Economic Analysis of Spatial-Temporal Patterns in Corn and Soybean Response to Nitrogen and Phosphorus

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ABSTRACT

Interactions among environmental factors, management decisions, and field characteristics cause temporal and spatial variability in corn (Zea mays L.) and soybean [Glycine max (L.) Merr.] yields. The objectives of this paper are (i) to test whether yield response of corn to N and P and of soybean to P are spatially and temporally stable, and (ii) to evaluate the profitability of a variable rate (VR) N and P fertility management strategy over a 5-yr, corn–soybean rotation using this response information. A field near Windom, MN, USA, was cropped with corn (1997, 1999, and 2001) and soybean (1998, 2000). Three replications of 13 N and P treatments were established in a split-plot arrangement of a randomized complete block design. Treatments were applied at constant rates in strips across the entire field. Fertilizer N treatments were 0, 67, 112, 157, and 202 kg ha⁻¹ applied at constant rates in strips across the entire field. Fertilizer N and P and of soybean to P are spatially and temporally stable. Results of an ex post profitability analysis found that average returns over the 5-yr period from the VR N and P management strategy were $28 ha⁻¹ higher than returns from a uniform application strategy.

It is well known that N and P needs of crops can differ within fields (Wibawa et al., 1993) and between fields (Carr et al., 1991; Wittry and Mallarino, 2004). If crop inputs are applied uniformly across a field where plant, soil, and input interactions are variable, the field will be underfertilized in some areas and overfertilized in others (Mamo et al., 2003). Overapplication of P may lead to excess runoff and eutrophication of waterways while P underfertilization can inhibit plant growth and translocation of carbohydrates, causing “purpling” in corn (Rehm and Schmitt, 2002). Overapplication of N causes nitrate leaching from the root zone while N underfertilization limits yields (Randall and Schmitt, 1993). In both cases, economic returns might be compromised if spatial aspects of soil–plant relations are not understood and inputs managed accordingly.

Until recently, multi-year on-farm production data from controlled variable rate technology (VRT) experiments have been limited. Studies that have looked at site-specific yield response to specific inputs over two or more growing seasons are more frequent because georeferenced panel production data are increasingly available. Finck’s (1998) article reported results from an on-farm VRT-NP experiment with corn and soybean over three seasons in Illinois, USA. Bongiovanni and Lowenberg-DeBoer (2002) analyzed variable rate nitrogen (VRN) profitability and site-specific crop response stability for corn over two growing seasons in Argentina. Kasper et al. (2003) used 6 yr of corn data to estimate corn response with respect to topography and elevation. Mamo et al. (2003) evaluated a VRN trial for corn and an N loss inhibitor over three corn production cycles in Minnesota, USA. Like Kasper et al., they found that corn yield response to N varied spatially and was temporally unstable. Swinton et al. (2002) analyzed a VRN trial on 14 farms in Michigan, USA, between 1999 and 2001 (in all, a total of 24 site years). They also found that corn yield response to N was not stable over time.

The objectives of this paper are twofold: (i) to test whether yield response of corn to N and P and soybean response to P exhibit stable temporal and spatial patterns, and (ii) to evaluate the profitability of a variable rate nitrogen and phosphorus fertility management strategy over a 5-yr corn–soybean rotation using this response information. The VRT study spans 5 yr of production data from a corn–soybean rotation in Minnesota, USA. The analysis focuses on the temporal and spatial variability of crop response rather than the temporal and spatial stability of yield itself. The spatial and temporal analysis of crop response to variable rate N and P identifies landscape positions where corn and soybean response to inputs is similar between production seasons. Variable rate application profitability is compared with a uniform (UNI) N and P management strategy with an ex post, multi-season net present value (NPV) comparison. Spatial correlation between yield observations is modeled using a geostatistically weighted regression (Cressie, 1993). The site-specific management (SSM) hypothesis (Whelan and McBratney, 2000) is tested over the production surface and between growing seasons. The null hypothesis states that the optimal risk management strategy for producers is a uniform input management strategy, given temporal variations in weather patterns. The findings have implications with respect to N and P fertilizer management and identification of areas where yield response is temporally stable and spatially homogenous. The results may interest corn and soybean...
producers, researchers, crop consultants, and the persons who advise them.

MATERIALS AND METHODS

Corn and Soybean Crop Response Data

The research site (Windom, MN, 43°90′ N lat; 85°05′ W long) was established in the fall of 1996. The 12.2-ha field had been in a corn–soybean rotation for the last 20 yr with no manure applied during that time frame. The site was extensively grid sampled in the fall of 1996. Soil organic matter content ranged from less than 2 to 10%. Phosphorus soil tests ranged from very low (<5 mg kg⁻¹ [ppm]) to very high (>15 mg kg⁻¹ [ppm]), and the soil pH ranged from 6 to 8. The site was dominated by Jeffers clay loam series (2–4% slopes, fine-loamy mixed [calcareous], mesic Typic Hapludolls), Clarion-Swanlake clay loams (6–12% slopes, fine-loamy mesic Typic and Entic Hapludolls), and Webster-Delft clay loam series (fine-loamy mesic Typic and Cumulic Hapludolls) soils.

The experimental design was 3 replications of 13 treatments in a split plot arrangement of a randomized complete block design where P was the main plot. Nitrogen rates were randomized within the P treatments (Fig. 1). The P treatments were randomized within each replication. Treatments were applied at constant rates in strips across the entire field. All treatments were repeated three times (a total of 39 strips). Fertilizer N treatments were 0, 67, 112, 157, and 202 kg ha⁻¹ of N as either anhydrous ammonia (fall 1996) or urea (fall 1998 and 2000), and P treatments were 0, 56, and 112 kg ha⁻¹ of P as triple super phosphate. In the fall of 1998 and 2000, P fertilizer was applied to the same areas of the field that had received the treatments in 1996. The experiment was designed to accommodate extra N treatment strips that would ensure zero N rate treatments were not placed in the same location as previous zero N rate treatments. Other N treatments were the same each corn year.

