DIELECTRIC REFERENCE MATERIALS FOR MATHEMATICALLY MODELING AND STANDARDIZING GRAIN MOISTURE METERS

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ABSTRACT. Dielectric-type instruments are the most widely used technology for grain moisture measurement. Measuring fundamental dielectric parameters (dielectric constant and loss factor) rather than instrument-specific parameters is advantageous in terms of understanding the physical basis of the measurement and in transferring calibrations; however, few grain moisture meter models report such fundamental parameters. One reason for this is the difficulty in calibrating instruments to measure dielectric parameters. High molecular weight alcohols are commonly used as dielectric reference materials for calibrating research-type instruments. However, grain moisture meters are designed to handle grain rather than liquids, so performing meaningful tests on alcohols is very difficult. Suitable granular dielectric reference materials are needed. This article describes research assessing the suitability of three alcohols (1-pentanol, 1-hexanol, and 1-decanol) and low-loss dielectrics (glass beads, plastic beads, plastic pellets, and air) as dielectric reference materials for grain moisture meters. Complex reflection coefficient measurements were performed from 1 to 501 MHz with an HP-4291A material/impedance analyzer and a transmission-line dielectric test cell. Mathematical models based on ABCD matrix methods were created to predict complex permittivity from the measurements of complex reflection coefficient. The low-loss materials were used successfully for calibrating the system to measure permittivity in spite of the fact that their permittivity values were unknown. Two characteristics were assumed: constant dielectric constant over the frequency range, and constant (near zero) loss factor over the frequency range. These two constraints (along with electrical test cell parameters defined by finite element analysis and the test cell physical dimensions) permitted unambiguous determination of all parameters for the test cell mathematical model. The resulting test cell model accurately predicted dielectric characteristics of alcohols.

Keywords. Alcohol, Dielectric, Glass, Grain, Moisture, Plastic, Radio frequency, Standardization.
cell (about 600 mm long) (Lawrence et al., 1999) that was used in developing the UGMA. Lawrence developed a signal flow graph (Kuhn, 1963; Pozar, 1998, pp. 213-217) to model the test cell. The original mathematical model for that test cell sought to achieve traceability to “absolute” dielectric measurements based on measurements of several high-purity alcohols and air (empty cell) (Funk, 2001). After several years of testing grain samples with the original test cell, we designed a smaller New Master test cell with dimensions more suitable for commercial grain moisture meters. Figure 1 compares the two test cells. Although the test results for grain were very similar for the two test cells, we needed more direct traceability to reproducible dielectric reference materials than that provided by comparisons to the old test cell, which had been calibrated with alcohols several years earlier (Funk, 2001). We needed a strategy to effectively and efficiently calibrate the New Master test cell and standardize (establish agreement among) the test cells of commercial grain moisture meters that would be based on the UGMA.

Liquids have some desirable characteristics as dielectric reference materials (Kaatz, 2007; NPL, 2003). Liquids fill the test cell uniformly regardless of its shape. A single Debye relaxation can describe their dielectric characteristics fairly closely. Debye parameters are available for several types of liquid dielectric reference materials. Some useful liquids that are not particularly hazardous are readily available in high-purity grades at moderate cost.

However, liquids can be tedious and difficult to use as dielectric reference materials, especially for test cells and associated mechanisms, electronics, and software such as those found in grain moisture meters that are designed to test relatively dry granular materials. To measure liquids with such a test cell, the cell must be sealed to prevent leakage, filled very carefully to achieve a fixed volume, and cleaned thoroughly before and after each test to avoid carry-over that could invalidate the reference measurements or subsequent measurements. The instrument operating system may require modification to allow manual operation rather than the normal automatic filling process. Some of the most useful liquids are quite hygroscopic, requiring fresh material for each test performed (NPL, 2003). Dielectric characteristics of lossy liquids are temperature sensitive, but the temperature coefficients are not generally available in the literature. For high accuracy, the temperature of the liquid reference and the test cell needs to be adjusted to the temperature at which the reference values were obtained. Of course, reference liquids that would react chemically with or dissolve the materials or adhesives used in the test cell must be avoided.

Furthermore, the absolute accuracy of the results (as indicated by the analysis of tests with different alcohols that Kurt Lawrence, USDA-ARS, performed with the original test cell) appeared to be no better than several percent (relative) (Funk, 2001). Achieving acceptable agreement among grain moisture meters (standardization) demands much better than 1% (relative) reproducibility in dielectric measurements. USDA-GIPSA requirements for official moisture meters (USDA-GIPSA, 1999) include agreement of better than ±0.05% moisture for “master” instruments for hard red winter wheat at 11% to 12.5% moisture content. (This standardization requirement, although stringent, is currently being met by a low-frequency dielectric moisture meter model with a parallel-plate test cell.) For measurements at 149 MHz, this tolerance corresponds to agreement in dielectric constant measurements of about ±0.01 units, or relative error of about 0.4%. Therefore, a better standardization strategy was needed, preferably one that did not rely on liquid dielectric reference materials.

