Tropical forest harvesting and taxation: a dynamic model of harvesting behavior under selective extraction systems

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ABSTRACT. A dynamic model of selective harvesting in multi-species, multi-age tropical forests is developed. Forests are predicted to exhibit different optimal harvesting profiles depending on the nature of their joint cost functions and own or cross-species stock effects. The model is applied to the controversy about incentives produced by various taxes. The impacts of specific taxes are shown to depend on the composition of the forest stocks, growth rates, and joint cost effects. Therefore, specific taxes may create different incentives and impacts in Indonesia than in Brazil or Malaysia, for example, suggesting that no single uniform forest tax policy will be appropriate for all countries or all forests.

1. Introduction
In this paper, we develop a rational choice model of selective cutting in a multiple-species forest. A renewed emphasis on natural forest management for tropical forests has emerged among scholars, governments, and development agencies (Sedjo, 1987a, 1987b, 1991; Hyde and Newman, 1991; Ingram and Buongiorno, 1997; Boscolo and Vincent, 2000; Dufournaud et al., 2000; Nieuwenhuyse et al., 2000; Reddy and Price, 1999). The vast majority of natural tropical forests owned by central governments, state and provincial governments, or by local communities are managed and developed under concession systems that mandate the use of selective

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1 For example, Sedjo and Lyon (1990) report that almost one-half of the world’s timber production derives from natural forests and that the importance of natural forests is not likely to change for the next 50-60 years.
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harvesting techniques (Grut et al., 1991; Ghani et al., 1992; Repetto and Gillis, 1988; Sedjo, 1987a, 1987b).

Most of the rich forest economics literature assumes clear-cutting strategies for single species, even-aged, temperate forests in developing models for determining optimal rotation periods for plantation forests.\(^2\) The economics literature on uneven-aged forest management has emphasized determining optimal stocking, cutting cycles, and minimum diameter limits on temperate forests dominated by one or only a few species.\(^3\)

The interaction of economics and biology in natural tropical forests is complicated by a number of factors not present in temperate forest plantations. First, different species have different contemporaneous and intertemporal outputs. Also, the species’ specific growth rates can be affected by the inter- and intra-species stocks. The presence of these types of externalities within and between species may affect harvesting strategies as well as the size and distribution of species remaining at the end of the planning period. Finally, costs may be affected through time by the presence of different stocks. For instance, the presence of relatively dense stands of commercial species, as are found in the dipterocarp forests of Indonesia and Malaysia, may reduce the cost of selective harvesting by allowing the logger to select trees which yield the same final output for a lower per unit cost. Finally, those species that are not harvested may affect the growth and costs of commercial species. Sedjo (1987a), for example, found that in the dipterocarp forests of Kalimantan and Sumatra, 267 out of 4,000 tree species have been designated as acceptable for commercial timber use. Such designations, whether commercial or government, may indicate how selective cutting is affected by the surrounding environment.\(^4\)

Each of these elements is included in our dynamic model of the tropical forest concessionaires’ problem. The model has characteristics similar to models of natural resource extraction where miners must determine the quality–quantity profile of output. Concessionaires are assumed to dynamically allocate the harvest from a fixed initial stock consistent with present value maximization. The major issue addressed is the determination of the dynamic quality–quantity harvest profile; that is, how the concessionaire determines which species and qualities within a species to harvest, when to harvest a particular species, and what volume to harvest both within and between time periods.


\(^4\) Recent research (Condit et al., 2000) indicates that average density of dipterocarp stems (even larger ones) in Malaysian forests is notably greater than earlier predicted by Janzen (1970).
Others have developed dynamic forest harvest and timber supply models, incorporating growth for single species models of temperate forest supply and demand. For example, Max and Lehman (1988) include a logistic growth function and recreation to develop a single species, two period model to examine optimal timber supply curves and how non-timber benefits such as recreation change the impact of various taxes on the US non-industrial private forest land owners (NIPF). Similarly, Hultkrantz and Aronsson (1989) include growth functions in their single species econometric study of roundwood from Swedish NIPF to examine factors that influence timber supply and demand. Kuuluvainen and Salo (1991) use a Fisherian two-period savings-consumption model to examine non-rationed short-term timber supply models. Ovaskainen (1992) extend the single species two-period model of timber consumption and harvesting by incorporating management intensity and use it to study the effects of alternative tax instruments on timber supply from single species, even-aged plantations in Finland. We extend these models by incorporating multi-species, intra- and inter-species influence on growth, over multi-time periods. These are features that distinguish typical tropical forest from temperate forest management problems.

Following the development and analysis of the model in sections 2 and 3, the model is applied in section 4 to the analysis of the relative incentives created by tax instruments commonly imposed on tropical forestry operations. The rents and royalties associated with commercial extraction of tropical timber is one of the few tropical forestry topics in which a specialized economics literature has developed over the past few years (Ghani et al., 1992; Gillis, 1980; Gray, 1983; Grut et al., 1991; Hyde and Sedjo, 1992; Page et al., 1976; Repetto and Gillis, 1988; Ruzicka, 1979; Vincent, 1990). Most argue that royalties may be too low, resulting in underpricing of the forest resource, reducing incentives for sustainable forest management, and generating windfall profits for concessionaires. In addition, incentives for rapid harvesting rates and ‘high-grading’ may occur when uniform royalties across species and grades are combined with inadequate concession time periods.

This literature has been criticized by Hyde and Sedjo (1992), Paris and Ruzicka (1991), Vincent and Binkley (1991), and Ghani et al. (1992). The debate over rents and royalties associated with tropical forest concessions has been based primarily on simple, graphical, marginal cost and price models. The application of our dynamic model to the tropical forest rents and royalties controversy shows that the effects of different taxes on the harvest profiles of concessionaires may be more complicated than previous analyses indicate. The impacts depend on the nature of the stocks, costs and growth effects, in addition to traditional economic variables such as the time path of prices, and suggests the need for additional empirical studies.