The 2001 growing season provided the fifth year of research information from this site, and the third year for corn production. Grain yield determination was made every 15 m through each treatment strip using a Model 8 Massey Ferguson plot combine (AGCO Corp., Duluth, GA, USA) equipped with a ground distance monitor and a computerized HarvestMaster weigh cell (HarvestMaster, Logan, UT, USA). Every 15 m, the combine was stopped, the position was georeferenced, and the harvest grain was weighed. The grain was cleared from the weigh cell after it was weighed at every location. Yield data locations are represented by the single squares in Fig. 1.

The experimental site was partitioned into 69 sub-blocks for purposes of spatial yield response analysis (Fig. 1). Previous applications of this methodology are found in Mamo et al. (2003), Hurley et al. (2003), and Malzer et al. (1996). For each corn year, 897 observations were available. Therefore, there were 13 observations available for estimating corn response to N and P treatments in each sub-block. Only 621 observations were collected with the research combine during the soybean years. Nine observations were available to estimate soybean response to P in each sub-block.

Because a yield response is estimated for each sub-block, economically optimal rates for N (EONR) and economically optimal rates for P (EORP) can be estimated for each sub-block, along with corresponding profit maximizing yields and profits. This facilitates ex post comparisons of variable rate fertilizer strategies with uniform fertilizer management strategies.

Corn and Soybean Yield Response Model

Because inputs are economically quantifiable, response functions facilitate comparison between input changes and the cost of making those changes. A quadratic response function is used to estimate corn response to N and P and soybean response to P. Quadratic functions have theoretical appeal because they are interpretable as second-order approximations of any function in general, and crop response functions to inputs in particular (Hurley et al., 2003). Alternatively, a quadratic response and plateau model could be considered. However, this is computationally onerous given the multiple response zones and semivariogram estimation. Most of the recent work estimating site-specific crop response functions used single, quadratic equations to estimate plant response to inputs over space (e.g., Bruulsema et al., 1996; Malzer et al., 1996; Bongiovanni, 2002; Swinton et al., 2002; Hurley et al., 2003; Mamo et al., 2003; Anselin et al., 2004; Lambert et al., 2004). For crop response economic analyses, quadratic
functions are suitable because they are concave functions with \( f' > 0 \) and \( f'' < 0 \), which permits diminishing marginal returns to inputs. The estimation of economically optimal input rates is also tractable because a closed-form solution to \( f' \) exists.

For each crop year, the response equations are:

\[
y(s) = \beta_{0,s} + \beta_{1,s}N_{s,t} + \beta_{2,s}P_{s,t} + \sum_{m=2}^{69} \left( \delta_{0,m}d_{s,m} + \delta_{1,m}d_{s,m}N_{s,t} + \delta_{2,m}d_{s,m}P_{s,t} \right) + \psi_j(s)
\]

where \( s \) is the Cartesian location of yield, \( y(s); \psi_j(s) \sim (0, \Sigma_j) \) is a disturbance term that may be spatially correlated; \( N \) and \( P \) are applied inputs at position \( s \); and \( d_{s,m} \) is a dummy variable identifying sub-blocks \( m = 2 \ldots 69 \), with the first block the reference group. Parameters of the sub-block dummy variables are constrained as \( \sum_{m} \delta_{0,m} = 0 \) to capture differences between site-specific crop response functions and the whole field (WF), average crop response. This constraint tests Whelan and McBratney’s (2000) SSM hypothesis because significant response differences are a precondition for VRT profitability. Significant \( t \) tests for these coefficients indicate whether the response in a sub-block is different from the average WF response in a given year. Rejection of the null hypothesis that a sub-block is not different from the WF response to \( N \) or \( P \) is evidence in favor of VRA management. The subscript \( t \) indicates which crop is grown in year \( t \), \( t = 1997 \ldots 2001 \), with corn starting in 1997 and soybean rotating in the even years thereafter. Because \( N \) was not the focus of the soybean study, \( P \) was the only fertilizer variable included in the soybean regression equation. Equation [1] was estimated separately for each year because the number of corn and soybean observations was not the same for each year.

### Modeling Spatial Covariance

A geostatistic regression model (Cressie, 1993) was used to estimate yield response when spatial correlation between observations was significant. Because yield observations were not equally dispersed, the assumption that equal weights could be placed on neighboring yield points was untenable. Therefore, a spatial econometric approach using a spatial weighting design based on a lattice contiguity matrix was not appropriate (for example, Anselin et al., 2004). Hurley et al. (2003) and Lambert et al. (2004) have used the GEO method to estimate crop response to variable rate inputs. Lark and Wheeler (2003) used a variant of this model to estimate yield response using yield monitor data. Robust empirical semivariograms (Cressie, 1993) were estimated using the corn or soybean residuals from Eq. [1]. The functional form used to model covariance between observations is \( \Psi(s) = c_0 + \xi s \) \( - \exp ( - ||s||^2 / a) \); where \( ||s|| \) is the Euclidean distance between observations; and \( c_0, \xi, \) and \( a \) are the nugget, sill, and range parameters, respectively. The innovation of this model is the shape parameter, \( \theta \). To ensure admissibility of this function, \( \theta \) is restricted to be positive. Experience with the Windom data has shown that empirical semivariograms are sometimes intermediate between Gaussian and exponential functional forms.