Some granular materials are compatible with test cells such as used with grain moisture meters. One criterion is that they should not readily pick up or leave residues. (However, even relatively inert materials can become contaminated from extensive use.) Grain, although often used as a transfer standard for testing moisture meters, is notoriously unstable because it exchanges moisture with the atmosphere during handling. In addition, grain dielectric properties are strongly dependent on temperature. Stable dielectric materials such as glass and plastics have relatively low dielectric constants and very low losses compared to grain. These can be significant limitations if the grain moisture technology to be calibrated does not provide successful extrapolation or is sensitive to the dielectric loss factor. The greatest problem with using a granular material as a dielectric standard, however, is that the absolute dielectric characteristics of the bulk material are undefined; the density of the bulk material and, therefore, its dielectric characteristics are slightly different each time the material is loaded in a test cell.

Materials and Methods

Test Cell

We contracted with Dickey-john Corporation (Auburn, Ill.) to fabricate three copies of our New Master test cell design for testing. Figure 2 shows the New Master test cell and a close-up of the transition region between the N-type connector and the center electrode. For this test cell, we maintained the basic electrode configuration of Lawrence and Nelson’s original design because that configuration demonstrated beneficial traits. The use of three parallel aluminum plates (3.2 mm thick × 76.2 mm high) helped achieve good repeatability because of: (1) uniform electric field strength throughout the sensed sample volume, and (2) consistent packing density within the sample volume and between tests. We previously described the process of optimizing the dimensions of the test cell (Gillay and Funk, 2006). The spacing between the three plates that comprise the transmission line structure was adjusted to 27.7 mm to yield the minimum achievable reflection coefficient with the test cell terminated with a precision 50 Ω load.
After the optimum spacing was determined from measurements, the glass-filled polyphenylene sulfide (PPS) divider plates that defined the grain-filled section of the transmission line were machined to fit tightly and were glued in place in slots in the electrodes. The hinged gate below the grain-filled section was also fabricated from PPS.

One disadvantage of the selected test cell configuration was the lack of inherent shielding. An aluminum base-plate (attached to the aluminum end-plates) effectively decoupled the test cell from conductive structures below the test cell, such as a metallic table frame.

Figure 3 identifies the significant electrical features and equivalent transmission line sections of the New Master test cell that correspond to the physical configuration shown in figure 2. Detailed drawings of all parts of the New Master test cell are available from the authors upon request.

**MODELING THE TEST CELL**

The original mathematical model was based on a signal flow graph, as mentioned above. We found the \( ABCD \) matrix representation (Pozar, 1998, pp. 206-213) to be more effective than the signal flow graph approach because it was easily adapted to specifically include all sections of the test cell. Assigning physically relevant parameters to each section of the test cell facilitated assessment of the effects of test cell dimensional tolerances. The signal flow graph model lumped many test cell sections together and did not allow for such sensitivity analyses.

Each of the sections of the transmission line test cell was modeled as a \( 2 \times 2 \) matrix. The matrix representing a transmission line section is (Mongia at al., 1999, p. 54):

\[
\begin{pmatrix}
A & B \\
C & D
\end{pmatrix} = \begin{pmatrix}
\cosh(\gamma \cdot d) & Z \cdot \sinh(\gamma \cdot d) \\
\sinh(\gamma \cdot d) & \cosh(\gamma \cdot d)
\end{pmatrix}
\]

where

\( A, B, C, D = \) elements of the \( ABCD \) matrix

\( \gamma = \) complex propagation constant (1/m)

\( d = \) physical length of the transmission line section (m)

\( Z = \) the characteristic impedance of the transmission line section (\( \Omega \)).

If the dielectric material surrounding the transmission line conductors is homogeneous, i.e., transverse electromagnetic (TEM) mode, then the apparent or measured relative permittivity (\( \varepsilon_{\text{eff}} \)) is equal to the relative permittivity (\( \varepsilon_r^* \)). However, if the space around the conductors is only partly filled (as in the case of the transmission line test cells considered here), then \( \varepsilon_{\text{eff}} \) is less than \( \varepsilon_r^* \) (quasi-TEM mode). A filling factor \( q \) can be used to relate a measurement of effective dielectric constant to the dielectric constant of the material that partly fills the space (Mongia at al., 1999, pp. 71-73):

\[
q = \frac{\varepsilon_{\text{eff}} - 1}{\varepsilon_r - 1}
\]

where

\( \varepsilon_{\text{eff}} = \) apparent or measured relative permittivity

\( \varepsilon_r = \) relative permittivity of the material in the test cell.

Furthermore, the effects of dielectric materials (such as the gate beneath the grain-filled section) and conductors in proximity to the transmission line were considered as additive factors that were independent of the permittivity of the measured material in the test cell. Therefore, these effects were described with the parameter \( Corr \) as:

\[
\varepsilon_{\text{eff}} = (\varepsilon_r^* - 1) \cdot q + 1 + Corr
\]

where \( Corr \) is the dielectric offset correction.
The permittivity of the material placed in the transmission line test cell affects both \( \gamma \) (eq. 5) and \( Z \) (eq. 6) according to the effective permittivity (eq. 4):

\[
\gamma = \sqrt{\varepsilon_{\text{eff}}} \cdot \frac{2\pi \cdot f}{c} \cdot j + \alpha \quad (5)
\]

\[
Z = \frac{Z_0}{\sqrt{\varepsilon_{\text{eff}}}} \quad (6)
\]

where
- \( \gamma \) = complex propagation constant (1/m)
- \( \varepsilon_{\text{eff}} \) = effective relative permittivity
- \( f \) = frequency (Hz)
- \( \alpha \) = attenuation due to loss mechanisms within the test cell itself (not the sample).