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5 Ghani et al. (1992) present the only known rigorous concession-level empirical analysis of concessionaire behavior. Page et al. (1976) and Ruzicka (1979) rely on data for average, hypothetical concessions.
2. The model

The model developed below is general in the sense that it may be applied to any type of natural forest where the forester must determine a selective harvest over a number of periods for different species. Present value maximization is assumed and the assumption of perfect foresight is maintained.

Assume that the forester has a planning horizon indexed by: \( t = 0, \ldots, T \). At date \( t = 0 \), there are \( S \) species of trees indexed by: \( s = 0, \ldots, S \). The initial stock of each species is known and defined by \( \bar{A}_0, s \). The harvest of any species, \( s \), in any period, \( t \), is defined as \( X_{t, s} \) and the stock remaining at the end of the period is defined by \( A_{t, s} \). The net stock remaining after the harvest of species \( s \) in period \( t \) is defined as

\[
N_{t, s} = A_{t, s} - X_{t, s}
\]

Trees of any species can grow or decay. We assume that the growth of any species, defined by \( G_{t, s} \) in any time period, is a function of the net stock remaining of that, or any other, species. That is

\[
\frac{\partial G_{t, s}}{\partial N_{t, s}} = \frac{\partial G_{t, s}}{\partial A_{t, s}} = -\frac{\partial G_{t, s}}{\partial X_{t, s}} > 0
\]

Growth can be positive, negative, or zero for both own-species effects and cross-species effects. Negative own-species effects might be common in climax forests, while positive own-species effects can be prevalent in younger forests. Cross-species growth effects are also possible. For instance, the presence of a particular species, which is a significant component of a canopy, can complement, or deter, the growth of another species that needs partial, or complete, sunlight. The growth function, defined in equation (2), combined with the initial stock in any period and the harvest in that period, will determine the stock in the subsequent period.
That is

\[ A_{t+1,s} = A_{t,s} - X_{t,s} + G_{t,s}(N_{t,0}, \ldots, N_{t,s}) \]
\[ = N_{t,s} + G_{t,s}(N_{t,0}, \ldots, N_{t,s}) \tag{3} \]

One implication of this lag structure is that, in general, the stock levels of any species will be a function of the initial stock of that species and the time profile of the harvest. Furthermore, the harvest in any period will affect the stocks in all future periods, as we will see below.

Prices are defined by species and time period. That is \( P_{t,s} \) is the price received by the forester for species \( s \) in time period \( t \). Harvesting costs may have both species-specific flow and stock components. That is

\[
C_t(X_{t,0}, \ldots, X_{t,s}; A_{t,0}, \ldots, A_{t,s})
\]
\[
\frac{\partial C_t}{\partial X_{t,s}} > 0
\]
\[
\frac{\partial C_t}{\partial A_{t,s}} < 0
\] (4)

Within-species costs may vary due to decreasing returns to scale. Cross-species costs may vary due to aggregate effects. Therefore, total harvesting costs and marginal species-specific costs may vary with the total amount of timber harvested, regardless of species. In addition, cross-species effects may vary because of height, volume, and/or canopy. Stock effects are included because costs may vary through time as depletion affects the forester’s choices. For example, foresters may resort to harvesting smaller trees as depletion (or accumulation) occurs.12

Profit in any period, \( t \), is defined as revenues less costs or

\[
\Pi_t = \sum_{s=0}^{S} P_{t,s} X_{t,s} - C_t(X_{t,0}, \ldots, X_{t,s}; A_{t,0}, \ldots, A_{t,s}) \tag{5}
\]

The residual value of the area at the end of the planning period may be a function of the remaining stocks of each species as well as residual land

11 Note that the form of equation (3) is similar to those used in other types of capital accumulation models such as Hall and Jorgenson (1947). In the current model, the stock at the beginning of the next period will be reduced (depreciated via harvesting (consumption)) and increased (or reduced) via growth (a type of exogenous next investment).

12 Marginal species-specific harvesting costs may be the same. Nothing in this formulation prevents this result. Total cost may be a function only of the total harvest and independent of species. Stock effects may also be irrelevant. The nature and size of cross-species and stock effects are empirical questions which should not be ruled out at this level of generality.
values benefits, which might be zero.

\[ V_{T+1}(A_{T+1,0}, \ldots, A_{T+1,S}) \quad \text{with} \quad \frac{\partial V_{T+1}}{\partial A_{T+1,s}} \geq 0 \quad (6) \]

Finally, define \( r \) as the discount rate. Based on the above equations, the forester’s decision problem can be defined as maximizing the present value of the harvest plus any residual value subject to the various definitions and non-negativity constraints, or

\[
\begin{align*}
\max PV &= \sum_{t=0}^{T} \frac{\Pi_t}{(1+r)^t} + \frac{V_{T+1}}{(1+r)^{T+1}} \\
\text{subject to } \Pi_t &= \sum_{s=0}^{S} P_{t,s} X_{t,s} - C_t(X_{t,0}, \ldots, X_{t,S}; A_{t,0}, \ldots, A_{t,S}) \forall t \\
V_{T+1} &= V_{T+1}(A_{T+1,0}, \ldots, A_{T+1,S}) \\
N_{t,s} &= A_{t,s} - X_{t,s} \forall t, s \\
A_{t+1,s} &= N_{t,s} + G_{t,s} \forall t, s \\
G_{t,s} &= G_{t,s}(X_{t,0}, \ldots, N_{t,S}) \forall t, s \\
A_{0,s} &\leq \bar{A}_{0,s} \forall s \\
\text{and } X_{t,s}, A_{t,s} &\geq 0 \forall t, s
\end{align*}
\]

This problem can be formulated as a discrete time optimization problem, or

\[
\mathcal{L} = \sum_{t=0}^{T} \left( \sum_{s=0}^{S} P_{t,s} X_{t,s} - C_t(X_{t,0}, \ldots, X_{t,S}; A_{t,0}, \ldots, A_{t,S}) \right) \frac{1}{(1+r)^t} \\
+ \frac{V_{T+1}(A_{T+1,0}, \ldots, A_{T+1,S})}{(1+r)^{T+1}} - \sum_{s=0}^{S} \lambda_{0,s}(\bar{A}_{0,s} - A_{0,s}) \\
- \sum_{t=0}^{T} \sum_{s=0}^{S} \lambda_{t+1,s}(A_{t+1,s} - N_{t,s} - G_{t,s}(N_{t,0}, \ldots, N_{t,S})) \quad (8)
\]

where \( \lambda_{t,s} \) is the Lagrangian multiplier for each time period and each species.