### Regression Estimation and Diagnostics

A Breusch-Pagan (Breusch and Pagan, 1980) Lagrange Multiplier (LM) test for heteroskedasticity was conducted for each crop response equation, each season. When the null hypothesis of homogenous variance between observations was rejected, Eq. [1] was re-estimated using General Method of Moments (GMM) with White’s robust standard errors. The GMM procedure is useful when the exact form heteroskedasticity is unknown (SAS Institute, 1999). Furthermore, no assumptions about the error distribution are required except that errors are independent and identically distributed (Mittelhammer et al., 2000).

Cressie’s (1993) weighted nonlinear least squares procedure was used to fit \( \gamma(s) \) to the regression residual semivariograms. The \( F \) test of each semivariogram regression indicates if there is significant spatial structure in the residuals of each response equation. When spatial structure was evident, Eq. [1] was estimated using geostatistically weighted GMM.

### Ex Post Marginal Analysis of Management Strategies

Uniform and VRT profitability is compared using a partial budget (Boelhje and Eidman, 1984), and marginal analysis is used to estimate net returns from applied \( N \) and \( P \) (Beattie and Taylor, 1985). Marginal analysis shows that profit is maximized when the value of the increased yield from added \( N \) or \( P \) equals the cost of applying an additional unit of fertilizer, or when the marginal value product equals the marginal factor cost.

The profitability analysis is ex post because it assumes that the producer knows, in hindsight, yield response and optimal \( N \) and \( P \) applications (Bullock and Bullock, 2000). Ex post analyses are useful as starting points in economic assessments of solutions to management problems (Anselin et al., 2004). While ideally such evaluations should be based on ex ante decision-making, ex post estimates provide insight into possible outcomes of VRT-UNI comparisons. If ex post results are not profitable, then ex ante results are unlikely to be profitable. Alternatively, if the ex post approach does show profitable results there still remains the question of how well it would do in an ex ante decision making context.

A spatial optimization framework proposed by Bullock et al. (2002) is modified to accommodate the multi-period corn-soybean rotation. Over the course of the experiment, \( N \) and \( P \) were only applied in the fall before corn was planted (except for the 2001 year when bad weather conditions postponed fertilizer application). Therefore, the P-for-soybean decision is made when fertilizer amounts are determined for corn.

The producer’s maximization problem in the mth corn-soybean rotation and zth location is

\[
E[\text{NPV}_{z,m}] = \max \left[ B^m_{z,m} \text{ynthesis} + B^{m+1}_{z,m} \text{ynthesis} \right]
\]

where \( B \) is expected net present value gained in position \( z \), rotation \( m, m = 1,3,5; \ldots B^{m+1} \) is a discounting factor with an opportunity cost of capital \( p \) of 7.5% (Chiarella et al., 2002); \( \gamma(z) \) is the estimated corn(soybean) yield from Eq. [1] in sub-block z, rotation m; \( \text{ynthesis} = \left[ P_{z,m}, N_{z,m} \right] \) is a \( k \times 1 \) input vector applied at the beginning of rotation \( m \) on sub-block \( z \); \( F \) are fixed costs; \( g \) and \( v \) are quasi-fixed, site-specific information and variable rate application costs, respectively; \( r \) is a \( k \times 1 \) vector of variable costs for the \( k \) inputs applied at the beginning of the corn year in rotation \( m \); and \( \text{ SYN } \) is an indicator variable equal to one when \( g \) and \( v \) costs are incurred by the producer, zero otherwise. For example, soil tests or maps \( g \) may only be needed every 4 yr or more, or \( v \) may be incurred only during corn years.

The sum of NPVs over the m corn-soybean rotations and z locations is the cumulative expected NPV over the course of the experiment. Here, total expected NPV is \( E[\text{NPV}_{\text{TOTAL}}] = \sum_{z} \omega_{z} E(\text{NPV}_{z,1997–1998}) + E(\text{NPV}_{z,1999–2000})+ E(\text{NPV}_{z,2001}) \), with \( \omega_{z} \) the portions of the field covered by sub-block
Price Data Used in the Ex Post Profitability Analysis

The average market prices (1997–2001; NASS, 2002) for corn and soybean observed in Cottonwood County, Minnesota, were used as output prices ($0.0787 and $0.175 kg⁻¹, corn and soybean, respectively). Input prices were $0.077 and $0.118 kg⁻¹ for N and P, respectively. Information costs for SSM included costs for map making and management zone development ($2.96 per map), and soil sampling, including lab and collection fees ($13.59 ha⁻¹). These are the costs reported in the 1997 Akridge and Whipker dealership survey. Variable rate application costs ($13.21 ha⁻¹) were the average of the VRT costs reported in Akridge and Whipker (1997, 1999) and Whipker and Akridge (2001). A uniform application cost of $9.88 ha⁻¹ is assumed for the UNI budget analysis (Aghib and Lowenberg-DeBoer, 1999). The input application costs for VRT and UNI are doubled because P and N are usually applied at different times. Because the UNI strategy uses soil test information to follow extension recommendations for P, a whole field soil test fee of $0.82 ha⁻¹ is charged under the uniform strategy. Mapping and soil test fees are charged once, in the 1997 year. These products are assumed to have a useful life of approximately 4 yr (Swinton and Lowenberg-DeBoer, 1998). Uniform and VRT fees are only charged before the corn production years. The NPV of the VRT strategy is compared with the NPV of returns from the UNI N and P fertilization strategy.