The loss factor of the dielectric material is contained within \( \varepsilon_{\text{eff}} \), but there are other energy losses due to the test cell itself. The effect of (non-zero) \( \alpha \) was not readily observed with a matched load on the test cell, but measurements with open and shorted terminations (where the magnitude of the reflection coefficient should be 1.00 independent of frequency) showed a small frequency-dependent attenuation (increasing loss with frequency, possibly due to radiation) that required a non-zero \( \alpha \) for precise modeling. The form of the correction (eq. 7) was empirically chosen to minimize modeling errors over a moderate frequency range for matched, open, and shorted test cell terminations:

\[
\alpha = \alpha_{149} \cdot \frac{f}{149} \cdot 10^6 \quad (7)
\]

where
- \( \alpha_{149} \) = attenuation constant or real part of empty-cell propagation constant at 149 MHz (see table 3)
- \( f \) = measurement frequency (Hz)

\( M_{T1} \) and \( M_{T2} \) (see fig. 3) represent the transitions where the N-type connectors are joined to the center test cell electrode. Transmission line discontinuities are generally modeled as “T” or “IT” sections in \( ABCD \) matrices, with the elements represented by the impedances or admittances of inductors and capacitors (Krauss, 1960). Because of the structure of the transitions in the test cell, we chose to model them as “L” networks with an inductor in series and a shunt capacitance. The electrical lengths of the N-type connectors are included in table 3, which lists all of the final model parameters.

\( M_{T1} \) was similarly calculated. For cascaded 2-port networks such as series-connected sections of transmission line, the \( ABCD \) matrix of the total network is simply the matrix product of all of the individual matrices (Pozar, 1998, p. 207). The matrices (identified in fig. 3) were multiplied together as:

\[
ABCD = M_{01} \cdot M_{T1} \cdot M_{1} \cdot M_{P} \cdot M_{G} \cdot M_{D} \cdot M_{T2} \cdot M_{02} \quad (9)
\]

From the \( ABCD \) matrix resulting from the matrix multiplication of all of the cascaded networks, the scattering parameters (\( S_{11}, S_{12}, S_{21}, S_{22} \)) can be computed with equation 10 (Frickey, 1994):

\[
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix} = \begin{bmatrix}
\frac{A \cdot Z_2 + B \cdot C - Z_1 \cdot Z_2 - D \cdot Z_1'}{A \cdot Z_2 + B \cdot C + Z_1 \cdot Z_2 + D \cdot Z_1} & 2 \cdot \frac{- \text{Re}(Z_1) \ \text{Re}(Z_2)}{2} \cdot (A - D) \cdot (B - C) \\
\frac{-A \cdot Z_2 + B \cdot C - Z_1 \cdot Z_2 + D \cdot Z_1}{A \cdot Z_2 + B \cdot C + Z_1 \cdot Z_2 + D \cdot Z_1} & \frac{A \cdot Z_2 + B \cdot C + Z_1 \cdot Z_2 + D \cdot Z_1}{A \cdot Z_2 + B \cdot C + Z_1 \cdot Z_2 + D \cdot Z_1}
\end{bmatrix}
\]

where
- \( Z_1, Z_2 \) = characteristic impedances (ratio of complex voltage and current) of the transmission lines connected to the source and load ends of the cascaded network (\( \Omega \)).

\( Z_1', Z_2' \) = complex conjugates of \( Z_1 \) and \( Z_2 \) (\( \Omega \)).

If the cascaded network is terminated in impedance \( Z_T = 50 \Omega \), then the reflection from the load end of the test cell is zero, and the reflection coefficient \( \Gamma \) seen at the source end of the test cell is:

\[
\Gamma = S_{11} \quad (11)
\]

Since there is no explicit solution for the relative permittivity of the measured material (\( \varepsilon_r' \)) as a function of this measured reflection coefficient, an iterative solver was used to find \( \varepsilon_r' \) for each material for each measurement frequency.

Quantifying all the parameters mentioned above is crucial in using the model. Some parameters are definable by geometrical measurements, and some electrical characteristics can be determined by finite element analysis (FEA). But a few parameters had to be quantified by calibration measurements using dielectric materials.

**Geometrical Dimensions**

The design, precision manufacturing, and mechanical rigidity of the New Master test cell made it possible to establish all of the physical dimensions of the test cell very precisely. The electrical lengths of the N-type connectors (which included Teflon bushings) were calculated, with allowances for the reference plane positions established through open/short/load/l ow-loss capacitor calibration of the Hewlett Packard HP-4291A material/impedance analyzer. The dimensions of the sections are included in table 3, which lists all of the final model parameters.
FINITE ELEMENT ANALYSIS

An open-source 2-D FEA package, FEMM (Meeker, 2008), performed very well for modeling the cross-section of the test cell. Simulations with different mesh sizes and different boundary conditions were performed to determine operating parameters that yielded acceptable accuracy in a reasonable length of time (about 10 s per simulation). In addition, the effects of beveled edges on the electrodes and other subtle dimensional effects were tested to be confident that the FEA results were stable.