Both the remaining stock, \( A_{t,s} \) and the harvest \( X_{t,s} \) can be considered choice variables and the Kuhn–Tucker conditions for this problem with
respect to these variables and the Lagrangian multipliers are

\[
\frac{\partial L}{\partial X_{t,s}} = \frac{P_{t,s} - \frac{\partial C_t}{\partial X_{t,s}}}{(1 + r)^t} - \lambda_{t+1,s} - \sum_{k=0}^{s} \lambda_{t,k} \frac{\partial G_{t,k}}{\partial N_{t,s}} \leq 0; \quad X_{t,s} \geq 0;
\]

\[
X_{t,s} \cdot \frac{\partial L}{\partial X_{t,s}} = 0 \quad \forall t, s
\]

\[
\frac{\partial L}{\partial A_{0,s}} = -\frac{\partial C_0}{\partial A_{0,s}} - \lambda_{0,s} + \lambda_{1,s} + \sum_{k=0}^{s} \lambda_{0,k} \frac{\partial G_{0,k}}{\partial N_{t,k}} \leq 0; \quad A_{0,k} \geq 0;
\]

\[
A_{0,k} \cdot \frac{\partial L}{\partial A_{0,k}} = 0 \quad \forall s
\]

\[
\frac{\partial L}{\partial A_{t+1,s}} = \frac{\partial V_{t+1}}{\partial A_{t+1,s}} - \lambda_{t+1,s} \leq 0; \quad A_{t+1,k} \geq 0; \quad A_{t+1,k} \cdot \frac{\partial L}{\partial A_{t+1,k}} = 0 \quad \forall s
\]

\[
\frac{\partial L}{\partial \lambda_{t,0}} = \bar{A}_{0,s} - A_{0,s} \geq 0; \quad \lambda_{0,s} \geq 0; \quad \lambda_{0,s} \cdot \frac{\partial L}{\partial \lambda_{t,0}} = 0 \quad \forall s
\]

\[
\frac{\partial L}{\partial \lambda_{t,s}} = \bar{A}_{t+1,s} - N_{t,s} - G_{t,s}(N_{t,0}, \ldots, N_{t,s}) \geq 0; \quad \lambda_{t,s} \geq 0;
\]

\[
\lambda_{t,s} \cdot \frac{\partial L}{\partial \lambda_{t,s}} = 0 \quad \forall t, s
\]

### 3. Analysis

The determination of the optimal harvesting profile represented by equation (9) will, in general, be a function of the initial stock, the nature of the cost function, the time paths of relative prices, and costs and the nature of the growth functions. A number of interesting results can be derived, however, from the model even at this level of generality. For instance, consider a positive harvest of any species \(s\) in any time period \(t\). The first-order conditions for that species in that time period are

\[
\frac{\partial L}{\partial X_{t,s}} = \frac{P_{t,s} - \frac{\partial C_t}{\partial X_{t,s}}}{(1 + r)^t} - \lambda_{t+1,s} - \sum_{k=0}^{s} \lambda_{t,k} \frac{\partial G_{t+1,k}}{\partial N_{t,s}} = 0
\]

\[
\frac{\partial L}{\partial A_{t,s}} = -\frac{\partial C_t}{\partial A_{t,s}} - \lambda_{t,s} + \lambda_{t+1,s} + \sum_{k=0}^{s} \lambda_{t+1,k} \frac{\partial G_{t+1,k}}{\partial N_{t,s}} = 0
\]
Manipulation of these conditions reveals

\[ \lambda_{t,s} = \frac{P_{t,s} - \frac{\partial C_t}{\partial X_{t,s}}}{(1 + r)^t} - \frac{\frac{\partial C_t}{\partial A_{t,s}}}{(1 + r)^t} \]  

(11)

That is, the opportunity cost attributable to an additional unit of the harvest (or change in stock) of species \( s \) in \( t \) is equal to the full marginal profit from the harvest, in present value terms, including stock effects, if any.13

Similar conditions hold for more than one time period and more than one species. In particular, consider the case where the same species is harvested in two concurrent periods. Substitution of the results from equation (11) into the first equation in (10) yields:

\[ \left( \frac{P_{t,s} - \frac{\partial C_t}{\partial X_{t,s}}}{(1 + r)^t} \right) - \left( \frac{P_{t+1,s} - \frac{\partial C_{t+1}}{\partial X_{t+1,s}} - \frac{\partial C_{t+1}}{\partial A_{t+1,s}}}{(1 + r)^{t+1}} \right) \left( 1 + \frac{\partial G_{t,s}}{\partial N_{t,s}} \right) \]

\[ - \sum_{k=0}^{s} \left( \frac{P_{t+1,k} - \frac{\partial C_{t+1}}{\partial X_{t+1,k}} - \frac{\partial C_{t+1}}{\partial A_{t+1,k}}}{(1 + r)^{t+1}} \right) \frac{\partial G_{t,k}}{\partial N_{t,s}} = 0 \]  

(12)

Some manipulation reveals

\[ \left( P_{t,s} - \frac{\partial C_t}{\partial X_{t,s}} \right) (1 + r) - \left( P_{t+1,s} - \frac{\partial C_{t+1}}{\partial X_{t+1,s}} \right) \left( 1 + \frac{\partial G_{t,s}}{\partial N_{t,s}} \right) \]

\[ = \sum_{k=0}^{s} \left( P_{t+1,k} - \frac{\partial C_{t+1}}{\partial X_{t+1,k}} - \frac{\partial C_{t+1}}{\partial A_{t+1,k}} \right) \frac{\partial G_{t,k}}{\partial N_{t,s}} - \frac{\partial C_{t+1}}{\partial A_{t+1,s}} \left( 1 + \frac{\partial G_{t,s}}{\partial N_{t,s}} \right) \]  

