Scale Economies and Inefficiency of U.S. Dairy Farms

Roberto Mosheim and C.A. Knox Lovell

This study employs data drawn from the 2000 Agricultural Resource Management Survey, a U.S. Department of Agriculture-sponsored farmers' survey. The article estimates returns to scale relationships across dairy farms of different sizes and across different regions, incorporating variables hypothesized to influence farm performance. Results point to significant scale economies in U.S. dairy farms and underscore the importance of taking account of inefficiency when estimating scale economies. Contrary to previous research, the preferred cost function specification does not show a region of decreasing returns to scale. This finding helps explain why the average size of dairy farms has been increasing.

Key words: dairy farms, inefficiency, scale economies, shadow cost function.

Structural changes taking place on dairy farms are an important policy concern in the United States and elsewhere. Dairy farm herd sizes and cow productivity have exhibited significant increases during the last twenty years. The demand for dairy products has only grown slowly, however, leading to an imbalance between supply and demand and a consequent reduction in the number of dairy farms. Despite the general trend of increasing farm size, a very heterogeneous pattern of structural change appears across regions that relates to costs of production, technology, weather, and geography among other factors (MacDonald et al. 2007; Wolf 2003). Blayney and Normile (2004) contend that the main drivers of these changes are a mixture of technological, efficiency, and scale changes, and they note a lack of empirical evidence on such key technology indicators as scale economies and their variation across geographical areas in the United States, specifically. This research seeks to fill that gap.

This study uses a data set for 619 dairy farms drawn from the 2000 Agricultural Resource Management Survey (ARMS2000) (USDA-ERS 2000),[1] a national survey of U.S. dairy producers, to estimate scale economies in such a way as not to confuse them with economic inefficiency or other influences in the cost–output relationship. It builds on analyses of scale economies (e.g., Alvarez and Arias 2003; Kumbhakar 1993; Moschini 1988; Tauer and Mishra 2006) and estimated efficiency in the dairy sector (e.g., Maietta 2000; Stefanou and Saxena 1988).

Costs can decrease when efficiency improves due, for example, to better training, more time on the farm, or better technology that lowers costs of production. Our study builds on earlier ones that have tackled the question of performance by either using nonfrontier approaches to modeling inefficiency (e.g., Maietta 2000; Stefanou and Saxena 1988) or frontier using various approaches. Examples include Tauer and Mishra's (2006) single-equation cross-section; Alvarez and Arias' (2003) panel data single-equation models with time-varying and cross-sectional variation in inefficiency; and system approaches (where first-order conditions and economic objectives are employed) as, for example, in Kumbhakar (1993) and Moschini (1988). The latter employed a nonfrontier framework to tackle economies of

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[1] Information on accessing ARMS can be obtained from http://www.ers.usda.gov/Briefing/ARMS/Access.htm.

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scale, one of our main concerns in this study.

Several earlier studies have analyzed scale economies and efficiency in dairy farms while investigating key variables that we also find important. Moschini (1988) found increasing returns to scale for Canada's Ontario dairy producers, with large levels of milk output but decreasing returns for the very largest ones. Factors affecting scale economies in that study were location, debt/equity ratio, milking techniques, building quality, cow type, education, and horsepower of the largest tractor. Tauer and Mishra (2006) also found increasing returns to scale, but they were quite small once accounting for inefficiency. In their study the higher cost of production on most small farms was caused by inefficiency rather than technology. They represented technology by number of cows and state dummies rather than by a production, cost, or profit function, however. Kumbhakar (1993) found that large farms have exploited short-run returns to scale to a greater extent than medium or small dairy farms. His study also noted that large farms were more technical, allocative, and scale efficient than the others. In addition, he found that off-farm income is negatively related to performance but least so for large farms, and the farmer's level of education contributes more for medium and large farms than for small ones. Unlike in our present study, he did not precisely disentangle allocative and technical inefficiency because of inconsistent distributional assumptions. Alvarez and Arias (2003, p. 141) found a U-shaped average cost curve and contended that "even if there are observed diseconomies of size, a large enough increase in managerial ability could outweigh the rising part of the average cost curve." They did not control for variation in allocative inefficiency. Using panel data, Maietta (2000) estimated long-run returns to scale as decreasing on average, and without providing specific sources for investigating allocative or technical inefficiency, she found a decrease in allocative inefficiency and an increase in technical inefficiency for larger farms. Finally, Stefanou and Saxena (1988) also contributed to the research on some key variables by finding that education and experience matter quite a bit for dairy farm profit efficiency. They employed a shadow profit system of equations that did take into account and explain both allocative and technical inefficiency variation.

Following and expanding on this literature, in this article we analyze farm-level data, allowing for variation in cost efficiency and incorporating variables commonly thought to influence farm performance. Our estimates are not plagued by the distributional inconsistencies and misspecifications of inefficiency faced by previous studies that have examined scale economies in dairy farming.

Structural Change and Scale Economies in U.S. Dairy Farming

The transformation of dairy operations is usually analyzed through changes in location, production system, herd size, total and per-cow production, and organizational shifts through time (see, e.g., Blayney and Normile 2004; MacDonald et al. 2007). Here, we focus instead on structural changes that have occurred during the last twenty years. Figure 1 shows the inverse relationship between the number of cows in the national herd and production of milk per cow. Given that demand growth for dairy products has not kept pace with the increase in milk production per cow, the national herd has declined 11% from 1987 to 2007. During this same period, milk production per cow increased 47%. These production trends have led to total milk production increasing by 30% (U.S. Department of Agriculture, National Agricultural Statistical Service [USDA-NASS] 2008).

Simple correlation analysis provides some evidence that scale economies are important determinants of productivity. There is a wide variation in milk produced per cow across states. The correlation between milk produced per cow and the number of milk cows per operation across dairy farms is strong and positive, indicating a potential role for scale economies in determining productivity. A simple correlation analysis using publicly available USDA-NASS data at the state level shows a correlation of 0.436 between milk produced per cow and cows per establishment in 1987 and 0.564 for 2007 (USDA-NASS 2008).

Figure 2 shows further evidence of scale economies. From 1998 to 2007, the number of dairy operations in the United States decreased by 39%. This decline, however, was not symmetrical across farm sizes, resulting in fewer small and a considerable increase of large dairy farms. The cow inventory of dairy farms with herd sizes between one and forty-nine declined from 14.1% to 7.4% of the total, and that of operations with 50–199 milk cows decreased from 43.6% to 28.8%. In contrast, dairy operations with 200–1,999 head increased their cow inventory from 35%