The FEA method was used to yield values for some of the test cell parameters, including the characteristic impedance of the empty cell ($Z_0$), the filling factor ($q$), and an initial estimate for the empty-cell dielectric offset ($\text{Corr}$), due to the presence of the dielectric gate and conducting base-plate.

If the capacitance per unit length ($C_0$) of a transmission line structure (surrounded by dielectric constant of unity) is known, then the characteristic impedance ($Z_0$) of the structure is (Mongia et al., 1999, p. 71):

$$Z_0 = (c \cdot C_0)^{1/4} \quad (12)$$

where

- $Z_0 =$ empty-cell characteristic impedance (\(\Omega\))
- $C_0 =$ empty-cell capacitance per unit length (F m\(^{-1}\))
- $c =$ velocity of light (299,792,458 m s\(^{-1}\)).

Although we had deliberately adjusted the electrode spacing to get the minimum reflection coefficient from the test cell (with 50 \(\Omega\) termination), FEA indicated that the characteristic impedance (with 27.7 mm spacing) was approximately 52.5 \(\Omega\). If one assumed that the characteristic impedance of the parallel-plate section of the test cell was other than 52.5 \(\Omega\), then it was possible to find (different) values for the equivalent transition inductances and capacitance to create a model that yielded dielectric predictions that were equivalent to using 52.5 \(\Omega\) for the characteristic impedance. Therefore, it was not possible to solve simultaneously for the characteristic impedance of the test cell and the values for the transition inductances and capacitance.

The capacitances (per unit length) of the transmission line structure were calculated by FEA using several values (1, 2, 3, 4, 5, 6, 7, 8) for the relative permittivity ($\varepsilon'_r$) in the grain-filled section of the test cell. The effective relative permittivity ($\varepsilon_{\text{eff}}$) was given by the ratio of the calculated capacitance with the dielectric present to the capacitance with unity relative permittivity present. The optimum values of the $q$ and Corr parameters were found from equation 4 based on the values of $\varepsilon_{\text{eff}}$ computed for multiple values of permittivity. The calculated characteristic impedances of the air-filled and grain-filled sections were different because the effects of the conductive base-plate were included in the capacitance estimate for the air-filled section and in the dielectric offset Corr for the grain-filled section.

Along with the careful measurements of the lengths of the various sections of the test cell identified in figure 3, the FEA results provided almost all the parameters that were needed for the $ABCD$ matrix model, without depending on measurements of alcohols or low-loss dielectric materials.

QUANTIFYING THE PARAMETERS OF TRANSMISSION LINE DISCONTINUITIES

One aspect of the test cell remained that could not be quantified with the available theoretical tools. The transitions (discontinuities) between the N-type connectors and the center electrode of the transmission line (figs. 2 and 3) presented difficulties in establishing a definitive test cell model. The available FEA software package was not able to estimate $L_{T1}$, $L_{T2}$, and $C_T$ in equation 8; these had to be determined by tests of dielectric materials.

ALCOHOL TESTS

The initial approach to calibrating the New Master test cell was to use high molecular weight alcohols, as had been done for the original test cell. Table 1 describes the alcohols that were tested. The values of the dielectric constant ($\varepsilon_r$) and loss factor ($\varepsilon''_r$) at frequency $f$ were computed from the Debye coefficients with equations 13 and 14 (Debye, 1929):

$$\varepsilon'_r = \varepsilon_s + \frac{\varepsilon_e - \varepsilon_s}{1 + \left( \frac{f}{f_r} \right)^2} \quad (13)$$

$$\varepsilon''_r = \frac{\varepsilon_e - \varepsilon_s}{1 + \left( \frac{f}{f_r} \right)^2} \quad (14)$$

where

- $\varepsilon_s =$ static dielectric constant (low frequency)
- $\varepsilon_e =$ high frequency (far above relaxation) value of dielectric constant
- $f_r =$ characteristic frequency of the relaxation (frequency of peak value of loss factor).

Figure 4 shows the theoretical dielectric constant and loss factor values for air and the three alcohols. For comparison, the dielectric constants of grain at 149 MHz are generally between 2 and 5, with extremely moist samples reaching only about 7. Therefore, the alcohol values (with the exception of decanol) are far outside the region of interest for grain moisture measurements.

<table>
<thead>
<tr>
<th>Alcohol</th>
<th>Source</th>
<th>Purity (%)</th>
<th>Debye $f_r$ (Hz)</th>
<th>Debye $\varepsilon_s$</th>
<th>Debye $\varepsilon''_s$</th>
<th>Reference Temp. (°C)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Pentanol</td>
<td>Sigma-Aldrich</td>
<td>≥99</td>
<td>1.90 × 10^6</td>
<td>15.66</td>
<td>3.33</td>
<td>20</td>
<td>Clarke, 1994</td>
</tr>
<tr>
<td>1-Hexanol</td>
<td>Sigma-Aldrich</td>
<td>≥99</td>
<td>1.52 × 10^6</td>
<td>12.9</td>
<td>3.3</td>
<td>20</td>
<td>Buckley and Maryot, 1958</td>
</tr>
<tr>
<td>1-Decanol</td>
<td>Sigma-Aldrich</td>
<td>99</td>
<td>8.00 × 10^7</td>
<td>7.98</td>
<td>3.3</td>
<td>21.5</td>
<td>Baker-Jarvis, 1996</td>
</tr>
</tbody>
</table>
The bottom gate of one of the New Master test cells (unit 2) was sealed with silicone cement. The test cell was cleaned and dried carefully before and after each test. The cell was filled by adding a calculated mass of the alcohol to the test cell (based on the density of the alcohol and the dimensions of the test cell). The density of each alcohol was tested with a precision volumetric flask to verify that the density was very close to the supplier’s stated value. Each side of the test cell was filled based on its volume as determined from careful dimensional measurements. The mass of the alcohol remaining in the flask after filling the test cell was determined, and that mass of alcohol was added separately to minimize errors. The complex reflection coefficient of the terminated test cell was measured with the HP-4291A material/impedance analyzer (single-port instrument) with one point every 2 MHz from 1 MHz to 501 MHz. The temperature of the room was adjusted to get the apparatus and the alcohol as close as possible to the reference temperatures at which the Debye factors from the literature (table 1) were determined.