RESULTS AND DISCUSSION

The 1997 and 2001 corn seasons were exceptional in terms of adverse weather conditions early in the growing season. The 1997 corn yield was affected by late snowfall and cold, wet conditions just before planting, causing relatively low yields in some areas of the field. Total seasonal precipitation (April–October) was highest for this year (533 mm), with 202 mm of rain falling in June. The early part of the 2001 growing season was also marked by very wet conditions, with 189 mm of rainfall in April before planting. This heavy precipitation produced significant hillside and low-lying seepages with effects persisting through the 2001 growing season (Fig. 2, Sites A, B, C, E, and F). As a result, there were many zero or near zero yields in 2001 where drainage was a problem. Sometimes these low or zero yields were in the low-lying areas of the field. In other parts of the field, hillside seepage also contributed to low yields. Low organic matter areas on the eroded hilltop area (Sites E, F) also contributed to low corn yields during the 2001 growing season in particular, and lower corn and soybean yields in general. These positions are dominated by the Clarion-Swanlake loam series soils (6–12% slopes). Lower yield patterns for corn and soybean are apparent on the eroded hilltop area (Sites E, F). Higher yield patterns generally occurred in the area corresponding with the field entry point of an old dairy barn (Site G). Phosphorus readings were highest in this portion of the field (>15 mg kg⁻¹ [ppm], Fig. 2).

Crop Yield Response Variability

Whole-field average response estimates are presented in Table 1. Linear and quadratic terms for N and P were consistent with expectations for corn and soybean response in all years. These are the estimates by which individual sub-block responses are statistically compared. The null hypothesis that errors were homoskedastic was rejected at the 5% level for all years, except for the corn 2001 season (LM = 426, P = 0.32). Subsequently, Eq. [1] was re-estimated using GMM with White’s robust standard errors for each season when error terms were not homoskedastic.

Empirical semivariograms for each season were estimated using residuals from Eq. [1]. For the 2001 season, OLS residuals were used to estimate the empirical semivariogram. Spatial structure in the semivariograms was detected with γ(s). The 1999 and 2001 corn years were the only years where spatial error dependence was significant (F test, F = 2.37, P = 0.08 and F = 4.76, P = 0.005, respectively). This is not surprising because spatial error dependence is usually considered an omitted variable problem (Anselin, 1988). The sub-block regressions are essentially fixed effects models because block-specific dummy variables identify intercept, linear, quadratic, and interaction response terms in each sub-block. Allowing each sub-block its own response explained most of the spatial error dependence in the corn 1997 and soybean 1998, 2000 years. Therefore, the GEO method was only used to estimate corn response in 1999 and 2001.

Spatial and Temporal Response Variability and Stability

A graphical approach is used to convey spatial and temporal relationships of corn response to N and P and soybean response to P (Figs. 3 and 4). The t test statistics associated with the linear N and P coefficients for corn and soybean in Eq. [1] determine whether crop response to N and P was significantly different from the average whole-field response. The bars associated with each point estimate are 90% confidence intervals. The complete set of response coefficients are found in Lambert (2005). Quadratic and interaction terms are important with respect to economic analysis of crop response, but only linear response terms are highlighted here. The linear term is the plant response to the first kilogram of fertilizer and, as such, is the key to understanding profitability. In profit maximization, the interpretation becomes how much profit is increased when an extra kilogram of input is applied. If the first kilogram of fertilizer is not profitable, then it is unlikely that any fertilizer will be profitable. The quadratic term is important for determining the rate at which response diminishes at higher application rates and, consequently, the peak yield. Linear and quadratic terms were highly negatively correlated for N and P in all years for both crops (Pearson’s r > 0.93). This means examination of the quadratic coefficients shows similar patterns, but in a mirror image.

The extent to which corn response to N and P varied spatially differed depending on the production year.
In 1997, corn response to N in a given block was different from the WF average response (7.28 kg ha\(^{-1}\) corn per first kg of applied N) to N in 55% of the sub-blocks at the 5% level. Spatial variability of corn response to N decreased to 38% of the sub-blocks being significantly different at the 5% level from the WF average (8.25 kg ha\(^{-1}\) corn per first kg of applied N) in the 1999 corn year. In the 2001 season, corn response to N was nearly uniform over the entire site; only 7% of the sub-blocks exhibited N responses significantly different from the WF average response (6.60 kg ha\(^{-1}\) corn per first kg of applied N). In general, patterns where corn yield response to N was significantly different from the whole-field average are not clearly discernable between years. Most of the sub-blocks nonresponsive to N were located in a high organic sink where Jeffers clay loams and Jeffers variant clay loam soils (fine-loamy, mesic Aeric Calciaquolls) with 2 to 4% slopes averaged about

![Fig. 2. Bare soil images (1996), hillside seepage areas (aerial photo, 2001), and kriged P and %OM soil tests. A = seepage area, P section 1; B = seepage area, P section 1; C = seepage area, P section 1; D = high organic matter (6–10%), with soil P tests between 7 and 11 mg kg\(^{-1}\) [ppm], P section 2; E1,F1 = hillside seepage area on the downward slope of the eroded hilltop area, and the eroded hilltop area, P section 2; E2,F2 = eroded hilltop area and hillside seepage area on the downward slope of the eroded hilltop, P section 3; G = high P area associated with an old dairy entrance (P > 15 mg kg\(^{-1}\) [ppm]). Field aspect and slopes are denoted by the vectors.]

Table 1. Whole-field, average response to N and P for corn and soybean (t tests in parentheses).  