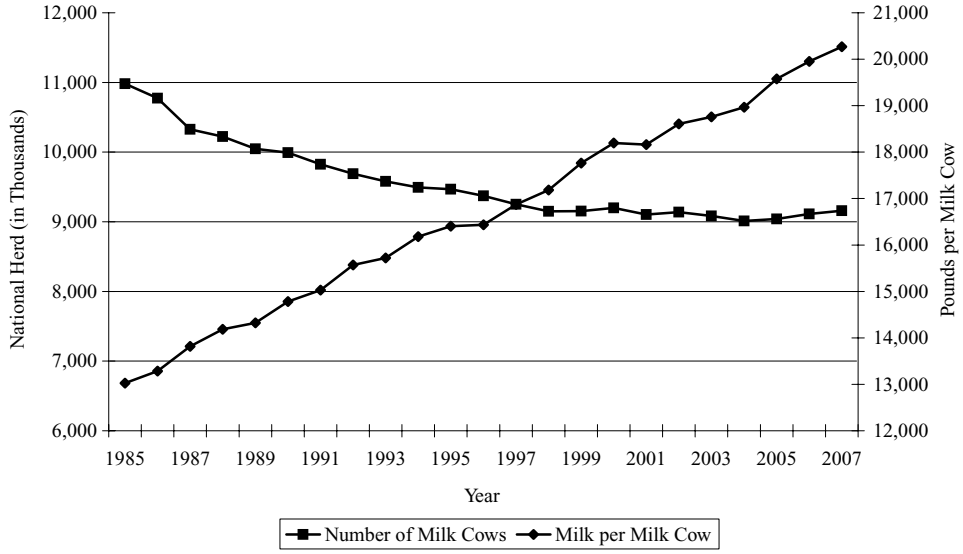

Figure 1. National herd and milk per cow in the United States

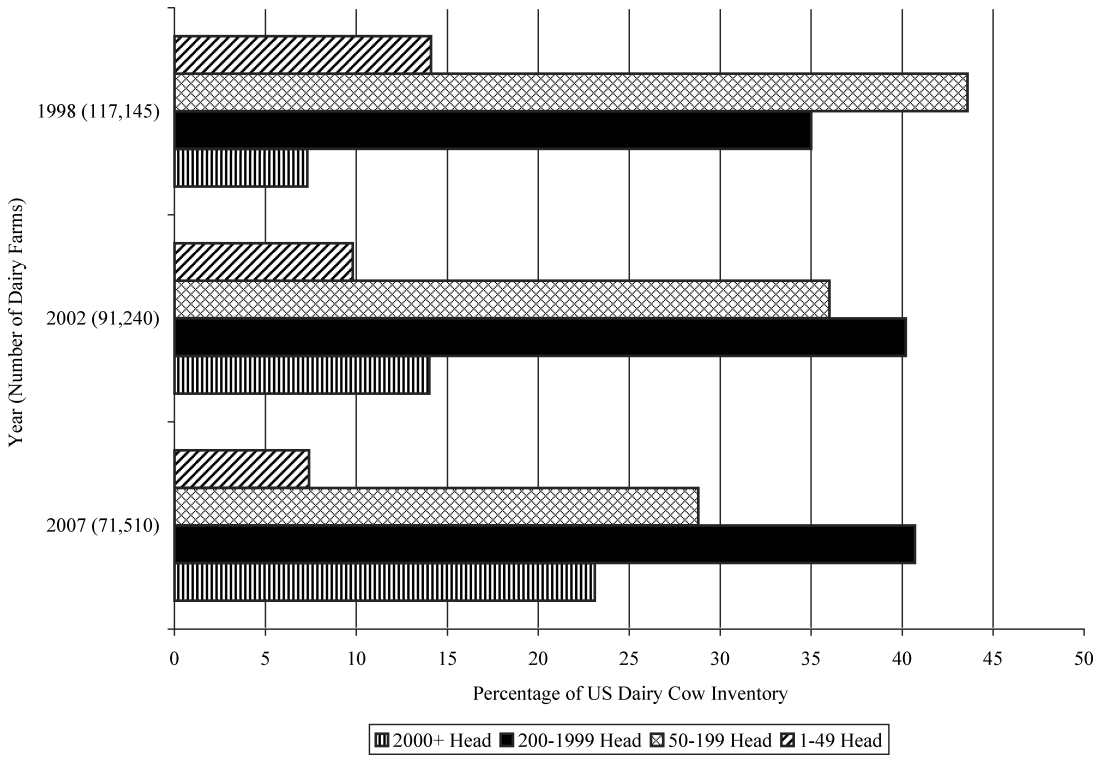

Figure 2. Dairy farm size distribution in the United States

to 40.17%, and operations with 2,000 or more head, from 7.3% to 23.1% (USDA-NASS 2008).

The change in size structure has not affected all regions of the country equally. A sense of the regional shifts that have occurred lately can be grasped by looking at the ranking of milk-producing states in 1987 and 2007. In 1987, the ten largest milk-producing states were, in order: Wisconsin, California, New York, Minnesota, Pennsylvania, Michigan, Ohio, Texas, Iowa, and Washington; in 2007, they were: California, Wisconsin, New York, Idaho, Pennsylvania, Minnesota, Michigan,

Texas, New Mexico, and Washington. In 1987, the top ten states produced 68% of the national milk supply, while in 2007 the top ten produced 73% (USDA-NASS 2008).

These regional changes also imply a shift in types of production systems.[2] Many operations in states like California, Idaho, and New Mexico, for example, have seen the emergence of so-called dry-lot systems (i.e., they rely on purchased feed). These emerge when low capital requirements and large herd sizes that enable exploitation of scale economies lower cost per unit of output. In 1987, in California, for example, the average number of milk cows per operation was 226, while in Idaho and New Mexico, it was fifty-three and fifty head, respectively. By contrast in 2007, California had an average of 824 cows per operation, Idaho had 684, and New Mexico had 814 cows. More traditional states increased their average size of operation but by a much smaller percentage. In Wisconsin, New York, and Pennsylvania, the average number of cows per operation in 1987 was 49, 57, and 39 cows, respectively; in 2007, it was 87, 101, and 65 cows. These states do not rely so much on purchased feed as on homegrown feed or pasture (USDA-NASS 2008); that is, they use capital differently.

Given the heterogeneity of the changes in dairy operation size across production technologies and regions, the question of the nature of scale economies becomes crucial. According to Chavas (2001), in general the average cost curve for the agricultural sector in developed countries tends to be L-shaped; while scale economies tend to exist for small farms, no strong evidence indicates that diseconomies of scale exist for large farms (i.e., there is a wide range in which scale economies are constant). In addition, Chavas emphasizes the importance of taking into account variables like the shadow value of unpaid labor. Morrison et al. (2004) have also examined the trend toward consolidation in U.S. agriculture generally and have found significant scale and efficiency advantages of large farms. For dairy specifically, Jones (1999) presents a similar picture in which scale economies are exhausted quickly. The variation in dairy operation sizes, moreover, can be explained by a myriad of variables internal and external to the dairy farm such as pecuniary economies, transaction costs, tax policy, regulation, and risk. Wolf (2003) argues that dairy farms in traditional areas such as Wisconsin, New York, and Pennsylvania face higher adjustment costs (because of high sunk costs) than emerging regions, a condition that constrains their growth and adoption of technology.

Shadow Cost Model

In modeling U.S. dairy farming, we have taken three important aspects into consideration. First, dairy farms employ a multioutput production technology. There is no reason to expect that the major types of output of the operation move together in response to price changes, and therefore, aggregation of these outputs is not justifiable. Second, according to Wolf (2003), dairy farming is characterized by fixity of farm assets and a slow farmer response to changes in technology and prices. The result is a quasi-fixed capital input quantity because of a combination of information asymmetries, transportation costs, and investment specificity that points to considerable adjustment costs that are region specific. Lastly, the high variation in the costs of dairy enterprises can be attributed to differences in scale of the operation and of technology employed and the efficiency with which it is applied.

We distinguish two outputs and three variable input prices and quantities as well as a fixed level of capital to model dairy farming sector costs. We identify livestock and crops as separate outputs $y = (y_1, y_2)$.[3] Milk represents 72% of the livestock variable, while crops and livestock account for a share of 20% and 80% of total output, respectively. The vectors $x = (x_1, x_2, x_3; K)$ and $w = (w_1, w_2, w_3)$ represent variable inputs and their prices, where 1, 2, and 3 correspond to labor, energy, and feed, and K represents the fixed level of capital. Because dairy farmers do not have the flexibility to adjust capital to their optimal proportions in the short run, we assume that the farm seeks to minimize variable cost, $\min w'x = VC(y, w, K)$. This function shows the smallest expenditures on variable inputs required to produce y, given the variable input price vector w and the level of fixed capital stock K. The resulting function $VC(y, w, K)$ is nonnegative and homogeneous of degree +1 in w. Given (y, K),

[2] Blayney and Normile (2004) distinguish three production systems: confinement, pasture-based, and dry-lot operations. The first two rely mainly on homegrown feed and the latter on purchased feed.

[3] A three-output model failed because of nearly fixed proportions between milk and other livestock products. Culled milk cows dominate the output of nonmilk livestock products on dairy farms, and in this data set milk and culled cows are highly correlated.

$VC(y, w, K)$ is concave in w, nondecreasing in y and w, and nonincreasing in K (Kumbhakar and Lovell 2000).

In order to model the variation in performance of dairy farms, we could use the traditional error components model:

$$(1) \qquad w'x = E = VC(y, w, K)\exp(v + u).$$

In this representation, the dairy enterprise's actual variable costs or expenditures E are modeled as the sum of the variable cost frontier, a term v that represents random noise, and a term u capturing cost inefficiency. The term u, in turn, represents the combined effect of allocative and technical inefficiency. Allocative inefficiency refers to the failure to combine variable inputs in optimal proportions, and technical inefficiency represents the unsuccessful minimization of variable input usage to produce the enterprise's outputs. Herein, we aim to estimate both of these types of inefficiency.