An iterative solver (written in MathCad, ver. 11, Needham, Mass.: Parametric Technology Corp.) was used to automatically find the optimum values of the two transition inductances \( L_{T1} \) and \( L_{T2} \) (allowed to be different), the transition capacitance \( C_T \) (assumed equal for both transitions), and the dielectric offset value \( C_{orr} \). The \( C_{orr} \) value was necessary in the calibration parameters to correct the offset in the result. The criteria for the optimum values were to minimize simultaneously (for all three alcohols and air) the sum of squared differences between the predicted and theoretical values of complex permittivity. Thirteen frequency points (of the 100 measured in that range) were selected for the optimization to reduce processing time.

**Granular Reference Materials**

The more nearly a model describes reality, the wider the range of conditions over which it should be valid. Similarly, one might assume that the wider the range of conditions (say frequency and dielectric constant) over which a model shows agreement between theory and measurement, the more likely it is to be correct, or at least good enough for measurements at one frequency as needed for the UGMA. Therefore, we attempted to match measurements to theory over a wide range of conditions.

But what is known about a granular dielectric material that could be useful for testing measurements against theory? Very low loss dielectrics should show nearly frequency-independent dielectric constant values and very nearly zero loss factors over a wide frequency range. The frequency independence of the dielectric constant of very low loss dielectrics is well known from the literature (Vanzura et al., 1994) and expected as a consequence of the Kramer-Kronig relations that couple the dielectric constant and loss factor (Fröhlich, 1958, pp. 8, 162).

We selected four inexpensive, readily available low-loss materials (fig. 5) with different dielectric constants: (air, soda-lime glass beads (Quackenbush, 1.2 mm dia.), soft air ammo (Crossman, 0.2 g, 6 mm dia.), and polyethylene pellets as used for injection molding.

**Granular Reference Material Tests**

Each material was loaded three times in each of the three New Master test cells for complex reflection coefficient measurements with the HP-4291A material/impedance analyzer. The sample volume was fixed by striking off the filled test cell (in the lengthwise direction) with a straight edge (Official Strike-Off Stick No. 65, Seedburo Equipment Co., Des Plaines, Ill.) using a zigzag motion. Sample mass was recorded for each test. Tests were performed with the test cells terminated with matched (50 \( \Omega \)), open, and shorted loads (once each before reloading). The results (reflection coefficients and sample mass) for the three repetitions for each condition were averaged.

Another iterative solver (written in MathCad, ver. 11, Needham, Mass.: Parametric Technology Corp.) was used to find the optimum values for the transition inductances \( L_{T1} \) and \( L_{T2} \), transition capacitance \( C_T \), and dielectric offset \( C_{orr} \). Thirteen frequency points (of the 100 measured in...
that range) were selected for the optimization to reduce processing time. The criteria for the optimum values were:

1. To simultaneously minimize the linear coefficient of slope for predicted $\varepsilon_r'$ versus frequency for each of the materials (frequency-independent dielectric constant).
2. To cause the predicted complex relative permittivity of air to be $1.000 - 0.1j$.
3. To cause the loss factor for all of the low-loss materials to be as close as possible to zero across the 50 to 250 MHz frequency range (except for glass beads, where a loss factor of 0.01 was the target).

**Repeatability Tests**

One concern regarding the use of these dielectric materials was the achievable repeatability of the dielectric results. The density of the material and the dielectric constant and loss factor were expected to vary depending on the loading method, and even from test to test when attempting to duplicate a single loading method. In the development of the UGMA, we had observed that the Landau-Lifshitz, Looyenga equation, as restated by Nelson (1992) (eq. 15), was effective in normalizing dielectric constant measurements to a common density. Furthermore, the residual errors due to loading method were smaller for nearly spherical granules than for highly elongated granules. Therefore, we tested the capability of equation 15 to reject the effects of loading differences and yield equivalent values of density-corrected dielectric constant for these dielectric reference materials for extreme cases of loading methods:

$$
\varepsilon_r = \left[ \frac{1}{\varepsilon_r^m - 1} \right] \frac{\rho_t}{\rho_m} + 1
$$

where 
- $\varepsilon_r$ = calculated dielectric constant at the target density
- $\varepsilon_r^m$ = measured dielectric constant at the measured density
- $\rho_m$ = measured density (g mL$^{-1}$)
- $\rho_t$ = target density (g mL$^{-1}$).