(13)

This expression is similar to the generalized Hotelling rule common in the non-renewable resource economics literature.14 That is, the difference in discounted marginal profit attributable to the harvest for a species between two periods is equal to the future marginal profit from the harvest of other species, adjusted for growth effects, plus the cost savings from the own-species stock effects, again adjusted for growth effects. Thus, the forester must take into account own-species stock effects as well as growth effects on other species resulting from the harvest of a particular species. Note also that marginal profit attributable to extraction, and the difference in discounted marginal profit, can be less than zero. That is, inter-species growth effects can be such that it is better to harvest a particular species now so that other species can grow even if the marginal, and perhaps total, profit today is less than zero.15

Some additional insight into the results of equation (13) can be gleaned from some special cases. For instance, marginal profit can rise at the rate of

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13 This value may be zero in cases where the marginal liquidation value is zero and/or other inter-temporal effects are absent. This is noted in some detail below.


15 For instance, it may be in the forester’s economic interest to remove the canopy from a forest so that light can increase the growth on other species. Alternatively, it may be economically rational to clear scrub trees from the area surrounding marketable trees in order to increase the growth of marketable trees for a later harvest.
interest if the right-hand side of equation (13) is zero. A sufficient condition for this result is the situation where both stock and inter-species growth effects are absent. Note also that the shadow price, \( \lambda_{t,s} \), is constant for all time when that species is harvested in every period and there are neither inter-species growth effects or own-species stock effects. The value of the shadow price is equal to the marginal liquidation value of the standing timber, \( \partial V / \partial A_{t,s} \), a value which may be zero.\(^{16}\)

Simple economic interpretations of the general conditions are possible from the results described above. First, if a species is not harvested in a particular period, then the value of the future harvest is greater than any present value at the margin. That is, the return from holding assets in the form of trees is greater than the current value of partial liquidation. If a species is harvested in a particular period, then the marginal internal rate of return is greater than or equal to the return from harvesting that species in any other period. That is, by changing the harvest profile, the forester may change the marginal returns from holding the stock as an asset. Finally, if the date of exhaustion is before the end of the planning period, then the return from holding stocks beyond the date of exhaustion is not sufficient enough to warrant holding assets in the form of a particular species.

These conditions apply to all periods and to all species. Thus, a forester applying this model is using a harvesting profile to adjust the returns from a portfolio of assets, which are standing timber of different species. That is, each species is a unique stock that can generate potential returns. These returns are endogenous to the forester relative to the cost of capital and, thus, the forester will determine when to harvest each species, how much to harvest, and when to exhaust a particular species, if at all, by equating the marginal returns from the elements in the forester’s portfolio.

Another interesting result is that the stock does not have to be physically exhausted for the forester to be willing to pay to increase (or decrease) the initial stock. Stock effects and inter-species growth effects may be sufficient to change the forester’s willingness to pay. For instance, if a forest contains a particularly valuable species, such as teak, a marginal change in the initial stock may decrease operating costs via the stock effect.

A determination cannot be made about optimal harvesting sequences, in general. To understand the nature of this statement, consider the case where two species \( f \) and \( g \) are harvested jointly in period \( t \). Further assume that the species \( f \) has a higher market value per unit of output. The following conditions hold in this case

\[
\left( P_t,f - \frac{\partial C_{t,f}}{\partial X_{t,f}} \right) \frac{1}{(1 + r)^t} - \left( P_{t+1,f} - \frac{\partial C_{t+1,f}}{\partial X_{t+1,f}} - \frac{\partial C_{t+1}}{\partial A_{t+1,f}} \right) \frac{1}{(1 + r)^{t+1}} \left( 1 + \frac{\partial G_{t,s}}{\partial N_{t,s}} \right) \right.
\]

\[
- \sum_{k=0}^{s} \left( P_{t+1,k} - \frac{\partial C_{t+1,k}}{\partial X_{t+1,k}} - \frac{\partial C_{t+1}}{\partial A_{t+1,k}} \right) \frac{1}{(1 + r)^{t+1}} \frac{\partial G_{t,k}}{\partial N_{t,s}} = 0
\]

\(^{16}\) This value, even if zero, is more in accord with the notion of user cost. See Scott (1953).
\[
\left( \frac{P_{t,g} - \frac{\partial C_t}{\partial X_t}}{(1 + r)^t} \right) - \left( \frac{P_{t,g} - \frac{\partial C_{t+1}}{\partial X_{t+1,g}} - \frac{\partial A_{t+1}}{\partial A_{t+1,g}}}{(1 + r)^{t+1}} \right) \left( 1 + \frac{\partial G_{t,s}}{\partial N_{t,s}} \right)
\]

\[- \sum_{k=0}^{S} \left( \frac{P_{t+1,k} - \frac{\partial C_{t+1}}{\partial X_{t+1,k}} - \frac{\partial A_{t+1}}{\partial A_{t+1,k}}}{(1 + r)^{t+1}} \right) \frac{\partial G_{t,k}}{\partial N_{t,s}} = 0 \quad (14)\]