According to Greene (2008), to identify and measure properly both technical and allocative inefficiency, a cost function together with a system of input demand equations needs to be employed. Despite the fact that the estimation of this system adds degrees of freedom and results in more efficient estimates, it also leads to a difficult dilemma, known as the Greene problem that occurs when using a flexible functional form of the cost function. To illustrate, in a translog cost setting, the model is

$$(2) \qquad \ln(w'x) = \ln E = \ln VC(y, w, K) + v + u \quad \text{and}$$

$$\frac{w_n x_n}{w'x} = S_n = S_n(y, w, K) + v_n + \eta_n \qquad n = 2, \ldots, N$$

where $S_n = \frac{\partial \ln VC}{\partial \ln w}$ follows from Shephard's lemma, and we have deleted one of the share equations. The terms v and v_n represent statistical noise. The introduction of the terms u and η_n converts the variable cost function model from the type originally estimated by Christensen and Greene (1976) to a frontier model. The terms u and η_n capture the increase in variable costs attributable to technical and allocative inefficiency. The usual distributional assumptions of equation (2) are that the error terms v and u are distributed independently of each other and of η_n. It follows, on the one hand, that these assumptions are only consistent if assuming allocative efficiency. As a result, estimating equation (2) provides no more information than estimating (1) would. In addition, the estimates of u are biased by the inappropriate allocative efficiency assumption. On the other hand, if η_n represents technical and allocative inefficiency, it cannot be distributed independently of u, because allocative inefficiency raises costs.

Given the difficulty in properly estimating and decomposing cost inefficiency using error components, we resort to the shadow cost function model,[4] in which the formulation of technical and allocative inefficiency is parametric. Hence, we assume that the dairy farmer employs a production function of the form $f(\phi x, K) = y$. The enterprise is technically efficient if it is not possible to radially reduce input usage x by lowering the magnitude of parameter ϕ, the maximum contraction being where ϕ is equal to one. In addition, the enterprise is allocatively inefficient if at ϕx the marginal rate of substitution $\frac{f_i}{f_j}$ is not equal to the market input price ratio $\frac{w_i}{w_j}$. The farmer in this case, nevertheless, is allocatively efficient relative to a shadow price ratio $\theta_{ij}\frac{w_i}{w_j} = \frac{w_i^*}{w_j^*}$, a relationship that the dairy farm uses instead of relative market prices to minimize input usage. The shadow price ratio takes account of all of the production constraints faced by the farmer but that are unknown to the observer. The enterprise, then, is allocatively efficient if the parameters θ_{ij} equal unity.

Following Kumbhakar and Lovell (2000), the unobserved shadow variable cost function for the dairy enterprise is

$$(3) \qquad VC\left(y, \frac{\theta w}{\phi}, K\right) = \min_{\phi x}\left\{\left(\frac{\theta w}{\phi}\right)^T (\phi x) : f(\phi x, K) = y\right\} = \frac{1}{\phi} VC(y, \theta w, K)$$

[4] This estimation strategy has been successfully used to address the problem (known as the Greene problem) of estimating and decomposing allocative and technical inefficiency in a translog system-of-equations (Fried, Lovell, and Schmidt 2008). This approach follows Kumbhakar and Wang (2006), who argue against lumping together allocative and technical efficiency in the estimation of cost frontiers, which biases both the cost function parameters and economic inefficiency and underestimates returns to scale and overestimate input price elasticities. Nevertheless, it is important to point out that their results as to returns to scale, for example, are based on a Monte Carlo study (i.e., a controlled experiment with a known data generating process) and not a detailed theoretical argument concerning the direction scale economies bias. We estimate this bias econometrically.

and applying Shephard's lemma, the variable input share equations are

$$(4)\qquad S_n = \frac{\partial \ln VC}{\partial \ln \theta w_n} = \frac{S_n(y, \theta w, K) \times (\theta_{n2})^{-1}}{\sum_{k=1}^{3} [S_k(y, \theta w, K) \times (\theta_{k2})^{-1}]} \qquad n = 1, 2, 3.$$

In the above formulation, not all of the $\theta_{ij}s$ can be identified because the variable cost function is linearly homogeneous in shadow variable input prices. We chose the price distortion for energy as a numeraire. We follow Atkinson and Dorfman (2006) and assume that the market for energy is at equilibrium at all times, and therefore, its market and shadow prices are the same. Therefore, we can assume that $\theta_{22} = 1$. This assumption allows us to calculate the cost savings of achieving allocative efficiency.

The shadow price vector, then, is $w^* = (\theta_{12} w_1, w_2, \theta_{32} w_3)$. If the input price vector w is used instead of w^* when estimating the shadow cost function, and the variables θ_{n2} are not equal to unity, the shadow cost function is misspecified. The parameters θ_{n2} represent the degree of departure from optimal proportions relative to the second input. If $\theta_{n2} > 1$, then x_n is underutilized; if $\theta_{n2} < 1$, then x_n is overutilized.

The technical and allocative inefficiency terms can be made functions of variables hypothesized to affect dairy farm performance. We employ the following variables, used in the literature, of input-oriented technical inefficiency and of allocative inefficiency: z_1, whether or not the enterprise is located in the traditional dairy area;[5] z_2, the farm's proportion of purchased feed to total feed; z_3, a dummy variable indicating a farmer operator with an educational attainment of college or more; and z_4, the farmer operator's years of management experience.

Input-oriented technical inefficiency is modeled as

$$(5)\qquad \ln \phi = \phi_1 z_1 + \phi_2 z_2 + \phi_3 z_3 + \phi_4 z_4.$$

If every ϕ_j is zero, then $\phi = 1$, and the farm is technically efficient. If $\phi_i > 1$, then the variable raises technical inefficiency. If $\phi_i < 1$, then the effect is positively related with technical efficiency. We hypothesize that the same variables we employed above affect the distortion factors according to the following positive exponential functional form

$$(6)\qquad \begin{aligned} \theta_{12} &= \exp(\theta_{120} + \theta_{121} z_1 + \theta_{122} z_2 + \theta_{123} z_3 + \theta_{124} z_4), \\ \theta_{32} &= \exp(\theta_{320} + \theta_{321} z_1 + \theta_{322} z_2 + \theta_{323} z_3 + \theta_{324} z_4), \quad \text{and} \\ \theta_{22} &= 1. \end{aligned}$$

If all θ_{12i} are equal to zero, then $\theta_{12} = 1$, and variable inputs 1 and 2 are allocated efficiently. If all θ_{32i} are zero, $\theta_{32} = 1$, and variable inputs 3 and 2 are allocated efficiently. A negative coefficient inside the exponential expression contributes to an overutilization of the input relative to input 2, and vice versa.

We estimate equations (3)–(6)[6] using the following translog[7] specification:

$$(7)\qquad \begin{aligned} \ln VC &= \beta_o + \ln \phi + \sum_{l=1}^{2} \alpha_l \ln y_l + \sum_{j=1}^{3} \beta_j \ln(\theta_{j2} w_j) \\ &+ \gamma_K \ln K + 1/2 \sum_{l=1}^{2} \sum_{m=1}^{2} \alpha_{lm} \ln y_l \ln y_m \\ &+ 1/2 \sum_{n=1}^{3} \sum_{k=1}^{3} \beta_{nk} \ln(\theta_{n2} w_n) \ln(\theta_{k2} w_k) \\ &+ 1/2 \sum_{j=1}^{3} \sum_{l=1}^{2} \mu_{jl} \ln(\theta_{n2} w_n) \ln y_k \\ &+ 1/2 \gamma_{KK} (\ln K)^2 \\ &+ \sum_{l=1}^{2} \eta_l \ln y_l \ln K + \sum_{j=1}^{3} \nu_j \ln(\theta_{j2} w_j) \ln K \\ &+ \ln \left\{ \sum_{n=1}^{3} (\theta_{n2})^{-1} \left[\beta_n + \sum_{k=1}^{3} \beta_{nk} \ln(\theta_{k2} w_k) \right.\right. \\ &\left.\left. + \sum_{k=1}^{2} \mu_{nk} \ln y_k + \nu_n \ln K \right] \right\}. \end{aligned}$$

[5] Traditional dairy states are: CT, DE, IA, IL, IN, MA, MD, ME, MI, MN, MO, NH, NJ, NY, PA, OH, RI, VT, and WI (MacDonald et al. 2007).

[6] This is based on the models presented in Kumbhakar and Lovell (2000) that introduce the shadow cost function approach to efficiency measurement as developed by Lau and Yotopoulos (1971), Yotopolous and Lau (1973), and Atkinson and Halvorsen (1980, 1984).

[7] See Christensen, Jorgenson, and Lau (1973).