The three methods tested were: funnel loading, slow pour loading, and fast drop loading (fig. 6). Triplicate tests were performed for each of the three granular dielectric reference materials for each of the three loading methods. The target densities used for the reference materials were the mean densities of the repeated measurements for each material.

**Temperature Stability Tests**

For a dielectric material to be suitable for routine use for calibrating test cells for grain moisture meters, it should be sufficiently stable with temperature that no great care is required to maintain the ambient temperature. We tested each of the three granular dielectric reference materials over the temperature range of 4°C to 20°C. Each material was chilled to approximately 1°C and then loaded into a special insulated transmission line test cell (similar in dimensions to the New Master test cell, but fabricated with plates made of 3.2 mm double-sided copper-clad epoxy-glass circuit board). A fiber-optic thermometer probe (Lxtron One, LunaSense Technologies, Santa Clara, Cal.) was inserted in the sample, and complex reflection coefficient values (1 to 501 MHz) were automatically collected at 0.2°C increments (with software provided by Dickey-john Corp., Auburn, Ill.) as the dielectric material warmed slowly to room temperature. The measured values of complex reflection coefficient were converted to dielectric constant using an appropriate $ABCD$ matrix model that was calibrated using low-loss dielectric materials as described in this article.

**ConfirmaTory Tests**

We assessed the reasonableness of the bulk dielectric constants for the granular reference materials via equation 15 by calculating the material dielectric constant ($\varepsilon_r$) from the measured bulk dielectric constant ($\varepsilon_r^m$), density ($\rho_m$), and the literature values for the material density ($\rho_t$). The literature values suggested considerable variations in density and dielectric constant depending on the composition of the materials, and values were not available for the specific materials tested.

We further evaluated the accuracy of the New Master test cell model by using an Agilent 4294A precision impedance analyzer with an Agilent 42941A coaxial probe (Agilent Technologies, Santa Clara, Cal.). The coaxial probe was connected to the New Master test cell via a SMA-male to N-type-male adapter. The system was calibrated with open, shorted, and 50 $\Omega$ N-type loads. The load end of the New Master test cell was open for these tests. Capacitance values (over the 44 Hz to 100 MHz frequency range) and sample mass were recorded for the three dielectric reference materials and empty cell. Each sample was loaded with the slow pour method and struck off as described previously (triplicate determinations).

The dimensions of the grain-filled section of the test cell were used to calculate its empty capacitance ($C_{cc} = 4.941$ pF) with the parallel-plate approximation (eq. 16):

$$
C = \frac{\varepsilon_0 \varepsilon_r' A}{d}
$$

Figure 6. Loading methods (left to right): funnel, slow pour, and fast drop.
where

\[ A = \text{area of facing parallel plates within the grain-filled section (0.01548 m}^2) \]

\[ d = \text{separation between plates (0.02774 m)}. \]

The stray capacitance \( (C_s) \) at each frequency was calculated by subtracting \( C_{ec} \) from the measured capacitance for the empty cell. The bulk dielectric constants for the dielectric materials at each frequency were calculated from the measured capacitance \( (C_m) \) with equation 17:

\[ \epsilon' = \frac{C_m - C_s}{C_{ec}} \]  

**RESULTS AND DISCUSSION**

**INITIAL ALCOHOL RESULTS**

Solving for the best agreement between predicted and theoretical permittivity values for alcohols yielded unique frequency-independent solutions for \( LT_1, LT_2, CT, \) and \( Corr. \) The results (predicted minus theoretical permittivity values) are shown in figure 7 (see fig. 4 for the theoretical values). Despite the precautions exercised in the alcohol measurements, the results were not very encouraging. The inconsistencies among alcohols (several percent, relative) were similar to those observed from Lawrence’s tests on the original test cell (Funk, 2001). Solving for the best overall agreement between theoretical and predicted values for alcohols resulted in non-physical results (dielectric constant less than one and negative loss factor) for air. Besides that, the difficulties encountered in performing the tests re-emphasized the impracticality of using liquids for routine calibration of test cells that were designed to test granular materials. Clearly, another strategy was needed.

**GRANULAR REFERENCE MATERIAL RESULTS**

The iterative solver process with granular dielectrics also yielded unique frequency-independent solutions for \( LT_1, LT_2, CT, \) and \( Corr. \) Figure 8 shows how precisely the optimization criteria were met. For clarity, the dielectric constant plot shows differences from the mean for each material rather than the actual values. Note that the axis scales for dielectric constant and loss factor differences are 100 times and 50 times smaller, respectively, than those for the alcohol results shown in figure 7.

Table 2 gives the mean values of the predicted bulk dielectric constants (column 2) for each of the low-loss materials along with the associated densities (column 3). These mean dielectric values and mean densities are important as target values for future standardization of other test cells.

Table 3 lists the final values for the model parameters for the New Master test cell. The values shown are for unit 1. The parameters (not shown) for units 2 and 3 were slightly different because of very small differences in plate spacing. Separate FEA tests were performed for each test cell based on precise mechanical measurements.

