been adopted.

For conventional land grading practices in Egypt, the standard deviation of surface elevations often exceeds 30 mm. With an application depth of 120 mm and $sr = 36$ mm, we get $CV_r = 0.3$. From figure 4, $DU_{lq}$ will be less than 0.6, and with a poor design can be as low as 0.5. This is not likely to offer any improvement over existing practices. However, observed laser-grading precision in Egypt showed $sr$ in the range 15 to 18 mm. Under these conditions, $DU_{lq}$ can reach 0.8, which is a more reasonable value. Once field levelness reaches a reasonable state, other practices such as the use of open secondary ditches can also be used to improve hydraulic performance.

Equation 10 provides a conservative estimate for the random variability in distribution uniformity. It does not include errors associated with advance. If the non-random component were added to equation 10, the difference from the 1:1 line in figure 3 would increase. Thus equation 10 should be adequate. See Clemmens and Solomon (1997) for a more thorough treatment of errors.

As expected, the uncertainty in our knowledge of $DU_{lq}$ from the equation 8 increases as $CV_r$ increases. This is demonstrated by figure 5 which shows the confidence intervals (±2$s$) from the Monte-Carlo simulation for $DU_{lq} = 0.85$. This does not include uncertainty associated with other measurements (e.g., advance).

The application of equation 8 to design is relatively straightforward. The design program of Clemmens et al. (1995) provides a value for average depth infiltration and $DU_{lq}$ for a given set of variables (e.g., flow rate, dimensions, etc.). For a given value of $sr$, equation 8 can be used to calculate a new estimate of $DU_{lq}$ and of the low quarter depth. A trial and error process would be needed to produce a design based on this low quarter depth (i.e., such that this low quarter depth equals the depth required).

If the statistical equation is applied to a level basin event which has been observed (i.e., an evaluation), then the influence of elevation variations on uniformity can be reasonably well predicted. However, to extrapolate the results of one observed event to an event with a different precision of land grading (i.e., computed with different $sr$) may be misleading since the change in advance times can not be predicted with these statistical equations. Further studies, such as those presented by Fangmeier et al. (1999), are needed to determine the influence of surface elevation variations on advance. However, analysis of field data collected in Egypt (Clemmens et al., 1999) suggest that advance in non-furrowed basins is not significantly influenced by grading precision.

CONCLUSIONS

The statistical equations presented here provide a reasonable estimate of the effect of soil surface elevation variations on distribution uniformity for an observed irrigation event. These results can be used for design, but caution should be exercised since the effect of poor leveling on advance in not considered.

For a coefficient of variation of soil surface elevations (standard deviation divided by mean infiltrated depth) less than about 0.1, the effect of these elevation variations on low quarter distribution uniformity is small. Laser land-grading typically provides this degree of soil-surface elevation uniformity, and thus makes low quarter distribution uniformity much more dependent on other uniformity components (e.g., opportunity time). As the grading precision gets worse, or the depth applied gets smaller, land grading precision has a larger influence on low quarter distribution uniformity. This may explain the tendency of U.S. farmers to relevel laser-graded level basins every few years, as grading precision deteriorates. It also may explain why Egyptian farmers with poor land-grading precision continue to break their fields into small blocks for irrigation.
ACKNOWLEDGMENT. Funding for this project was provided by the National Agricultural Research Project (NARP), a joint research program between the Egyptian Ministry of Agriculture and Land Reclamation and the U.S. Agency for International Development, Project No. A-037.

REFERENCES