In equation (7) symmetry $\beta_{nk} = \beta_{kn}$, $k \neq n$ is imposed as required by Young's theorem. In addition, the variable cost function must be linearly homogeneous with respect to shadow input prices, which requires the following restrictions:

$$(8)\quad \begin{aligned} &\sum_{j=1}^{3} \beta_j = 1, \\ &\sum_{k=1}^{3} \beta_{nk} = 0, n = 1, 2, 3, \\ &\sum_{j=1}^{3} \mu_{jl} = 0, l = 1, 2, \quad \text{and} \\ &\sum_{j=1}^{3} \nu_j = 0. \end{aligned}$$

The shadow cost shares derived from equation (7) are

$$(9)\quad S_n = \frac{(\theta_{n2})^{-1}\left[\beta_n + \sum_{k=1}^{3} \beta_{nk} \ln(\theta_{k2} w_k) + \sum_{k=1}^{2} \mu_{nk} \ln y_k + \nu_n \ln K\right]}{\sum_{j=1}^{3} (\theta_{j2})^{-1}\left[\beta_j + \sum_{k=1}^{3} \beta_{jk} \ln(\theta_{k2} w_k) + \sum_{k=1}^{2} \mu_{jk} \ln y_k + \nu_j \ln K\right]}.$$

The elasticity of shadow variable cost with respect to the output vector and the shadow shares must also be nonnegative, and the Hessian matrix of second-order derivatives of shadow variable costs with respect to shadow prices must be negative semi-definite. We checked these monotonicity and concavity properties of the cost function for each observation using the parameter estimates as well as the mean of the data both assuming and not assuming inefficiency.

Given asset fixity, in order to estimate a measure of scale elasticity (*SCE*) from a variable shadow cost function, we follow Caves, Christensen, and Swanson (1981), Caves, Christensen, and Tretheway (1984), and Garcia and Thomas (2001) and define:

$$(10)\quad SCE(y, w^*, K) = \frac{1 - \partial \ln VC(y, w^*, K)/\partial \ln K}{\sum_{i=1}^{2} (\partial \ln VC(y, w^*, K)/\partial \ln y_i)}.$$

If $SCE(y, w^*, K) > 1$, average variable cost is decreasing in y, and returns to scale are increasing. If $SCE(y, w^*, K) < 1$, average variable cost is increasing in y, and returns to scale are decreasing. $SCE(y, w^*, K) = 1$ signals locally constant returns to scale.

Data Sources and Variable Construction

In order to construct the variables for the shadow variable cost function model (equations (5)–(9)), we obtained most of the raw data from the 2000 Agricultural Resource Management Survey Phase III Version 4 for dairy (ARMS2000) (USDA-ERS 2000). ARMS is a nationally representative, annual USDA survey of farms, with a probability-based sample that is stratified within states according to commodity mix and farm sales (Banker, Green, and Korb 2001). Because it is a stratified sample, farms have varying sample weights (e.g., larger farms are sampled at higher rates and hence have smaller weights). The survey includes several questionnaire versions, two of which pertain to this study. Version 4 was directed to dairy farms, with specific questions related to the farm's dairy enterprise. Most of the data for our analysis are drawn from this questionnaire, which gathered information from 848 individual dairy farms. We derived some supplemental information from version 1, which was directed to a broad cross-section of all types of farms.

We applied a set of rules to "clean up" the data, resulting in a data set composed of 619 dairy farms. Inconsistencies in production and marketing, farmer refusal to provide information, missing variables, negative operating profits, or suspiciously large or small entries were used as criteria for elimination. The sample's structure before and after cleaning is roughly comparable. In the original raw data, 23.59% of the observations covered herd sizes of milk cows 1–49, 55.43% for herd sizes 50–199, 17.21% for herd sizes 200–999, and 3.78% for herd sizes of more than 1,000. After cleaning, the sample was for herd sizes: 1–49, 21.81%; for 50–199, 57.19%; for 200–999, 16.96%; and for more than 1,000, 4.03%.

Variable cost *VC*, as defined in the previous section, is calculated as total expenditure for labor, feed, and energy. We constructed all of the price indexes for the outputs and variable inputs using the multilateral Tornqvist price index proposed by Caves, Christensen, and Diewert (1982, p. 78). This procedure

compares the price faced by firm k to the geometric mean of prices:

$$(11) \quad \ln P_k^{CCD} = \frac{1}{2}\sum_{i=1}^{N}(\omega_{ik} + \bar{\omega}_i)(\ln p_{ik} - \overline{\ln p_i})$$

where $k = 1, 2, \ldots, N$, are the number of firms

$i = 1, 2, \ldots, M$, are the number of commodities

$\omega_{ik} = \frac{p_{ik}q_{ik}}{\sum_{i=1}^{M} p_{ik}q_{ik}}$ is the value of the ith commodity for the kth firm, and

$$\bar{\omega}_i = \frac{1}{N}\sum_{k=1}^{N}\omega_{ik} \text{ and } \overline{\ln p_i} = \frac{1}{N}\sum_{k=1}^{N}\ln p_{ik}.$$

Thus, we constructed an implicit livestock quantity index y_1 by dividing total livestock revenues by a price index p_1, using price available in nonpublished form from USDA's Economic Research Service (ERS)[8] for the fifteen livestock and livestock commodities identified in ARMS2000. In a similar fashion, we constructed a crop quantity index y_2 for the thirty-one crop commodities identified in the ARMS2000 by using prices available from NASS and ERS and by deflating total crop revenue by a Tornqvist multilateral price index. ARMS2000 did not collect all prices and quantities for all of the commodities used to construct the Tornqvist indexes. Where missing, we used state averages or internal ERS data to separate the revenues of a particular commodity into price and quantity. Hence, ARMS2000 data allow us to calculate either implicit or direct quantity indexes. The choice between Tornqvist direct or implicit quantity indexes presents a problem because the index fails the factor reversal test as quantity indexes defined directly result in different magnitudes than those defined implicitly. We follow Allen and Diewert's (1981) recommendation of using the implicit quantity index if the individual quantity ratios exhibit more variability than the individual prices ratios. Price and quantity ratios are defined as p_{mi}/p_{mj} and q_{mi}/q_{mj}, respectively, where $M = 27$ (the total number of crop and livestock commodities that are produced by two dairy farms or more). We then calculated the standard deviation of a total of 1,243,646 price and quantity ratios to decide which type of index to use. We found the standard deviation of the output ratios to be 17.39 and that of the price ratios 1.03. The use of implicit quantity indexes is thus justified. The derivation of the labor prices and quantities was a great deal more challenging because the necessary information is not directly available from ARMS2000. We generated a labor price index w_1 by using the cost of unpaid and paid labor for the farm operator, spouse, and full- and part-time workers. Because ARMS2000 Version 4 does not distinguish between earned and unearned income, the compensation of the operator, spouse, and other family members was calculated as the marginal increase in total income from an extra hour of work controlling for variables such as location, assets, and education.

Obtaining this marginal increase took several steps. First, we estimated a log-linear regression with the logarithm of off-farm income as a dependent variable and variables representing operator and spouse off-farm hours worked as independent variables, as well as others to be described subsequently. We also used the squares and interactions of these two variables. We further employed characteristics such as unemployment rates for the U.S. commuting zones (CZ), as defined by ERS (Tolbert and Sizer 1996), where the dairy farm is located. We compiled information for the delineated CZs that included manufacturing, services, construction, retail, and wholesale employment. We merged this information with ARMS2000 data on variables such as education and age of operator and nonfarm assets of operator and spouse. We also used state dummy variables and interaction terms. We followed El-Osta and Ahearn (1996) to decide on a broad set of variables to include in the regression and then refined the selection of these variables by specification tests. Second, because we detected heteroscedasticity in these estimations, we employed the Harvey–Godfrey methodology to correct this problem. The adjusted R^2 of the unweighted regression was 0.600 and that of the feasible generalized least squares was 0.870, reflecting a much better fit. We then derived the implicit value of an hour of work for spouse and operator separately. We aggregated the four types of labor (implicit value of operator's and spouse's unpaid labor and paid part and full-time paid employees) using a Tornqvist multilateral index.

[8] Information on accessing ARMS can be obtained from http://www.ers.usda.gov/Briefing/ARMS/Access.htm.

In order to generate a price index for energy w_2, we needed prices and quantities by type of fuel, which is not available in ARMS2000 Version 4 but is in Version 1. We used the latter to estimate fuel demand by energy type for the dairy enterprise employing a seemingly unrelated regression model. We then allowed the parameters of this model to predict energy consumption by type in ARMS2000 Version 4. We also constructed a feed price index w_3 for the twenty-six types of purchased, sixteen types of homegrown, and the five types of pasture feed identified in ARMS2000. Finally, the different types of energy were aggregated using, again, a Tornqvist multilateral index.

Capital stock is only available for the dairy enterprise in ARMS2000 Version 4, but the unit of observation is the whole farm, which may have several crop or livestock enterprises. We approximated the farm-level price of capital stock by a weighted average cost of capital (WACC). In this formulation the cost of capital is a weighted sum of the cost of debt and cost of equity. Cost of debt is the interest rate that farmers actually paid, and we determined the cost of equity using the capital asset pricing model.