**FINAL ALCOHOL RESULTS**

The test cell parameters (table 3) were applied to the reflection coefficient data for the alcohols that were previously measured in the New Master test cell. Figure 9 demonstrates the agreement between predicted and
Table 2. Mean results for low-loss dielectric tests and comparison of calculated material dielectric constants to published values. Published values were not available for the specific materials tested.

<table>
<thead>
<tr>
<th>Dielectric</th>
<th>Mean Measured Dielectric Constant, $\varepsilon_m$</th>
<th>Mean Measured Bulk Density $\rho_m$ (g mL$^{-1}$)</th>
<th>Calculated Material Dielectric Constant, $\varepsilon_t$</th>
<th>Published Material Dielectric Constant</th>
<th>Published Material Density $\rho_t$ (g mL$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soda-lime glass beads</td>
<td>3.425</td>
<td>1.5471</td>
<td>6.15</td>
<td>6.82$^{[a]}$</td>
<td>2.537$^{[b]}$</td>
</tr>
<tr>
<td>Soft air ammo</td>
<td>2.221</td>
<td>1.0675</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>1.700</td>
<td>0.5682</td>
<td>2.27</td>
<td>2.2 to 2.35$^{[c]}$</td>
<td>0.92$^{[c]}$</td>
</tr>
<tr>
<td>Air</td>
<td>1.001</td>
<td>0</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

$^{[a]}$ Source: Vanzura et al., 1994.
$^{[b]}$ Source: Schumacher et al., 2001.
$^{[c]}$ Source: Goodfellow, 2010.

Table 3. Test cell parameters for New Master test cell (unit 1).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedances</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Source impedance</td>
<td>50 $\Omega$</td>
<td>Assumed</td>
</tr>
<tr>
<td>First connector impedance</td>
<td>50 $\Omega$</td>
<td>Assumed</td>
</tr>
<tr>
<td>First air-filled section impedance</td>
<td>52.1245 $\Omega$</td>
<td>FEA</td>
</tr>
<tr>
<td>Grain-filled section impedance</td>
<td>52.56 $\Omega$</td>
<td>FEA</td>
</tr>
<tr>
<td>Second air-filled section impedance</td>
<td>52.1245 $\Omega$</td>
<td>FEA</td>
</tr>
<tr>
<td>Second connector impedance</td>
<td>50 $\Omega$</td>
<td>FEA</td>
</tr>
<tr>
<td>Termination impedance</td>
<td>50 $\Omega$</td>
<td>Standard</td>
</tr>
<tr>
<td>Dimensions</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First connector length</td>
<td>0.0155 m</td>
<td>Measured and corrected</td>
</tr>
<tr>
<td>First air-filled section length</td>
<td>0.0635 m</td>
<td>Measured</td>
</tr>
<tr>
<td>Divider thickness</td>
<td>0.001575 m</td>
<td>Measured</td>
</tr>
<tr>
<td>Grain-filled section length</td>
<td>0.1017 m</td>
<td>Measured</td>
</tr>
<tr>
<td>Second air-filled section length</td>
<td>0.0635 m</td>
<td>Measured</td>
</tr>
<tr>
<td>Second connector length</td>
<td>0.0204 m</td>
<td>Measured and corrected</td>
</tr>
<tr>
<td>Attenuation coefficient (radiation), $\alpha_{149}$</td>
<td>0.014 Np m$^{-1}$</td>
<td>From “open” tests</td>
</tr>
<tr>
<td>Divider dielectric constant, $\varepsilon_r'$ (glass-filled PPS)</td>
<td>3.8</td>
<td>Literature (Mark, 1999)</td>
</tr>
<tr>
<td>First transition inductance, $L_{T1}$</td>
<td>4.4781 nH</td>
<td>Low-loss dielectric tests</td>
</tr>
<tr>
<td>Second transition inductance, $L_{T2}$</td>
<td>4.1382 nH</td>
<td>Low-loss dielectric tests</td>
</tr>
<tr>
<td>Transition capacitance, $C_T$</td>
<td>2.407 pF</td>
<td>Low-loss dielectric tests</td>
</tr>
<tr>
<td>Air-filled section dielectric constant, $\varepsilon'_r$</td>
<td>1</td>
<td>Assumed</td>
</tr>
<tr>
<td>Filling factor, $q$</td>
<td>0.7694</td>
<td>FEA</td>
</tr>
<tr>
<td>Dielectric offset factor, $Corr$</td>
<td>0.062</td>
<td>Low-loss dielectric tests</td>
</tr>
<tr>
<td>Test cell volume</td>
<td>429.58 mL</td>
<td>Calculated from dimensions</td>
</tr>
</tbody>
</table>

Figure 9. Differences between measured and theoretical dielectric constants and loss factors versus frequency for alcohols using test cell model parameters based on finite element analysis and tests of low-loss dielectric materials.

Figure 10. Effectiveness of Landau-Lifshitz, Looyenga density correction for three loading methods (fast drop, funnel, and slow pour) without and with density correction.
Table 4. Standard deviations of predicted dielectric constants for three materials pooled across three loading methods (fast drop, funnel loading, slow pour) without and with density correction.