It is clear from these conditions that such a strategy may be optimal as long as

\[
\frac{P_{t,f} - P_{t,g}}{(1 + r)^t} = \left( \frac{\partial C_t}{\partial X_{t,f}} \right) + \left( \frac{P_{t,f} + \frac{\partial C_{t+1}}{\partial X_{t+1,f}} - \frac{\partial A_{t+1}}{\partial A_{t+1,f}}}{(1 + r)^{t+1}} \right) (1 + \frac{\partial G_{t,s}}{\partial N_{t,s}})
\]

\[+ \sum_{k=0}^{S} \left( \frac{P_{t+1,k} + \frac{\partial C_{t+1}}{\partial X_{t+1,k}} - \frac{\partial A_{t+1}}{\partial A_{t+1,k}}}{(1 + r)^{t+1}} \right) \frac{\partial G_{t,k}}{\partial N_{t,s}} - \left( \frac{\partial C_t}{\partial X_{t,g}} \right)(1 + \frac{\partial G_{t,s}}{\partial N_{t,s}})
\]

\[\left( \frac{P_{t,g} - \frac{\partial C_{t+1}}{\partial X_{t+1,g}} - \frac{\partial A_{t+1}}{\partial A_{t+1,g}}}{(1 + r)^{t+1}} \right) \left( 1 + \frac{\partial G_{t,s}}{\partial N_{t,s}} \right)
\]

\[- \sum_{k=0}^{S} \left( \frac{P_{t+1,k} - \frac{\partial C_{t+1}}{\partial X_{t+1,k}} - \frac{\partial A_{t+1}}{\partial A_{t+1,k}}}{(1 + r)^{t+1}} \right) \frac{\partial G_{t,k}}{\partial N_{t,s}} \geq 0 \quad (15)\]

Optimal selective cutting depends on the time path of costs and the time path of stock effects, as well as inter-species growth effects. Thus, the forester must examine more than simply the time path of relative output prices in order to determine the optimal harvesting sequence. A similar result can be obtained with respect to the optimal harvest of a species. That is, it is not necessarily the case that the species will be harvested in the greatest quantity, or even at all, in the period with the highest discounted price, because stock effects and the time path of factor prices combined with output prices determine the optimal profiles. It is clear from this example that the forester may even harvest a species with no market value. If the removal of this species will reduce future cost, or increase the stock of marketable timber, then the present value of the operation can be increased by disposal of output with zero market value now.\(^\text{17}\)

Finally, marginal user cost can be negative. This can occur when inter-species growth effects are negative and/or future costs increase due to larger stocks. Increases in costs as a function of stocks may be due to crowding, which makes selective cutting difficult. The basic rule holds in all types of situations, however. That is, the marginal return from holding stocks in future periods will be equalized across time and species. Current marginal

\(^\text{17}\) This is equivalent to weeding a garden where the farmer removes weeds with no market value to increase the yield and reduce the cost of producing marketable output.
profit attributable to the harvest may be negative, which in turn implies that future profits (both total and marginal) will be greater by at least the amount of the increased loss today.

In summary, the model presented reflects one method the concessionaire may use to determine the optimal harvesting profile for a particular forest. In effect, the forester will engage in portfolio adjustments by changing stocks for each species within and between periods to equalize marginal returns with those which accrue via liquidation and investment in other assets. Some stocks may be physically exhausted, but physical exhaustion is not a necessary condition for significant inter-temporal user costs. Inter-temporal user costs can be positive because of the presence of own- and inter-species growth effects as well as stock effects. The importance of initial stocks is also critical. Different harvesting profiles will be generated relative to changes in initial stocks. That is, in applied situations, the harvesting portfolio will be unique to a particular forest. Thus, an optimal profile for a forest in Kalimantan will not necessarily be optimal for a forest in Thailand for the same species because of different initial conditions.

4. Taxation
The model developed in the last section is used to examine the incentives created by particular taxes. This exercise is presented to illustrate one potential application of the model as well as to highlight the potential interactions between economic and non-economic variables such as growth effects and stock effects. Output or yield taxes (both per unit and ad valorem) are examined.

The introduction of taxation into the selective harvesting model can have a number of effects. An incremental tax change can induce the forester to cease operations altogether, change the inter-temporal allocation of species and quantities, or induce the forester to reduce (increase) the yield from a particular species. Marginal changes in tax rates can affect the inter-temporal harvesting profile by changing the time path of relative prices and costs. Thus, a change in a tax rate may create incentives to reallocate quantities and species both within and throughout time. The marginal incentives are discussed here.

A two-period–two-species model is used to facilitate the analysis. The model is further restricted to the case where one species is harvested in both periods and exhausted in the second period. It is assumed that the second species is harvested in the second period only and not exhausted. Furthermore, it is assumed that harvesting costs are a function of the total

18 The analysis which follows is related to traditional tax analysis. That is, implications cannot be drawn with respect to factor payment policy (the charge for the right to harvest trees) or with respect to any externalities. Thus, it is assumed that, as property owner, the government has already imposed an appropriate charge for the right to harvest. The relationship between factor payments and taxes is an important issue for governments that hold resource rights. See Conrad (1989) for further discussion of this issue.
current harvest, independent of species. That is:

\[ C_t = C_t(X_{t,0} + X_{t,1}, A_{t,0}, A_{t,1}) = C_t(X_t, A_{t,0}, A_{t,1}) \]  

(16)

Note that \( G_{00} = G_{00}(X_{00,0}) = G_{00}(X_{00}) \) and \( G_{01} = G_{01}(X_{00,0}) = G_{01}(X_{00}) \). Either growth function might be affected, in theory, by harvesting of the second species. The second species is not harvested by assumption in this example, however, making this particular effect irrelevant. There are four distinct cases even with these restrictive assumptions. The cases are:

1. No stock or growth effects are present; that is, \( C_t = C_t(X_{t,s}) \) and \( G_{t,s}() \) does not exist for all \( t \) and \( s \).

2. Growth effects are present, but there are no stock effects, that is, \( C_t = C_t(X_{t,s}) \) and \( G_{t,s}() \) is not necessarily equal to zero for all \( t \) and \( s \). The derivatives of the growth effect \( G_{t,s}() \) can be any sign.

3. Stock effects are present, but there are no growth effects; that is, \( C_t = C_t(X_t, A_{t,0}, A_{t,1}) \) and \( G_{t,s}() \) does not exist.

4. Both stock effects and growth effects are present.

The Lagrangian for this problem in the no-tax situation and for the general case is

\[
\mathcal{L} = P_{0,0} X_{0,0} - C_0(X_{0,0}, A_{0,0}, A_{0,1}) + \frac{P_{1,0} X_{1,0} + P_{1,1} X_{1,1}}{1 + r} \\
- \frac{C_1(X_{1,0} + X_{1,1}, A_{0,0} - X_{0,0} - G_{0,0}(X_{0,0}), A_{0,1} + G_{0,1}(X_{0,0}))}{1 + r} \\
+ \lambda_0 (A_{0,0} - X_{0,0} - G_{0,0}(X_{0,0}) - X_{1,0})
\]  

(17)

For convenience, results along with some sufficient conditions are reported in tables 1 and 2 respectively for each case discussed. For convenience, the relevant second derivatives are presented in table 3.