Following Coelli et al. (2003):

$$(12) \quad WACC = [(1 - g) \times r_e] + [g \times r_d]$$

where the leverage g is equal to debt/(debt + equity), r_e is the cost of equity capital, and r_d is the cost of debt capital. ARMS2000 Version 4 also provided the data on debt and equity capital. Cost of debt capital is the Moody's seasoned Baa corporate bond yield. We calculated the cost of equity capital through the capital asset pricing model (CAPM):

$$(13) \quad CAPM = r_e = r_f + \beta_e \times (r_m - r_f)$$

where r_f is the return of a three-month U.S. treasury bill minus the rate of inflation for 2000; β_e is the revenue-weighted average of livestock and crop industry betas; and r_m is the compounded annual returns for a ten-year holding period minus the rate of inflation for 1991–2000 (see Kaplan and Peterson 1998 for industry betas and Ibbotson Associates 2005 for inflation and returns).

We assume that the optimal level of capital K can be best calculated as operating profit divided by a rate of depreciation plus the WACC.[9] We assume further that the availability of profits is an important determinant of investment behavior and hence the optimal level of capital. We follow the cash flow model (see, e.g., Berndt 1991, p. 239) and work by Grunfeld (1960), who assumed that the optimal level of capital is a linear function of the firm's value, which we approximate as discounted profits. Elliott (1973) also provides a comparison of this approach with more conventional ones, such as in Jorgenson and Siebert (1968). We tested two models with different measures of capital. In one we used the value of the firm as a proxy for capital and in the second total physical assets used in the farm. The latter measure is statistically insignificant, which we take to mean that it is an inferior representation of capital relative to our current measure. Therefore, the former is our current measure.

Operating profit is equal to crops and livestock sold minus explicit variable costs. We calculated the rate of depreciation as $\delta = \frac{S}{A}\delta_S + \frac{E}{A}\delta_E + \frac{L}{A}\delta_L$, where A is total assets, S is the value of the farm structures and buildings, E is the value of machinery and equipment, and L is the value of land. The rates of depreciation for structures and equipment are, respectively, 0.0237 and 0.1179 (see Jorgenson and Yun 1991, p. 82). We do not depreciate land. This method of estimating capital stock also draws on Bhattacharyya, Parker, and Raffiee (1994), Morrison (1999a), and Coelli et al. (2003). We described the variables hypothesized to influence dairy farm performance at the end of the previous section and present descriptive statistics for these variables in table 1.

Estimation and Results

In this section, we test first for the most appropriate functional form for the shadow cost function, then the four models incorporating various specifications of inefficiency, and next the regularity properties of the preferred specification. We choose the preferred specification by likelihood ratio tests. In the second part of this section, we employ the preferred model, model 4, to calculate various indexes of inefficiency, and we then return to the original four models in order to derive the scale elasticities implied by these various specifications. Finally, we derive marginal and average cost

[9] Capital is calculated to be the residual of revenue less variable cost divided by the opportunity cost of capital, as in Bhattacharyya, Parker, and Raffiee (1994).

Table 1. Weighted Summary Statistics of Variables Used in Shadow Variable Cost Function

Variable and Units	Mean	Std Dev	Min	Max
Variable cost (VC) (in \$)	\$246,327	\$2,839,423	\$30,655	\$6,557,535
Livestock (y_1) quantity index	322,112	6,085,638	24,733	16,359,100
Crops (y_2) quantity index	112,955	4,256,090	0	13,235,647
Labor (w_1) price index	\$1.134	9.000	0.249	18.336
Energy (w_2) price index	1.032	1.541	0.723	1.488
Feed (w_3) price index	1.004	1.401	0.542	1.706
Capital (K) in \$	\$926,160	\$10,556,362	\$66,061	\$27,391,187
Variables hypothesized to influence dairy farm performance (Technical ϕ, allocative θ efficiency)				
Traditional dairy (z_1)	0.275	3.832	0	1
Purchased feed proportion (z_2)	0.432	2.241	0	1
Experience (z_3)	26	111	1	74
Education (z_4)	0.097	2.539	0	1

indexes for model 4 under various inefficiency restrictions.

Specification of the Variable Shadow Cost Function[10]

In order to test the appropriate specification of the variable cost function, we estimated the econometric model consisting of the shadow variable cost function, share equations, distortion function, and homogeneity restrictions—that is, equations (7), (9), (5), (6), and (8), respectively. We dropped one share equation (for energy) because otherwise they would sum to unity. A symmetric error term is appended to equations (7) and (9). Because the errors of these equations are correlated, we estimated the model by nonlinear iterated seemingly unrelated regression techniques. The resulting estimates approximate maximum likelihood estimates when they converge.

Table 2 reports the hypothesis tests conducted while estimating various specifications of the shadow cost model. Section A presents specification test results for models 1, 2, 3, and 4. Model 1 estimates a variable cost function without the technical and allocative inefficiency terms or the explanatory variables z. Model 2 includes the distortions θ but no ϕ or inefficiency explanatory variables z. Model 3 integrates θ and ϕ, the latter as a function of the z variables. Model 4 specifies both allocative and technical inefficiency as functions of the explanatory variables.[11] Hypotheses A(1), A(2), and A(3) test the restrictions on model 4 implied by models 1, 2, and 3. These hypotheses are soundly rejected, and hence, model 4 is preferred.

Table 3 shows parameter estimates for model 4. The first-order parameters can be interpreted as elasticities because we divided each observed variable by its sample mean. Regarding technical inefficiency, a positive or negative sign for a parameter ϕ_j indicates an increase or decrease, respectively, in variable costs related to the magnitude of the factor affecting farm performance. A positive sign for a given allocative inefficiency parameter θ_{i2j} indicates that the variable input is underutilized relative to energy, or vice versa. The strength of these results shows how flexible and appropriate our model is for analyzing the dairy farm sector. Unlike what usually happens with such a model, almost all parameters are significant at the 1% level, and the share equations are highly significant.

In general, we can distinguish a weak test of regularity conditions from a strong test of regularity conditions. For example, Chavas (2008) reports monotonicity and curvature test results only at the sample mean, a weak test that can be misleading. Therefore, we report strong test results, not just at the sample mean but also at every data point, for both of the cases where efficiency and inefficiency are assumed. We find strong support for most theoretical properties,

[10] We pay close attention to how we specify the variable cost function so as not to confuse inefficiency with scale economies.

[11] The general forms of the variable cost functions for models 1, 2, 3, and 4 are, respectively, $VC(y, w, K)$, $VC(y, \theta w, K)$, $VC(y, \theta w, K, \phi(z))$, and $VC(y, \theta(z)w, K, \varphi(z))$.

Table 2. Shadow Variable Cost Function Hypotheses Tests

Hypothesis	LR Test Statistic[a]	P-value
A. Model choice (model 4 against models 1, 2, 3, respectively)		
(1) $H_o : \phi_1 = \phi_2 = \phi_3 = \phi_4 = \theta_{120} = \theta_{320} = \theta_{121} = \theta_{321} = \theta_{122} = \theta_{322} = \theta_{123} = \theta_{323} = \theta_{124} = \theta_{324} = 0$	108.4	<0.0001
(2) $H_o : \phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$	58.3	<0.0001
(3) $H_o : \theta_{121} = \theta_{321} = \theta_{122} = \theta_{322} = \theta_{123} = \theta_{323} = \theta_{124} = \theta_{324} = 0$	15.93	0.04
B. Technology specification tests for model 4		
(1) $H_o : \alpha_1 = 0$	175.5	<0.0001
(2) $H_o : \alpha_2 = 0$	24.9	<0.0001
(3) $H_o : \beta_1 = 0$	145.7	<0.0001
(4) $H_o : \beta_2 = 0$	18.4	<0.0001
(5) $H_o : \beta_3 = 0$	36.0	<0.0001
(6) $H_o : \gamma_K = 0$	6.6	0.001
(7) H_o : Cobb–Douglas	1,396.1	<0.0001
(8) H_o : Homothetic	203.9	<0.0001
C. Technical inefficiency specification tests for model 4		
(1) $H_o : \phi_1 = \phi_2 = \phi_3 = \phi_4 = 0$ (overall)	58.3	<0.0001
(2) $H_o : \phi_1 = 0$ (traditional)	2.4	0.1
(3) $H_o : \phi_2 = 0$ (purchased feed proportion)	30.0	<0.0001
(4) $H_o : \phi_3 = 0$ (education)	0.0	1.0
(5) $H_o : \phi_4 = 0$ (experience)	28.9	<0.0001
D. Allocative inefficiency specification tests for model 4		
(1) $H_o : \theta_{120} = \theta_{320} = \theta_{121} = \theta_{321} = \theta_{122} = \theta_{322} = \theta_{123} = \theta_{323} = \theta_{124} = \theta_{324} = 0$ (overall)	40.0	<0.0001
(2) $H_o : \theta_{121} = 0$ (traditional 1)	0.0	1.000
(3) $H_o : \theta_{321} = 0$ (traditional 2)	2.1	0.15
(4) $H_o : \theta_{122} = 0$ (purchased feed proportion 1)	0.0	1.000
(5) $H_o : \theta_{322} = 0$ (purchased feed proportion 2)	0.0	1.000
(6) $H_o : \theta_{123} = 0$ (education 1)	0.03	0.867
(7) $H_o : \theta_{323} = 0$ (education 2)	1.5	0.225
(8) $H_o : \theta_{124} = 0$ (experience 1)	8.7	0.003
(9) $H_o : \theta_{324} = 0$ (experience 2)	4.7	0.03

[a] $LR = -2(L_R - L_U) \sim \chi^2_q$.

and we attempt to explain failure of remaining properties where they occur.