<table>
<thead>
<tr>
<th>Dielectric Material</th>
<th>Without Density Correction</th>
<th>With Density Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass beads, 1.2 mm dia.</td>
<td>0.074</td>
<td>0.0190</td>
</tr>
<tr>
<td>Soft air ammo, 0.2 g, 6 mm dia.</td>
<td>0.031</td>
<td>0.0027</td>
</tr>
<tr>
<td>Polyethylene molding pellets</td>
<td>0.025</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

Table 5. Measured temperature coefficients for dielectric constants of granular dielectric reference materials.

<table>
<thead>
<tr>
<th>Dielectric Material</th>
<th>Temperature Coefficient (°C⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass beads, 1.2 mm dia.</td>
<td>0.00047</td>
</tr>
<tr>
<td>Soft air ammo, 0.2 g, 6 mm dia.</td>
<td>0.00019</td>
</tr>
<tr>
<td>Polyethylene pellets</td>
<td>-0.00001</td>
</tr>
</tbody>
</table>

Theoretical values based on those parameters. The agreement between predicted and theoretical values was essentially equal to that achieved by calibrating to the alcohol tests directly (fig. 7). Additionally, the non-physical results for air (empty cell) were avoided.

**REPEATABILITY RESULTS**

The repeatability tests verified the effectiveness of the Landau-Lifshitz, Looyenga equation (eq. 15) for density-correcting the measured dielectric constants for these granular dielectric reference materials to the mean density for each material. Figure 10 shows calculated dielectric constant values without and with density correction for each of the three materials with the three extremely different loading methods. Without density correction, the measured dielectric constant obviously varied with density; density correction virtually eliminated that dependency. Table 4 shows the standard deviations of dielectric constant results for each material (pooled across repeated tests for all three loading methods) without and with density correction. Density correction reduced the variability by almost a factor of 10. This suggests that, with reasonable control of loading methods, repeatability for multiple tests should be very good.

**TEMPERATURE STABILITY RESULTS**

The temperature stability test data were analyzed by calculating the slope of linear regression between the predicted dielectric constant and temperature for each dielectric material. Table 5 shows the calculated slopes (temperature coefficients). The temperature coefficients were all less than 0.0005 °C⁻¹, thus allowing for reasonable variations in ambient temperature during calibration testing with such materials.

**CONFIRMATORY TEST RESULTS**

Table 2 (columns 4 and 5) compares the calculated material dielectric constants and published values for soda lime glass and low-density polyethylene. The calculated material dielectric constant for the soda-lime glass beads was within 10% of the published value for a soda-lime glass (NIST SRM 710a), but no published value was available for the beads to permit direct comparison. The calculated value for polyethylene was within the wide published range, but again no published values were available for the specific polyethylene pellets that were tested. Therefore, these comparisons were inconclusive.

The second set of confirmatory tests was much more useful. The results from the Agilent 4294A tests were unreliable below 10 kHz (because of increased noise) and above 10 MHz (because of the inadequacy of the simple lumped-parameter model to account for the electrical length of the test cell), so only the 10 kHz to 10 MHz data were used to estimate the bulk dielectric constants for the reference materials. Density correction (eq. 15) was applied to the calculated dielectric constants (and measured sample masses) to normalize them to the respective target densities shown in column 3 of table 2.

Figure 11 shows the measured dielectric constant values (points) and their logarithmic extrapolations to 149 MHz. The slopes were -0.03, -0.01, and +0.00007 units per decade of frequency for glass beads, soft air ammo, and polyethylene pellets, respectively. These results supported the original assumption that the dielectric constant should be nearly constant with frequency. The relative errors (extrapolated values at 149 MHz minus values measured with the high-frequency model (column 3 of table 2)) were +0.253%, -0.215%, and +0.059% for glass beads, soft air ammo, and polyethylene pellets, respectively. Note that these errors were better than required to meet the USDA-GIPSA moisture meter specification (approx. 0.4% relative) cited above.

**APPLYING THE TEST CELL MODEL**

This detailed ABCD matrix model offered some important benefits besides providing a precise set of equations for converting measured complex reflection coefficient to dielectric constant for purposes of grain moisture prediction. The model permitted estimation of errors caused by mechanical and electrical tolerances in the test cell, thus facilitating the design of commercial UGMA-compatible moisture meters.

**CONCLUSION**

The method described here, which combined the ABCD matrix method, precise mechanical measurements, careful finite element analysis of the test cell cross-section, and tests of low-loss dielectrics, was able to determine parameters for a dielectric test cell model that gave good agreement with theoretical dielectric characteristics for three high molecular...
weight alcohols. The model parameters were further confirmed by comparing high-frequency measurements of low-loss dielectrics to extrapolated values from independent low-frequency measurements of the same materials. The ABCD matrix method made feasible the incorporation of many subtle elements of the test cell to achieve very good performance over a wide frequency range. Such a robust model lends itself to system error analysis to help with the design of practical commercial instruments. The repeatability of density-corrected dielectric constant measurements for the reference materials (with different loading methods) was between 0.0027 and 0.019 (one standard deviation). Temperature tests confirmed the expected high stability over a modest temperature range. These low-loss dielectrics appear to be suitable for routine calibration and standardization of test cells and systems that are designed for compatibility with the Unified Grain Moisture Algorithm.

ACKNOWLEDGEMENTS

We wish to thank Mr. Steven Burton, USDA-GIPSA, for his major contribution to this project through his conscientious work in collecting much of the data.

REFERENCES

Clarke, R. N. 1994. Personal communication.