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<tr>
<td>Intercept</td>
<td>5695.76</td>
<td>2578.52</td>
<td>6618.63</td>
<td>1805.70</td>
<td>5071.84</td>
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<tr>
<td></td>
<td>(95.26)</td>
<td>(58.97)</td>
<td>(45.53)</td>
<td>(69.01)</td>
<td>(4.69)</td>
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<tr>
<td>(\beta_N)</td>
<td>7.28</td>
<td>8.25</td>
<td>9.57</td>
<td>2.24</td>
<td>6.60</td>
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<td>(9.57)</td>
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<tr>
<td>(\beta_{N2})</td>
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<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
<td>-0.01</td>
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<tr>
<td>(\beta_P)</td>
<td>9.12</td>
<td>3.07</td>
<td>23.86</td>
<td>11.84</td>
<td>14.13</td>
</tr>
<tr>
<td></td>
<td>(8.19)</td>
<td>(3.08)</td>
<td>(7.74)</td>
<td>(22.22)</td>
<td>(0.70)</td>
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<tr>
<td>(\beta_{P2})</td>
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<td>-0.001</td>
<td>-0.08</td>
<td>-0.03</td>
<td>-0.04</td>
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<td></td>
<td>(–3.18)</td>
<td>(–0.01)</td>
<td>(–5.66)</td>
<td>(–15.95)</td>
<td>(–0.67)</td>
</tr>
<tr>
<td>(\beta_{N\times P})</td>
<td>-0.01</td>
<td>0.003</td>
<td>0.01</td>
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<td></td>
<td>(–6.51)</td>
<td>(1.87)</td>
<td>(0.91)</td>
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8% organic matter. In the lower end of this section, percentage organic matter was also relatively high (~5%). These findings were expected because soil high in organic matter has a high potential to supply N through mineralization (Mamo et al., 2003).

Spatial patterns where corn response to P was significantly different from the WF average response were clearly discernable (Fig. 3), especially in the bare hilltop area dominated by a Clarion-Swanlake loam series soil with a 6 to 12% slope (Sections 2, 3, Rows 10–17), the high organic matter portion (Section 2, Rows 1–6), and in Section 1 in the low-lying seepage areas of the field (Sections 2 and 3, Rows 1–8, Jeffers variant clay loams series) for the 1997 and 1999 corn years. In the 1997 and 1999 corn years, 32 and 40% of the sub-blocks exhibited P response levels significantly different from the WF average (Table 1). In the 2001 season, P response heterogeneity for corn increased to 49%. Locations where corn response to P was lowest were in areas where soil organic matter or P soil test levels were relatively high in Sections 2 and 3 (Fig. 2).

Corn response to P in Section 3, Rows 19 to 23 was temporally and spatially stable. The point estimates of all years rest in the same confidence interval band and were highly significant (Fig. 3). On the other hand, corn response to P was not temporally stable in Section 2, Row 5 because the confidence intervals of those point estimates do not overlap. Likewise, 1999 corn response to P in Section 2, Row 4 was significantly different from the 1997 corn response in Section 2, Rows 20 to 23 in the same year with respect to location. Corn response profiles along the P treatment sections suggest that although yield response was significantly different between crop rotations, similar patterns were evident in physically distinct portions of the field (Fig. 3).

In the low-lying seepage area dominated by Jeffers clay loam and Clarion loam (2–4% slopes) series soils (Site A) and hillside seepage areas (Sites B, C, Clarion loams, 2–4% slope and Webster-Delft clay loam soils, respectively) in P treatment Section 1, the corn response pattern to applied P in 2001 was highly variable compared with the 1997 and 1999 seasons (Fig. 3). The high variability associated with P in Section 1 was most likely due to the fact that only three P levels were used in the treatment block, and that this portion of the field has

![Fig. 3. Spatial and temporal corn responses to N and P. Bars are 90% confidence intervals.](image-url)
poor drainage capability. In Section 3, soil in the portion of the field where the old dairy entrance was located (Site G) had a high P content. Corn response to P in this landscape position was remarkably similar across all corn years. The south end of Section 2 is dominated by a Canisteo clay loam series soil (fine-loamy mixed [calcareous] Typic Haplaquolls, Rows 20–23), and corn response to P was also stable across all years. But in Section 1, the corn response pattern to applied P in 2001 was highly variable compared with the 1997 and 1999 seasons. This area of the site corresponds with high soil organic levels (>4.5%) and soil test P levels of 5 to 11 mg kg\(^{-1}\) [ppm]. In P treatment Sections 2 and 3, a prominent physical feature is the eroded hilltop area composed of a Clarion-Swanlake loam series soil with 6 to 12% slopes (Sites E2 and F2; between Rows 10 and 17). Hillside seepage areas dominated by a Jeffers variant clay loam series soil (2–4% slope) are also present on the downhill slope of these transects (Site E1, F1, Rows 10–14). During the 2001 corn year corn response to P was highly variable in these spots, which are associated with low organic matter and low soil P. The organic sink in the north end of Section 2 (Site D) also exhibited stable and relatively small P response profiles for corn where soil organic matter ranged between 5 and 9% and soil test P levels were between 8 and 11 mg kg\(^{-1}\) [ppm].

When spatial P response variability was considered across the corn years, corn response to P was similar in 51% of the blocks. Here, response similarity means that the SSM null hypothesis for a given sub-block was equally rejected across all years. For soybean, 57% of the blocks responded similarly to P between growing seasons. But for corn response to N, only 26% of the sub-blocks responded similarly between growing seasons. The relative instability of corn response to P in 2001 was likely caused by the interaction between the number of P treatments and the poor drainage capability of some parts of the field. The relative stability of P response was likely the result of its mobility in soils and the physiological effects of P on root system growth.

Similarities in corn response to P can be expected due to P soil carryover dynamics and placement, but consistent spatial response patterns are less discernable for corn response to N. Nitrogen is more susceptible to hydrologic conditions that are affected by annual precipitation and topography interactions. These findings for N correspond with the Mamo et al. (2003) N study; temporal inconsistencies in response can be attributed to organic matter mineralization, denitrification, and water availability, which in turn are influenced by topography. These findings for N are also consistent with the Fiez et al. (1994) conclusions about N response and topography relations.