4.1. Per unit output taxes

Consider a per unit output tax in a fixed monetary amount per unit of harvest regardless of quality and time; that is, \( \kappa_{t,s} = \kappa \) for \( t = 0, 1 \) and \( k = 1, 2 \). This tax will reduce the vector of prices but not in a proportional way. That is, the output tax will be lower, in present value terms, in the future relative to the present. The Lagrangian for this problem is

\[
\mathcal{L} = (P_{0,0} - \kappa) X_{0,0} - C_0(X_{0,0}, A_{0,0}, A_{0,1}) + \frac{(P_{1,0} - \kappa) X_{1,0} + (P_{1,1} - \kappa) X_{1,1}}{1 + r} \\
- \frac{C_1(X_{1,0} + X_{1,1}, A_{0,0} - X_{0,0} - G_{0,0}(X_{0,0}), A_{0,1} + G_{0,1}(X_{0,0}))}{1 + r} \\
+ \lambda_0 (A_{0,1} - X_{0,0} - G_{0,0}(X_{0,0}) - X_{1,0})
\]  

(18)
The first-order conditions are

\[
\frac{\partial \mathcal{L}}{\partial X_{0,0}} = P_{0,0} - \kappa - \frac{\partial C_0}{\partial X_{0,0}} + \frac{\frac{\partial G_{0,0}}{\partial X_{0,0}}}{1 + r} \frac{\partial A_1}{\partial X_{0,0}}
\]

\[-\lambda_0 \left(1 + \frac{\partial G_{0,0}}{\partial X_{0,0}}\right) = 0\]

\[
\frac{\partial \mathcal{L}}{\partial X_{1,0}} = \frac{P_{1,0} - \kappa - \frac{\partial C_1}{\partial X_{1,0}}}{1 + r} - \lambda_0 = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial X_{1,1}} = \frac{P_{1,1} - \kappa - \frac{\partial C_1}{\partial X_{1,1}}}{1 + r} = 0
\]

\[
\frac{\partial \mathcal{L}}{\partial \lambda_0} = A_{0,0} - X_{0,0} - G_{0,0}(X_{0,0}) - X_{1,0} = 0
\]

Two incentives are created by this tax. First, the fixed per unit tax will be lower in present value terms in the future relative to the present. Thus, an incentive is created to shift production from the present to the future. This tendency is confirmed in cases 1 and 2, where output of the species to be exhausted falls in the first period and increases in the second period. Second, marginal cost is increased (or marginal revenue is decreased) by a constant amount, creating an incentive to reduce total recovery. Again, this incentive is confirmed in the first two cases because output of the less valuable species is reduced. The presence of growth (cases 2 and 4) complement or offset, in part, the reallocation incentive. Note that the growth effect (the \(D\) term – (see table 3 and definitions) will complement the reallocation effect if growth is positive and will offset, but not completely, the reallocation effect if growth is negative,19

The presence of stock effects, cases 3 and 4, is somewhat more complicated, depending on the sign of the change in marginal cost associated with a change in the stock. That is, an increase (decrease) in the harvest this year will decrease (increase) the stock next year, which in turn may change the marginal cost of harvesting next year. The sign of this effect cannot be predicted.20 Sufficient conditions for the incentive effects to be the same as those for the cases without stock effects include a positive cross effect. This would tend to reinforce the incentive to reallocate the harvest to the future. That is, reallocating extraction to the future will increase the stock in the future and, thus, lower marginal harvesting costs. This effect could be so

19 Note that the term \(D\) is the unit tax effect plus the growth effect (see table 2). This entire term must be positive to satisfy the necessary and sufficient conditions for profit maximization.

20 It might be the case that an increase in the stock will decrease marginal cost in a manner similar to non-renewable resources because it might be less expensive at the margin to harvest when a significant stock is available. Alternatively, a lower stock might be associated with lower marginal costs because of easier access, cheaper hauling, and other production-related factors.
Table 1. Per unit tax

<table>
<thead>
<tr>
<th>Case</th>
<th>$\frac{\partial X_{0,0}}{\partial \kappa}$</th>
<th>$\frac{\partial X_{1,0}}{\partial \kappa}$</th>
<th>$\frac{\partial X_{1,1}}{\partial \kappa}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-\frac{C}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td>2</td>
<td>$-\frac{C}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td>3</td>
<td>$-\frac{C + B (1+r)}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $B &gt; 0$ or</td>
<td>if $B &gt; 0$ or</td>
<td>if $B &gt; 0$ or</td>
</tr>
<tr>
<td></td>
<td>if $C &lt; \frac{B}{(1+r)}$</td>
<td>if $C &lt; \frac{B}{(1+r)}$</td>
<td>$-\frac{A}{(1+r)} - \frac{\partial X_{1,0}}{\partial \kappa}</td>
</tr>
<tr>
<td>4</td>
<td>$-\frac{C + B (1+r)}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $B &gt; 0$ or</td>
<td>if $B &gt; 0$ or</td>
<td>if $B &gt; 0$ or</td>
</tr>
<tr>
<td></td>
<td>if $C &lt; \frac{B}{(1+r)}$</td>
<td>if $C &lt; \frac{B}{(1+r)}$</td>
<td>$-\frac{A}{(1+r)} + \frac{\partial X_{1,0}}{\partial \kappa}</td>
</tr>
</tbody>
</table>

Note: See table 3 for exact expressions for second derivatives: A, B, C, and D.

strong that even the harvest of the second species, the species which is less valuable, is increased which in turn implies that the total yield from the forest is increased. Such an outcome may not be likely in practice because the change in marginal costs associated with the stock effect must be greater than the direct effect of the tax on increasing marginal cost.