We checked monotonicity and curvature properties for each observation and at the mean of the data. Table 4 summarizes the properties of the estimated equations. We denote elasticity by the symbol ε. Monotonicity implies that shadow shares and output elasticities must be positive, and variable cost must be nonincreasing with respect to capital. The concavity condition requires that the shadow cost function's matrix of second-order derivatives with respect to shadow prices be negative semi-definite. At the mean when efficiency is assumed ($\phi = 0$ and $\theta = 1$), the shadow variable cost function satisfies all of these regularity conditions.

We further conducted observation-by-observation tests of the monotonicity and concavity properties despite the well-known problem that flexible functional forms violate the regularity conditions implied by economic theory (see, e.g., Caves and Christensen 1980).[12] These results are displayed in table 4. The monotonicity property with respect to y_1 is met at every observation but becomes a problem at large herd sizes for y_2. Large herd farms have relatively low crop output. As to the monotonicity properties of inputs x_1, x_2, and x_3, we do not encounter a significant number of violations of the regularity conditions for x_2 and x_3, but we do find violations at the largest herd sizes for x_1. An explanation might be that dairy farms use more paid labor as they get bigger. Regarding the expected nonincreasing effect of K on VC, we find that at low herd size levels, this property is not met,

[12] In general, we attempt to minimize the number of regularity condition violations, especially concavity, when specifying a model. We do not impose curvature as in Diewert and Wales (1987), but we do test it.

Table 3. Parameter Estimates of the Preferred Shadow Variable Cost Function

β_o	−0.16*	β_{23}	0.04***	γ_{KK}	0.29***		AI
	(−1.80)		(2.90)		(2.85)		
α_1	0.66***	β_{33}	0.07***	η_1	−0.28***	θ_{120}	−2.96***
	(13.25)		(3.63)		(−3.44)		(−13.39)
α_2	0.07***	μ_{11}	−0.15***	η_2	−0.001	θ_{121}	1.29***
	(4.99)		(−11.82)		(−0.33)		(9.83)
β_1	0.52***	μ_{21}	0.05***	α_{11}	0.36***	θ_{122}	1.46***
	(12.07)		(3.04)		(4.91)		(7.83)
β_2	0.20***	μ_{31}	0.10***	α_{22}	0.003***	θ_{123}	−0.03
	(6.80)		(5.92)		(4.89)		(−0.16)
β_3	0.28***	μ_{12}	−0.0006	α_{12}	−0.0008	θ_{124}	0.02***
	(5.08)		(−0.89)		(−0.31)		(4.69)
γ_K	−0.12***	μ_{22}	0.002***		TI	θ_{320}	−2.46***
	(−2.56)		(2.76)				(−7.52)
β_{11}	0.23***	μ_{32}	−0.001**	ϕ_1	−0.04***	θ_{321}	0.11*
	(40.04)		(−2.28)		(−1.54)		(1.62)
β_{12}	−0.13***	ν_1	0.03***	ϕ_2	−0.29***	θ_{322}	−0.004***
	(−6.80)		(2.54)		(−5.47)		(−0.03)
β_{13}	−0.10***	ν_2	0.002	ϕ_3	0.002	θ_{323}	0.11
	(−5.73)		(0.20)		(0.06)		(1.28)
β_{22}	0.89***	ν_3	−0.03	ϕ_4	0.005***	θ_{324}	0.005**
	(3.55)		(−3.58)		(5.43)		(2.16)
Adj R^2	lnVC	Share1	Share3	LLF	1,815		
	0.86	0.69	0.66				

Note: t-statistics in parentheses.
Single asterisk (*), double asterisks (**), and triple asterisks (***) denote significance at 0.1, 0.05, and 0.01 levels, respectively.
TI and AI denote technical inefficiency and allocative inefficiency, respectively.

Table 4. Shadow Variable Cost Function at Mean (Preferred Model) Calculated Indices[a]

Herd Size (No. of Obs)	$\hat{\varepsilon}_{y_1} \geq 0$	$\hat{\varepsilon}_{y_2} \geq 0$	$\hat{s}_1^* \geq 0$	$\hat{s}_2^* \geq 0$	$\hat{s}_3^* \geq 0$	$\frac{\nabla^2 V\hat{C}}{\nabla \hat{w}^{*2}}(NSD)$ Data Mean Inefficient	$\hat{\varepsilon}_K < 0$
Herd < 30 (32)	0%	0%	0%	0%	0%	70% (*76%*)	37%
30 ≤ Herd < 50 (103)	0%	2%	0%	0%	2%	66% (*82%*)	53%
50 ≤ Herd < 100 (207)	0%	1%	0%	0%	0%	68% (*79%*)	40%
100 ≤ Herd < 200 (147)	0%	8%	0%	0%	0%	75% (*65%*)	34%
200 ≤ Herd < 500 (70)	0%	17%	0%	0%	0%	86% (*60%*)	11%
500 ≤ Herd < 1,000 (35)	0%	14%	2%	0%	0%	76% (*48%*)	14%
1,000 ≤ Herd < 2,000 (16)	0%	13%	6%	0%	0%	99% (*54%*)	1%
2,000 ≤ Herd (9)	0%	78%	12%	0%	0%	100% (*52%*)	0%
Data mean efficient	0%	0%	0%	0%	0%	0%	0%

[a]Observation-by-observation violations of monotonicity, nonnegativity, and concavity properties.
Note: Data mean efficient implies calculation of properties using average input price data but imposing technical and allocative efficiency; data mean inefficient, in contrast, allows for inefficiency to vary by observation, but it is evaluated at the mean of the input price data. Results of this last calculation are in italics in parenthesis. To the left of the parentheses are the results of evaluating the properties at the prices faced by each individual operation allowing for observation-specific inefficiency.

perhaps because the dairy farms with small herd sizes are not as capital intensive as large operations (i.e., the proportion of purchased feed goes up as the dairy farm gets larger). Hence, a key finding is that the structure of capital does not remain optimal as the dairy farm expands. We also tested violations in concavity at the mean of price data assuming observation-by-observation variation in inefficiency, and we tested curvature violations assuming observation-by-observation variation in both inefficiency and price.

Once established that the variable shadow cost function meets the required regularity conditions (conditional on what we have said above), we return to tables 2 and 3 to present hypothesis tests on the structure of technology and efficiency based on the preferred model 4

and the impact of variables hypothesized to affect inefficiency. We performed likelihood ratio tests on the first-order parameters, hypotheses B(1) through B(6), in order to assure their statistical significance given the complex survey design of ARMS. Specification tests, hypotheses B(7) and B(8), soundly reject a Cobb–Douglas functional form as well as a homothetic translog variable cost function as the preferred specification.

Hypotheses C(3) and C(5) indicate that parameters ϕ_2 and ϕ_4 are highly significant. Table 3's information on parameter ϕ_2 suggests that a higher proportion of purchased feed (an indicator of relative capital intensiveness) implies a lower level of technical inefficiency, which is as expected. In addition, the coefficient ϕ_4 says that the more experience that the farmer operator has managing a dairy farm, the higher his or her variable cost is. This latter effect captures the fact that more experienced (and older) dairy farmers manage less capital-intensive operations. We did not attempt to separate the impact of age and experience. The ϕ_1 and ϕ_3 parameters are insignificant (hypothesis C(2) and C(4)), suggesting that technical inefficiency differences between traditional and nontraditional dairy areas are unimportant in our model. Educational attainment does not matter either. The two effects, hypotheses D(8) and D(9), are highly significant, however. We interpret the negative signs for the parameters θ_{124} and θ_{324} as more experienced operators overutilizing labor and feed relative to energy. These latter effects reinforce the point about the impact of experience on inefficiency—that is, more experienced, older farmers use less capital-intensive technologies. Overall technical and allocative inefficiency, hypotheses C(1) and D(1), respectively, do matter when analyzing dairy farm costs, and therefore, the cost function needs to take account of these types of inefficiency in order to be specified correctly.