The most striking temporal response trends are observed in the soybean response profiles (Fig. 4). The curves of soybean response trends within transects are remarkably similar. Although the magnitude of soybean yield response to P was different between years, the same underlying effects of P response dynamics are present. These relations were strongest in Sections 2 and 3 (Fig. 4). Soybean response to P was different from the WF average in 33% of the sub-blocks in 1998, and different from the WF average in 62% of the sub-blocks in the 2000 season. Soybean response to P was significantly different from the WF average in the high organic matter portions of the field and the eroded hilltop area for the 2000 season. For both soybean years, response to P was significantly different from the WF average along the entire transect of Section 3, excluding the southeastern upper portion of the field where the entrance to an old dairy barn was once located (Site G). Because of the localized distribution of manure from the barn, P soil test values were very high in this position. These findings are consistent with Wittry and Mallarino’s (2004) VRT-P study for corn and soybean. They found that plant response to P was only significant in areas of the field recording <15 mg kg\(^{-1}\) [ppm] P. For P, 33 and
62% of the sub-blocks reported significant response to the first kilogram of P for soybean. As a response sensitivity test, N was included in the soybean response equations. Soybean marginal response to the first kilogram of N was significant in only 4 and 14% of the sub-blocks for the 1998 and 2000 soybean years, respectively.

A side-by-side comparison of corn and soybean response patterns to N and P is possible by standardizing the response coefficients for both crops across all years as $\sim N(0,1)$ with their respective averages and standard deviations (Fig. 5). In the third P section, similar response patterns to P are observed in corn and soybean. Likewise, similar response patterns are also discernible in the second P section (Site E1). In the first section, comovement of response patterns is less noticeable but still present, particularly in the areas between the hillside seepage areas (Sites B, C, Rows 12–19). Comovement of N across corn years is less discernable, except perhaps in Section 2, Rows 1 to 15. By these measures the spatial patterns of temporal (in)stability become clear. For corn response to N, each year is different and response patterns appear to be random. For P, moving west to east, P response over time for corn and soybean converges toward a stable trend over landscape position and cropping years.

**Marginal Analysis Scenarios**

Four ex post scenarios compare returns from VRA management strategy combinations to returns from a uniform (UNI) NP management strategy. The first scenario (S1) compares a VRA-NP strategy to a UNI strategy. In the next scenario (S2), information costs and input application costs are omitted from the budget. The value of this scenario is for comparison with prior studies that have ignored information and application costs of VRA technologies (for a review, see Lambert and Lowenberg-DeBoer, 2000). This scenario might reflect the changing nature of VRA technology with the expectation that, in the long run, VRA costs will not be a constraint with respect to technology choice as VRA equipment becomes relatively more common and less expensive over time. The third scenario (S3) compares VRA-P coupled with a uniform N application program to a uniform NP management strategy. The last scenario (S4) compares a VRA-N combined with a UNI-P...
strategy. All information costs are charged in the VRA and UNI strategies in S3 and S4. In the UNI strategy, sub-block response functions are evaluated at the University of Minnesota extension recommendation rates of 90 kg ha\(^{-1}\) N (Randall and Schmitt, 2002) and 67 kg ha\(^{-1}\) P (Rehm and Schmitt, 1993).

### Economically Optimal Nitrogen and Phosphorus Rates

In S1, the field average level of applied N under the VRA program was higher than the extension recommendation amount for all years by 15, 44, and 27% for 1997, 1999, and 2001, respectively (Table 2). The EORP was lower for the 1997 and 2001 years by 2 and 8%, but only slightly higher in the 1999–2000 rotation by 1%. On average, EONR was higher than the extension N recommendation (mean ± SD, 115 kg ha\(^{-1}\) ± 13), but EORP was slightly lower (64 kg ha\(^{-1}\) ± 5) than extension recommended P amounts for Scenarios 1 and 2 (Table 2). In the third corn year, 13% of the cells had zero EONRs. One location for these recommendations was in the lower corner section of Section 1 (Rows 21–23), which is dominated by a Webster-Delft clay loam series with poor drainage capacity. Other zero N levels occurred on the margins of the eroded hilltop area, and the downward slope just north of this position (Section 2, Row 8 and 15). This indicates that in these areas the mineralization potential of N by organic matter is substantial. In the control strip, corn yield was 7843 kg ha\(^{-1}\) in this section, indicating the fertility of the position. Other zero N rates occurred along the slopes of the eroded hilltop area (Sites E1, F1). This landscape position is prone to seepage problems during wet years and is dominated by a Jeffers clay loam soil series relatively high in organic matter. In these positions, the mineralization potential of N by organic matter content is also high.

In all scenarios, the EORP decreased during the very wet year of 2001. When P is applied uniformly and N is spatially managed, EONRs are lower for the first and last corn years compared with the VRA-NP strategy. This is in part explained by the interaction terms between N and P. Only 27% of the coefficients explaining the N × P interaction were positive, contrary to the usual expectations of N and P interaction effects on yield. As a result, the expected amount of applied N decreased by 46 kg ha\(^{-1}\). The 1997 and 1999 EORPs of S1, S2, and S3 indicate that the extension recommendation of 67 kg ha\(^{-1}\) approximates optimal P application for this field.

The 1997 EONR rate in S4 is much lower than the S1 optimization EONR by, on average, 45%. This difference is also due to the N × P interaction term. The cause of this difference is the negative sign on the N × P interaction term in the 1997 corn year in S4 (Table 1). When P (or N) is not allowed to vary, the first-order conditions of the optimization problem are quite different. UNI evaluated at 90 and 67 kg ha\(^{-1}\) for N and P, respectively.