Nevertheless, the net effect is an empirical question. This is in contrast to Ovaskainen (1992) who found that a gross yield tax for temperate forests under even-aged management (i.e. clear cutting) is never neutral because the gross yield tax ‘reduces the marginal return on investment but leaves their marginal costs unaffected’ (p. 32) and that short-run timber supply is reduced through increases in both short-run harvests and reforestation incentives.

21 The welfare cost of the tax is still positive even if the total yield is increased. That is, a volume of timber greater than the socially efficient amount will be harvested if there are no externalities, this tax is imposed and the yield is increased.

22 One referee noted that Ovaskainen’s analysis is not restricted to even-aged management. We concur and cite only one specific instance in the text.
4.2. Ad valorem output taxes

Now consider the case of ad valorem taxes, which are a function of the value of the output. The tax is equal to $\alpha P_{t,j}$ where $\alpha$ is the tax rate, which is assumed to be time invariant.

The Lagrangian for this problem is

$$\mathcal{L} = (1 - \alpha) P_{0,0} X_{0,0} - C_{0}(X_{0,0}, A_{0,0}, A_{0,1})$$

$$+ \frac{(1 - \alpha)(P_{1,0} X_{1,0} + P_{1,1} X_{1,1})}{1 + r}$$

$$- \frac{C_{1}(X_{1,0} + X_{1,1}, A_{0,0} - X_{0,0} - G_{0,0}(X_{0,0}), A_{0,1} + G_{0,1}(X_{0,0}))}{1 + r}$$

$$+ \lambda_{0}(A_{0,1} - X_{0,0} - G_{0,0}(X_{0,0}) - X_{1,0})$$

(20)

and the first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial X_{0,0}} = (1 - \alpha) P_{0,1} - \frac{\partial C_{0}}{\partial X_{0,0}} + \frac{\frac{\partial C_{1}}{\partial A_{0,0}} (1 + \frac{\partial G_{0,0}}{\partial X_{0,0}}) + \frac{\partial C_{1}}{\partial A_{0,1}} \frac{\partial G_{0,1}}{\partial X_{0,0}}}{1 + r}$$

$$= 0$$

$$\frac{\partial \mathcal{L}}{\partial X_{1,0}} = (1 - \alpha) P_{1,0} - \frac{\partial C_{1}}{\partial X_{1,0}} + \lambda_{0} (1 + \frac{\partial G_{0,0}}{\partial X_{0,0}}) = 0$$

(21)

$$\frac{\partial \mathcal{L}}{\partial X_{1,1}} = (1 - \alpha) P_{1,1} - \frac{\partial C_{1}}{\partial X_{1,1}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{0}} = A_{0,0} - X_{0,0} - G_{0,0}(X_{0,0}) - X_{1,0} = 0$$

Similar to the fixed per unit tax situation, two effects are present when this tax is introduced. First, there is an incentive to reallocate the harvest away from the periods with a higher marginal tax in present value terms. The period with the higher marginal tax is the first period in the present situation if the price this year is greater than the ‘discounted price difference’ in the future, a situation one might expect but for which the opposite situation cannot be ruled out in theory.\(^2^3\) This result is illustrated in cases 1 and 2 and reported on table 2. A decrease (increase) in the harvest of the best species this year will lead to an increase (decrease) in the harvest of the best species next year, accompanied by a decrease (increase) in the harvest of the lower-quality species next year, other things equal. Thus, the tax savings this year will be compensated, in part, by a tax increase next year of the difference between the species’ prices in the future. The reallocation effect

\(^2^3\) Note that the result would depend on the difference between discounted prices when one species is harvested.
Table 2. Ad valorem tax

<table>
<thead>
<tr>
<th>Case</th>
<th>$\frac{\partial X_{0,0}}{\partial \alpha}$</th>
<th>$\frac{\partial X_{1,0}}{\partial \alpha}$</th>
<th>$\frac{\partial X_{1,1}}{\partial \alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-C \left( \frac{P_{0,0} - (P_{1,0} - P_{1,1})}{1 + r} \right)$</td>
<td>$C \left( \frac{P_{0,0} - (P_{1,0} - P_{1,1})}{1 + r} \right)$</td>
<td>$-AP_{1,1}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $P_{0,0} &gt; \frac{(P_{1,0} - P_{1,1})}{1 + r}$</td>
<td>if $P_{0,0} &gt; \frac{(P_{1,0} - P_{1,1})}{1 + r}$</td>
<td>if $\frac{\partial X_{0,0}}{\partial \alpha} &lt; 0$</td>
</tr>
<tr>
<td>2</td>
<td>$-C \left( \frac{P_{0,0} - D(P_{1,0} - P_{1,1})}{1 + r} \right)$</td>
<td>$DC \left( \frac{P_{0,0} - D(P_{1,0} - P_{1,1})}{1 + r} \right)$</td>
<td>$-AP_{1,2}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $P_{0,0} &gt; \frac{D(P_{1,1} - P_{1,2})}{1 + r}$</td>
<td>if $P_{0,0} &gt; \frac{D(P_{1,1} - P_{1,2})}{1 + r}$</td>
<td>if $\frac{\partial X_{1,0}}{\partial \alpha} &lt; 0$ and $B &gt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $P_{0,0} &gt; \frac{(P_{1,0} - P_{1,1})}{1 + r}$ and $B &gt; 0$</td>
<td>if $P_{0,0} &gt; \frac{(P_{1,0} - P_{1,1})}{1 + r}$ and $B &gt; 0$</td>
<td>if $\frac{\partial X_{1,0}}{\partial \alpha} &lt; 0$ and $B &gt; 0$</td>
</tr>
<tr>
<td>3</td>
<td>$-C \left( \frac{P_{0,10} - (P_{1,0} - P_{1,1})}{1 + r} \right) + B \left( \frac{P_{1,1}}{1 + r} \right)$</td>
<td>$\frac{-C \left( \frac{P_{0,10} - (P_{1,0} - P_{1,1})}{1 + r} \right) + B \left( \frac{P_{1,1}}{1 + r} \right)}{</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $P_{0,0} &gt; \frac{(P_{1,0} - P_{1,1})}{1 + r}$ and $B &gt; 0$</td>
<td>if $P_{0,0} &gt; \frac{(P_{1,0} - P_{1,1})}{1 + r}$ and $B &gt; 0$</td>
<td>if $\frac{\partial X_{1,1}}{\partial \alpha} &gt; 0$ and $B &gt; 0$ and if $P_{0,0} &gt; \frac{D(P_{1,0} - P_{1,1})}{1 + r}$</td>
</tr>
<tr>
<td>4</td>
<td>$-C \left( \frac{P_{0,0} + D(P_{1,0} - P_{1,0})}{1 + r} \right) + B \left( \frac{P_{1,1}}{1 + r} \right)$</td>
<td>$D \left( \frac{C \left( \frac{P_{0,0} + D(P_{1,0} - P_{1,0})}{1 + r} \right) + B \left( \frac{P_{1,1}}{1 + r} \right)}{</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{</td>
<td>H</td>
<td>} &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>if $P_{0,0} &gt; \frac{D(P_{1,0} - P_{1,1})}{1 + r}$ and $B &gt; 0$</td>
<td>if $P_{0,0} &gt; \frac{D(P_{1,0} - P_{1,1})}{1 + r}$ and $B &gt; 0$</td>
<td>if $\frac{\partial X_{1,1}}{\partial \alpha} &gt; 0$ and $B &gt; 0$ and if $P_{0,0} &gt; \frac{D(P_{1,0} - P_{1,1})}{1 + r}$</td>
</tr>
</tbody>
</table>