Inefficiency and Scale Economies

To analyze scale economies, an average total cost function is usually employed. Multioutput technologies pose a challenge because there is not a single output by which to divide costs. Baumol, Panzar, and Willig (1982) proposed the average incremental cost (AIC) function in order to confront this problem and to characterize scale economies in multioutput technologies. An AIC function is a partial average total cost function in that other outputs are held constant. The curvature of this function indicates scale economies in the same way that a single output average total cost curve would.

To estimate the AIC function, we first needed to derive a total cost function. To do so, we assumed that total shadow costs can be represented as the sum of shadow variable costs plus implicit capital expenses: $TC = VC + IP_K K$.[13] The variable IP_K in turn is equal to θW_K, where W_K is the *observed* opportunity cost of capital. Thus, we have the following relationship: $IP_K = \theta_K W_K = -\frac{\partial VC}{\partial K} \equiv \frac{VC}{K} \times \frac{\partial \ln VC}{\partial \ln K}$. In the framework that we employ here, the firm invests according to $\theta_K W_K$, the unobserved shadow value of capital due to constraints and inefficiency, and not according to the observed W_K. A value of 1 for θ_K will mean that the farm reached an optimal level of capital.[14] The elasticity of capital in turn is defined as $\frac{\partial \ln VC}{\partial \ln K} = \gamma_K + \gamma_{KK} \ln K + \sum_{l=1}^{2} \eta_l \ln y_l + \sum_{j=1}^{3} \nu_j \ln w_j^*$. Hence, the magnitude of $-\frac{\partial VC}{\partial K}$ also depends on the efficiency with which variable inputs are used.

We can then define the dairy farm's incremental cost of producing y_1, given that it produces the entire livestock, livestock products, and crop vector $y = (y_1, y_2)$, as: $IC_1(y_1, w^*) \equiv TC(y, w^*) - TC(y_{\hat{2}}, w^*)$, where $y_{\hat{2}} = (0, y_2)$. AIC for y_1 is then defined as

$$(14) \qquad AIC(y_1, w^*) \equiv \frac{C(y, w^*) - C(y_{\hat{2}}, w^*)}{y_1}.$$

We chose the mean levels of the crop output and the variable input prices to derive the AIC for y_1, using table 3's estimated parameters.

We derived technical, allocative, and average incremental cost *efficiency* indexes by imposing or relaxing restrictions on equations (5) and (6) while calculating equation (15) with the parameters obtained from estimating model 4. Thus, we derived four magnitudes for AIC: first AIC_{OE} (overall efficiency) obtained by imposing the restrictions of no technical inefficiency ($\ln \phi = 0$) or allocative inefficiency ($\theta_{12} = 1$ and $\theta_{32} = 1$); second, AIC_{TI} (technical inefficiency), derived by imposing allocative but not technical efficiency; third, AIC_{AI} (allocative inefficiency), derived by imposing

[13] We follow an approach different from that recommended in Morrison (1999b) in that we use the opportunity cost of capital to approximate the long-run cost function instead of first determining the optimal level of capital based on prices faced by the dairy farm.

[14] For a variable cost function the optimal amount of capital is found where marginal benefit, variable cost reduction, equals the marginal cost of capital (Chambers 1988; Morrison 1999b).

Table 5. Implied Technical, Allocative, and Overall Efficiency for Preferred Model[a]

Herd Size (% of Sample Cow Inventory)	*TE*	*AE*	*OE*
Herd < 30 (1%)	0.72	0.53	0.39
30 ≤ Herd < 50 (3%)	0.73	0.53	0.39**
50 ≤ Herd < 100 (11%)	0.74	0.55***	0.41***
100 ≤ Herd < 200 (16%)	0.75	0.58***	0.44***
200 ≤ Herd < 500 (18%)	0.80***	0.62***	0.49***
500 ≤ Herd < 1,000 (19%)	0.82**	0.66***	0.54***
1,000 ≤ Herd < 2,000 (16%)	0.83	0.66	0.55
2,000 ≥ Herd (17%)	0.86	0.71*	0.60
Average	0.75	0.56	0.42

[a] H_o: diff = $\text{mean}_i - \text{mean}_j = 0$.
Note: Differences across herd sizes significant at single asterisk (*) 10%, double asterisks (**) 5%, and triple asterisks (***) 1%.

technical but not allocative efficiency; and finally, AIC_{OI} (overall inefficiency) was derived by imposing neither allocative nor technical efficiency. Technical inefficiency (TI) is defined as $\frac{AIC_{TI}}{AIC_{OE}}$ and technical efficiency (TE) in turn as $\frac{1}{TI}$. Because TI is greater or equal to 1, the index TE varies between 0 and 1, with 1 being the most efficient. We defined allocative inefficiency (AI) as $\frac{AIC_{AI}}{AIC_{OE}}$ and allocative efficiency (AE) as $\frac{1}{AI}$. The index AE also varies between 0 and 1. Last, we define overall inefficiency (OI) as $\frac{AIC_{OI}}{AIC_{OE}}$ and overall efficiency (OE) as $\frac{1}{OI}$. Because OI is greater or equal to 1, the index OE varies between 0 and 1, with 1 being the most efficient. Overall efficiency can also be calculated as $TE \times AE$. Table 5 presents our implied *efficiency* results. Hotelling's T^2 generalized means tests ascertained the significance of means differences for the various efficiency measures across herd sizes, at the same time taking account of ARMS's complex survey design. Table 5 shows the increase in AIC efficiency for dairy enterprises, with herd sizes between fifty and 1,000. The AIC overall efficiency results are driven by the increased allocative efficiency of dairy enterprises as they get larger. The heavier reliance on paid labor in large farms could explain this increase. We also find significant differences in technical efficiency between small dairy farms and those of more than 200 cows.

To investigate further model 4's connection between inefficiency and herd size, we analyzed the shape of the total cost function according to the different specifications implied by models 1, 2, 3, and 4 using equation (10). The long-run scale elasticity results by herd size and model are shown in table 6. We also employed Hotelling's T^2 generalized means tests to determine significance levels of scale elasticity across herd sizes. Differences within models and across herd sizes are not significant for either small or large herd sizes. Notably, however, differences between models 1 and 4 within herd sizes are significant at the 1% level. This last significance test means that scale economies will be underestimated if inefficiency is not taken into account (e.g., if model 1 is estimated instead of model 4). Differences within models by herd size exhibit an L-shaped pattern in that the differences are not significant at both tail ends, which indicates small statistically insignificant decreases. Most striking of all, models 1 through 4 show increasing returns to scale up to the 1,000 to 2,000 herd size category and an insignificant increase in scale elasticity thereafter. Model 4 shows the largest significant scale elasticity.

Table 6. Scale Economies in U.S. Dairy Farming

Long-Run Scale Elasticity Under Different Specifications				
Herd Size (% of Sample Cow Inventory)	Model 1	Model 2	Model 3	Model 4 (Preferred Model)
Herd < 30 (1%)	2.33	2.48	2.01	2.42
30 ≤ Herd < 50 (3%)	2.37	2.60	2.00	2.48
50 ≤ Herd < 100 (11%)	1.77***	1.96***	2.43	1.98***
100 ≤ Herd < 200 (16%)	1.57**	1.67	1.57	1.81
200 ≤ Herd < 500 (18%)	1.28***	1.35***	1.33***	1.48***
500 ≤ Herd < 1,000 (19%)	1.16***	1.21***	1.22***	1.34***
1,000 ≤ Herd < 2,000 (16%)	1.10**	1.16**	1.17**	1.29*
2,000 ≥ Herd (17%)	1.10	1.15	1.15	1.25
Average	1.89	2.06	2.01	2.06

Note: H_o: diff = $\text{mean}_i - \text{mean}_j = 0$.
Differences across herd sizes significant at single asterisk (*) 10%, double asterisks (**) 5%, and triple asterisks (***) 1%.
All differences in scale elasticity between models 1 and 4 for each herd size are significant at the 1% level.