### Table 2. Ex post partial budget NPV analysis and average EONR/EORP for block response analysis.

<table>
<thead>
<tr>
<th>Production cycle</th>
<th>EONR‡</th>
<th>EORP‡</th>
<th>Yield (VRA)</th>
<th>Annual cash flow (VRA)</th>
<th>Yield (UNI)</th>
<th>Annual cash flow (UNI)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997 (corn)</td>
<td>103</td>
<td>66</td>
<td>8 239</td>
<td>419.41</td>
<td>7959</td>
<td>431.97</td>
<td>−12.56**</td>
</tr>
<tr>
<td>1998 (soybean)</td>
<td>129</td>
<td>68</td>
<td>10 070</td>
<td>472.48</td>
<td>9579</td>
<td>464.22</td>
<td>8.26*</td>
</tr>
<tr>
<td>1999 (corn)</td>
<td>126</td>
<td>67</td>
<td>9 534</td>
<td>371.92</td>
<td>7349</td>
<td>363.82</td>
<td>8.10</td>
</tr>
<tr>
<td>2000 (corn)</td>
<td>144</td>
<td>58</td>
<td>7 843</td>
<td>303.71</td>
<td>7349</td>
<td>308.76</td>
<td>21.38**</td>
</tr>
<tr>
<td>2001 (corn)</td>
<td>101</td>
<td>67</td>
<td>7 440</td>
<td>286.86</td>
<td>7349</td>
<td>289.00</td>
<td>21.78§</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2: No additional information or application cost (UNI and VRA)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997 (corn)</td>
<td>103</td>
<td>66</td>
<td>8 239</td>
<td>466.73</td>
<td>7959</td>
<td>452.54</td>
<td>14.19**</td>
</tr>
<tr>
<td>1998 (soybean)</td>
<td>129</td>
<td>68</td>
<td>10 070</td>
<td>498.91</td>
<td>9579</td>
<td>483.98</td>
<td>14.93**</td>
</tr>
<tr>
<td>2000 (soybean)</td>
<td>144</td>
<td>58</td>
<td>7 843</td>
<td>330.14</td>
<td>7349</td>
<td>308.76</td>
<td>21.38**</td>
</tr>
<tr>
<td>2001 (corn)</td>
<td>101</td>
<td>67</td>
<td>7 440</td>
<td>293.21</td>
<td>7349</td>
<td>289.00</td>
<td>21.78§</td>
</tr>
<tr>
<td>S3: Use VRA-P with UNI N rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997 (corn)</td>
<td>90</td>
<td>112</td>
<td>7 988</td>
<td>419.10</td>
<td>7959</td>
<td>431.97</td>
<td>−21.78§</td>
</tr>
<tr>
<td>1998 (soybean)</td>
<td>90</td>
<td>93</td>
<td>9 667</td>
<td>465.69</td>
<td>9579</td>
<td>464.22</td>
<td>1.47</td>
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<tr>
<td>2000 (soybean)</td>
<td>90</td>
<td>83</td>
<td>7 406</td>
<td>370.52</td>
<td>7349</td>
<td>363.82</td>
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<td>67</td>
<td>7 440</td>
<td>286.86</td>
<td>7349</td>
<td>289.00</td>
<td>21.78§</td>
</tr>
<tr>
<td>S4: Use VRA-N with UNI P rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997 (corn)</td>
<td>57</td>
<td>67</td>
<td>7 760</td>
<td>407.63</td>
<td>7959</td>
<td>431.97</td>
<td>−24.34**</td>
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<tr>
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<td>67</td>
<td>9 899</td>
<td>451.77</td>
<td>9579</td>
<td>451.77</td>
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<td>9579</td>
<td>464.22</td>
<td>6.66**</td>
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<tr>
<td>2001 (corn)</td>
<td>101</td>
<td>67</td>
<td>7 440</td>
<td>286.86</td>
<td>7349</td>
<td>289.00</td>
<td>−2.14</td>
</tr>
</tbody>
</table>

* Difference (E[NPV\text{VRA}]) − E[NPV\text{UNI}] is significant at the 5% level (paired t test).
** Difference (E[NPV\text{VRA}]) − E[NPV\text{UNI}] is significant at the 1% level (paired t test).
† UNI evaluated at 90 and 67 kg ha\(^{-1}\) for N and P, respectively.
‡ These are the average of the 69 sub-block EORPs and EONRs.
§ Difference (E[NPV\text{VRA}]) − E[NPV\text{UNI}] is significant at the 10% level (paired t test).
different than those of the joint N and P optimization problem. For N, holding P constant, solving the FOC for the optimal N rate is

\[ N^{\text{opt}}_{\text{P}} = \frac{-r_{\text{P}} \beta_{\text{P}}^{\text{corn}} + p \beta_{\text{P}}^{\text{corn}} (P - r_{\text{N}} - r_{\text{S}}) B}{2 p \beta_{\text{P}}^{\text{corn}} (P - r_{\text{N}} - r_{\text{S}}) B} \]

with B the discount factor, \( r_{\text{S}} \) the nitrogen price, \( p \), the corn price, \( P \) the phosphorus extension recommendation, and the subscripts on the betas indicating \( N \) linear (N) and quadratic (\( N \times N \)), and \( N \times P \) interaction terms. When \( \beta_{\text{P}}^{\text{corn}} < 0, \beta_{\text{N}}^{\text{corn}} < 0, \) and \( \beta_{\text{N}} > 0, N^* \) will decrease. This only happens in the 1997 year of S4, and is why the EONR is much lower than the S1 optimization scenario. For the N-UNI, variable P scenario, the FOC is slightly different due to the additional soybean term. Assuming that soybean linear (quadratic) terms are positive (negative), P increases because of the revenue boost from soybean, while N is held constant.

**Marginal Analysis: Ex Post VRT/UNI Yield and Profitability Comparison**

The SSM null hypothesis was rejected in the first and second scenarios (Table 2). The NPV of the VRA-NP strategy was significantly greater than the UNI strategy by $28.38 \text{ ha}^{-1}$ in the S1 (baseline) comparison (Paired t test, \( T = 3.06, P = 0.003 \)). The largest negative cashflow for VRA occurred in the 1997 corn season ($12.56 \text{ ha}^{-1}$). Cashflows were lower for this season in part because of initial capital outlay to maps, management zone identification, and soil test costs, combined with the VRA-P application fee. In a cost sensitivity analysis, when the initial costs were annualized over the 5 yr, the returns for VRA exceeded those of the uniform application every season. Differences between UNI and VRA-NP cashflows were significant only for the corn years. In the extremely wet year of 2001, the cashflow margin between VRA-NP and UNI was even greater than the previous seasons.