Note: See table 3 for exact expressions of second derivatives A, B, C, and D.
can be complementary or offset, in part, if growth effects are present (see cases 2 and 4).

An output effect is also present when this tax is imposed. The future harvest of the lower-quality species is reduced in cases 1 and 2 because the tax will increase marginal harvest costs (or decrease marginal revenue) and reduce marginal profit. Stock effects can once again offset, or complement, this incentive, depending on the indirect effect of the change in the current harvest on future marginal costs via the stock effect. These results are similar to the analyses of single-species even-aged management models for temperate forests. For example, Chang’s (1981) analysis suggested that ad valorem taxes reduce rotation length and optimal planting densities. Similarly, Ovaskainen (1992) concluded that ad valorem property taxes unambiguously increases short-run timber supply through increased timber harvesting but that the impact on management intensity is ambiguous and long-term supply negative.

5. Summary
A model of concessionaire behavior under selective harvesting systems in forests characterized by multi-species and age-class distributions has been developed in this paper. Such conditions might prevail in natural tropical forests. The interaction of stock and growth effects (including potential species elimination) is included so that the species quality–quantity profile of the harvesting strategy through time might be analyzed. A number of issues may be addressed with this model, including but not limited to:

(a) the determination of economically recoverable harvests,
(b) the interactive effects of harvesting and growth,
(c) the effects of differences in the time profile of prices and costs on the harvest strategy, and
(d) when to cease operations.

Three potentially significant elements in the concessionaire’s decision problem which have been incorporated are:

(a) the nature of the joint cost function in the multi-species environment,
(b) the degree to which stock levels affect costs, and
(c) how species growth may be affected by the stock level of that, or other, species.

An understanding of how these elements interact with traditional economic variables will enhance the development of a more rational forest policy. These policies might vary by forest because conditions vary across forest types and countries.

24 Profits tax, in the sense of a tax on pure profits, will have no effect given the structure of the problem and thus results for this tax are not reported here. However, this does not necessarily imply that the profits tax as applied will have no effect on incentives. The effect of the profits tax on investment and related decisions could affect the harvest.
The Hessian composed of the second-order conditions for this problem under consideration is:

\[
H = \begin{vmatrix}
A & B & B & D \\
B & C & C & D \\
B & C & C & 0 \\
D & D & 0 & 0
\end{vmatrix}
\]

where:

\[
A = -\frac{\partial^2 C_0}{\partial X^2_{0,0}} + \frac{1}{1 + r} \left\{ \frac{\partial^2 C_1}{\partial A_{1,0} \partial X_{1,1}} \left( 1 + \frac{\partial G_{0,0}}{\partial X_{0,0}} \right)^2 - \frac{\partial^2 C_1}{\partial A_{1,1} \partial X_{0,0}} \left( \frac{\partial G_{0,1}}{\partial X_{0,0}} \right)^2 \right\} - \lambda_0 \frac{\partial^2 G_{0,0}}{\partial X^2_{0,0}} < 0
\]

\[
B = -\frac{1}{1 + r} \left\{ \frac{\partial^2 C_1}{\partial A_{1,0} \partial X_{1,1}} \left( 1 + \frac{\partial G_{0,0}}{\partial X_{0,0}} \right) - \frac{\partial^2 C_1}{\partial A_{1,1} \partial X_{1,1}} \left( \frac{\partial^2 G_{0,1}}{\partial X_{1,1} \partial X_{0,0}} \right) \right\} < 0
\]

\[
C = \frac{1}{1 + r} \left\{ \frac{\partial^2 C_1}{\partial X_{1,1}} \right\} < 0
\]

\[
D = \frac{1}{1 + r} \left\{ 1 + \frac{\partial G_{0,0}}{\partial X_{0,0}} \right\} > 0
\]

The letters for each relevant term are reported in tables 1 and 2.

The model was applied to taxation to illustrate its potential usefulness and to study the controversy over the extent to which various concession and tax systems induce ‘high-grading’ of tropical forests (see Ghani et al., 1992; Hyde and Newman, 1991; Repetto and Gillis, 1988; Vincent, 1990). The extent of high grading and related allocative effects will depend on the three factors noted above. For instance, the presence of stock effects may affect the standard results of both inter-temporal reallocation and recovery. The need for empirical work in an inter-disciplinary environment is one implication of this paper. As noted, an awareness of the interaction between economic and environmental variables is needed to understand the full range of potential effects in particular situations.

References
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