Table 7. AIC Under Different Inefficiency Restrictions[a] for Preferred Model and MC

Herd Size (% of Sample Cow Inventory)	AIC_{OE}	AIC_{TI}	AIC_{AI}	AIC_{OI}	MC_{OI}
Herd < 30 (1%)	\$30.61	\$42.45	\$59.53	\$82.91	\$31.78
30 ≤ Herd < 50 (3%)	\$20.73***	\$28.84***	\$39.66***	\$55.46***	\$22.58***
50 ≤ Herd < 100 (11%)	\$15.10***	\$20.66***	\$27.60***	\$37.88***	\$18.01***
100 ≤ Herd < 200 (16%)	\$10.78***	\$14.35***	\$18.58***	\$24.78***	\$13.88***
200 ≤ Herd < 500 (18%)	\$9.00***	\$11.31***	\$14.56***	\$18.33***	\$12.35**
500 ≤ Herd < 1,000 (19%)	\$7.15***	\$8.69***	\$10.91***	\$13.25***	\$10.11***
1,000 ≤ Herd < 2,000 (16%)	\$6.81	\$8.18	\$10.14	\$12.18	\$9.72
2,000 ≥ Herd (17%)	\$7.13	\$8.30	\$10.00	\$11.66	\$10.12
Average	\$16.76	\$19.82	\$31.06	\$42.72	\$19.27

[a]OE = no technical or allocative inefficiency; TI = technical inefficiency and allocative efficiency; AI = technical efficiency and allocative inefficiency; OI = technical and allocative inefficiency.
Note: H_0: diff = $\text{mean}_i - \text{mean}_j = 0$.
Differences across herd sizes significant at single asterisk (*) 10%, double asterisks (**) 5%, and triple asterisks (***) 1%.
Differences of AIC_{OE} and AIC_{OI} across models within a herd size are all significant at 1% level. All differences of AIC $_{OI}$ and MC are significant at 1%.

The preferred model's results contradict research that has found diseconomies of scale in dairy (e.g., Alvarez and Arias 2003). Our findings are bolstered by the trends shown in figure 2 and in general by the trend toward larger dairy farms in the U.S. sector (see, e.g., MacDonald et al. 2007).

In order to validate the results of this study, we also employed the average incremental cost function to test the structure of scale economies in dairy farming. Table 7 presents these results, imposing different inefficiency restrictions on model 4: AIC_{OE}, AIC_{TI}, AIC_{AI}, and AIC_{OI} defined above. Table 7 also illustrates the significance levels for mean differences across herd sizes using the same means difference test employed above (test results between models 1 and 4 within a herd size are all significant at 1%). The preferred specification for model 4 shows a statistically significant reduction in average costs stopping at herd sizes up to 1,000 cows. No significant decreases in average incremental costs are shown thereafter. Nevertheless, that differences between the models are statistically significant again leads to the conclusion that scale economies will be greatly underestimated if inefficiency is not taken into account. We also calculated long-run marginal cost for the preferred model, defined as average incremental cost over long-run scale elasticity. It can be seen from table 7 that, despite the fact that marginal cost is still beneath average incremental cost, marginal cost has started increasing, which points to eventual decreasing returns to scale. Again, and importantly, our results did not find decreasing returns to scale for the sample we have, but we did find that eventually decreasing returns will happen, all other things equal.

Figure 3 shows four AIC curves for y_1 corresponding to the imposition of the technical, allocative, and overall inefficiency. This graph also displays two average revenue curves for 2000: milk and milk with livestock products. The chart shows several striking results. First, scale economies are definitely the driving force toward consolidation in the sector. As costs decrease rapidly, scale economies are increasing (or at least not decreasing) in each inefficiency model. Second, the costs employed to graph the various AIC curves are significantly different from each other and so, again, inefficiency matters. Third, comparing the AIC curves with the various average curves indicates that economic costs are not fully recovered for small herd sizes. Two reasons might explain this result: first, the opportunity cost of labor is particularly high for small dairy operations; second, adjustment costs hinder smaller dairies from adopting new technology that ultimately would lower their inefficiency and enhance their potential to exploit the cost-cutting economies of scale. These findings help to explain why smaller operations do not simply get bigger, and they harken back to Wolf's (2003) argument of adjustment costs faced by small dairy.

In short, we have shown that it is crucial not to confuse farm operation above the average variable cost (inefficiency) with the shape of the average variable cost curve (scale economies). Tables 6 and 7 show a significant understatement of scale economies for small, medium, and large sizes across inefficiency models. Table 5 displays a low overall

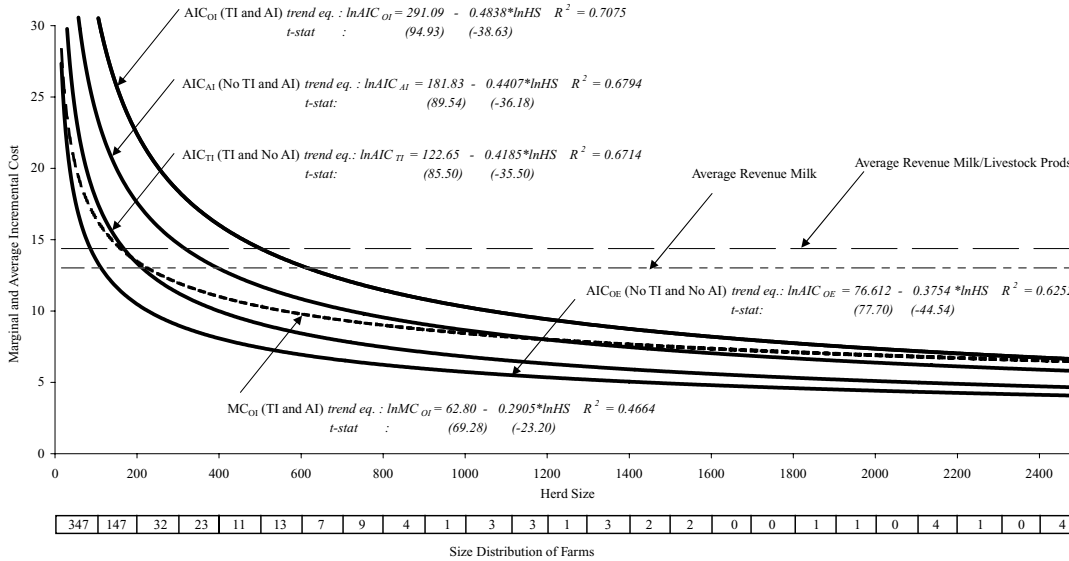

Figure 3. Scale economies and inefficiency in dairy farming in the United States

efficiency for small and medium farms. This cost penalty diminishes in magnitude as size increases, but it remains significantly important. This result is entirely consistent with the trends in the dairy farming sector. In table 7 the preferred model 4 does not allow for inefficiency points to decreasing returns to scale (though insignificant). Using the same data and allowing for inefficiency, model 4 also shows that the cost of inefficiency declines with the size of the dairy enterprise but shows a significantly larger elasticity of scale and increasing returns to scale than model 4 with efficiency imposed. In sum, if scale economies are important in dairy production—and they are—they must be measured so as to distinguish them from both the costs of allocative and technical inefficiency at the same time that important variables like the opportunity cost of labor are taken into account.

Summary and Conclusions

Our substantive findings are found in tables 5–7 and figure 3. We took care in estimating a function that met the regularity conditions of monotonicity in output quantities and shadow input prices as well as concavity in shadow input prices. Also, the elasticity of capital stock with respect to variable costs is negative. We selected the variable cost function incorporating technical and allocative inefficiency coefficients through parametric tests. This research employs a national farm-level sample of dairy producers in the United States. Table 7 exactly decomposes average incremental costs into its technical and allocative components and gives a clear picture of the respective costs. Table 6, in contrast, shows results from specification tests, the most important conclusion being that the significant differences in scale elasticity results from estimating a model with or without inefficiency. Table 5, based on the results of table 7, indicates that as the dairy farm gets larger, it becomes more efficient, a finding confirmed in the literature. As noted in the literature, our finding that the relationship between marginal costs and average cost points to eventual decreasing returns to scale needs to be qualified by saying that technological advances and human capital investment might delay this trend.

The title of this article mentions both scale and efficiency, but are they equally important? We do not think so. We argue that efficiency and its patterns are interesting and important, and they influence the patterns of scale economies, but scale economies are far more crucial. From this perspective, tables 7 and 6 matter more than table 5. Economies of scale drive structural change in dairy farming; hence, the model must specify different types of inefficiency correctly. Previous findings (and assertions) use models plagued by various types of specification error, perhaps the most serious being the failure to incorporate properly (or to incorporate at all) technical and allocative

inefficiencies. At the very least, the model we present here comes closest to accounting for these problems in estimating scale economies in U.S. dairy farming. Our substantive finding is that this more precise model supports a conclusion that returns to scale are larger at all levels of output than previously believed.

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