It is widely known that information and application costs have constrained widespread adoption of VRA technologies (Griffin et al., 2004). It is also generally recognized that applying the right amount of inputs in the right locations will produce higher yields and increases in returns over input costs compared with uniform application of inputs. But increased returns may not cover information and VRA costs with current technology. In S2, the SSM null hypothesis is rejected because the expected cumulative NPV of the VRA-NP strategy was significantly higher than returns to the UNI-NP strategy by $68.47 \text{ ha}^{-1}$ (\( T = 7.38, P < 0.0001 \)) (Table 2). When VRA does not require higher information or application costs, all rotation NPVs were higher under the VRA-NP strategy compared with the UNI strategy.

The third (S3) and fourth (S4) scenarios compare VRA-P or VRA-N strategies combined with constant N or P amounts applied at the extension recommendation, respectively (Table 2). The SSM null hypothesis is not rejected in Scenario 3. Returns to VRA were still higher than the UNI strategy ($0.25 \text{ ha}^{-1}$), but not significantly (\( T = 0.08, P = 0.93 \)). In S4, the SSM null hypothesis is not rejected because the profit margin significantly favored the UNI strategy by $19.81 \text{ ha}^{-1}$ (\( T = 13.27, P < 0.0001 \)). The lack of VRA-N profitability is likely due to the lower corn yield response to site-specific N in 1997 and 2001. The profitability of site-specific P is increased by returns in soybean years. However, the VRA-P uniform N, net present value would be negative were it not for the revenue gained from the soybean years. Additionally, N is more volatile than P, making it more difficult to spatially manage over multiple growing seasons without recourse to address spatial and temporal variability of soil mineralization or weather effects on response.

Because of the unbalanced nature of the experimental design (3 yr of corn and 2 yr of soybean), the partial budget results may be biased in favor of VRA fertilizer application. To test this assumption, the marginal analysis was conducted using the corn years only. Results showed that although the nature of site-specific relations was limited with soybean, they do not change the results of S1 or S2: VRA profits were greater than UNI by about $11$ and $18 \text{ ha}^{-1}$, respectively. However, the results do change in S3: VRA was no longer profitable compared with the uniform recommendation (difference = $-6.76 \text{ ha}^{-1}$). When considering the corn years only, applied P decreased, respectively, in the 1997 and 1999 years by 10 and 15% in S1. Optimal P applications decreased by 51 and 44% for the 1997 and 1999 years, respectively, in Scenario 3. This occurred because the first-order condition terms for soybean P drop out of the corn–soybean rotation optimization problem. When expected revenue from soybean is not jointly considered with corn in the planning horizon, conclusions about VRT as applied to multiple corn–soybean rotations may be underestimated.

**CONCLUSIONS**

Corn response to N and P and soybean response to P varied spatially and temporally. The response patterns for corn and soybean to P were temporally stable in some field positions but not in others, rejecting the SSM hypothesis. Soil characteristics such as percent organic matter, P soil tests, field history, topography and hydrology, and soil series were useful in the identification of such areas, as well as landscape positions susceptible to erosion and seepage. Many of the problems associated with poor drainage in the study site might be reduced by proper tiling. Response patterns for N were less stable between seasons, reflecting the difficulties associated with the temporal and spatial management of this relatively dynamic input. Economically optimal N applications were generally higher than recommended whole-field fertilizer amounts, indicating that extension recommended N levels underfertilize this field. On average, extension recommended P levels are close to the EORP for this field.

In an ex post cash flow analysis, discounted returns and corn–soybean yields from a VRA-NP fertilizer program were $28 \text{ ha}^{-1}$ greater than returns and yields from a UNI management strategy. Corn and soybean yields also increased under the VRA-P uniform N strategy (8354 and 2939 kg ha$^{-1}$, respectively) compared with the UNI strategy (8296 and 2881 kg ha$^{-1}$, respectively). However, the $0.25 \text{ ha}^{-1}$ profit margin between the VRA-P and UNI strategy was not significant. In this study, returns to
a VRA-N strategy were significantly less than the UNI strategy, indicating that spatial management of this input over multiple growing seasons is more difficult than spatial management of P.

Ignoring the opportunities gained from knowing timely nutrient level information may exaggerate the temporal level of yield response risk, as well as interactions between temporal and spatial yield response to risk. With respect to corn yield response, N clearly drives the profitability results. Today, presidedress nitrate tests (PSNT) are commercially available. The PSNT is just one example of many that might be used to minimize temporal risk. Although these tests are designed to predict plant N needs, they do not always work as well as most producers would like. The relative success of these tests is in part due to climatic and soil variability from one region to the next. But when PSNT tests are on the mark, they may substantially reduce the temporal risks of N management. Later-season N applications would be another strategy to deal with the risk caused by N instability over multiple growing seasons. For farmers whose income depended on revenue from corn production only, appropriate timing and placement of N would play more of a role with respect to risk management. But, if response functions are relatively flat (as they are in many parts of the field in this study), then variable rate N application will not substantially reduce risk, even if these parameters could be determined.

Future research could uncover the temporal and spatial mechanisms regulating crop nutrient removal and nutrient carryover dynamics. This next step would require estimation of nutrient carryover functions jointly with site-specific estimation of crop response functions. Results of this analysis are a starting point for management zone delineation, based not only on yield patterns or soil characteristics, but also on response variability and fertilizer carryover dynamics.

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