



Economically Optimal Nitrogen Rates of Corn: Management Zones Delineated from Soil and Terrain Attributes

Dan B. Jaynes,* Thomas C. Kaspar, and Tom S. Colvin

ABSTRACT

Much of the NO_3 found in streams in the Midwest comes from leaching of N fertilizers applied to corn (*Zea mays* L.). To reduce this leaching, N fertilizer must be used more effectively. Dividing fields into areas that respond more uniformly to management is one approach for improving N fertilizer use. The objective of this study was to determine and compare economically optimal N fertilizer rates for corn in a corn–soybean [*Glycine max* (L.) Merr.] rotation within management zones. A method based on soil and landscape characteristics was used to divide three fields into management zones roughly equivalent to toeslope, footslope, backslope, and shoulder landscape positions. We applied N fertilizer at six rates within replicated small plots (15.1 by 4.5 m) located within each management zone. Yield vs. N rate for each management zone was fitted to the Mitscherlich equation using nonlinear methods. Final Mitscherlich parameters, economically optimal N rates, and their confidence limits were computed for each management zone. Yields at the highest N rate generally followed the pattern of toeslope \geq footslope \geq backslope \gg shoulder among the management zones. Conversely, economically optimal N rates followed the reverse pattern and varied from 23 to 247 kg ha⁻¹. Thus, yield at the highest N rate was a poor predictor of the optimal N rate for the management zones delineated in this study and N fertilizer application would have been optimized by applying relatively less to the toeslope and footslope management zones and relatively more to the shoulder and backslope management zones.

WITHIN THE MIDWEST Corn Belt, NO_3 concentrations in surface waters often exceed the 10 mg L⁻¹ maximum contaminant level for drinking water set by the USEPA (Mitchell et al., 2000; Jaynes et al., 1999). Excessive NO_3 in the Mississippi River has been identified as a leading cause of hypoxia in the northern Gulf of Mexico (USEPA Science Advisory Board, 2007; Rabalais et al., 1996). Numerous studies at the field and watershed scale (David et al., 2010; Royer et al., 2006; Goolsby et al., 2001; Jaynes et al., 1999) have shown that much of the NO_3 in surface waters of the Midwest comes from corn and soybean production.

Numerous suggestions have been made about how to reduce NO_3 leaching from row crop lands in the Midwest (USEPA Science Advisory Board, 2007; Dinnes et al., 2002). One of these is to use precision farming methods to apply N fertilizers at variable rates across a field rather than at a uniform rate (Raun and Schepers, 2008). There are essentially two approaches to variable-rate application of N fertilizer: dividing fields into smaller, more uniform zones and varying man-

agement by zone, or using crop sensors to determine the N requirement during or before application of fertilizer.

For the development of management zones, two approaches have been used in the literature. The first approach divides the field into more uniform areas based on yield information from previous crops (Harmel et al., 2004; Boydell and McBratney, 2002). These zones can represent uniformly high- or low-yielding areas, areas that yield high in wet years and low in dry years, or vice versa. The second approach uses information on the spatial distribution of landscape and soil properties thought to impact yield, such as slope and soil texture, and may include surrogate properties that are related to soil properties important to yield, such as soil electrical conductivity (Kaspar et al., 2004; Fraisse et al., 2001; McCann et al., 1996). The field is then divided into zones based on how crop yields are assumed to respond to variation in those soil properties (Sawyer, 1999).

There have been few studies verifying the success of management zones constructed in either manner for managing N fertilizer (Derby et al., 2007; Schmidt et al., 2002). One possible cause for the limited success is the lack of information on how to manage N by zone. Most studies have managed N in each zone based on expected yield or the yield goal for that zone and a statewide N fertilizer recommendation (Ping et al., 2008; Ruffo et al., 2006; Bullock et al., 2002). It has been found, however, that yield is often a poor predictor of the N fertilizer requirement (Scharf et al., 2006). Thus, N rate prescriptions based on yield and existing N recommendation algorithms cannot be expected to optimize the N rate no matter how many zones a field is divided into (Ferguson et al., 2002).

In this study, we determined experimentally the economically optimal N rate (N_e) for corn in management zones developed

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Abbreviations: GPS, global positioning system; UAN, urea–ammonium nitrate.

using the technique described in Jaynes et al. (2003), based on past yield patterns and soil and landscape information. We hypothesized that N_c would differ by zone, providing the possibility to improve N fertilizer management for corn. Management zones were determined in three fields and N_c determined for corn in a corn–soybean rotation for each zone from 2001 to 2006.

MATERIALS AND METHODS

Site Description

The research was conducted on three private fields in central Iowa, within 15 km of the city of Ames (42.023° N, 93.625° W) where long-term weather data were available for an Iowa State University station. All fields were located within the Des Moines lobe, which is characterized by low-relief swell and swale topography. Soils were in the Clarion (fine-loamy, mixed, superactive, mesic Typic Hapludolls)–Niccollet (fine-loamy, mixed, superactive, mesic Aquic Hapludolls)–Webster (fine-loamy, mixed, superactive, mesic Typic Endoaquolls) association, formed in calcareous glacial till, and have surface soil organic matter contents ranging from 9 to 18% in lower landscape positions to about 3% in the hilltop positions (Soil Conservation Service, 1984). All field operations except N fertilization and plot harvesting were conducted by collaborating farmers following their typical practices. These included a 2-yr corn and soybean rotation, with P and K applied at maintenance rates in the fall after soybean harvest. Tillage was by chisel plow and field cultivator for at least the 10 yr before the study, with a 76-cm row spacing used for corn and soybean. Before the study, anhydrous NH_3 was applied to all fields in the fall before corn.

The first field, the 32-ha Baker field, has been described in Karlen and Colvin (1992) and Colvin et al. (1997) and was used in Jaynes et al. (2003) to determine multiyear yield patterns for corn. Jaynes et al. (2003) found that the corn yield in this field during the 1989 to 1999 period could be grouped or clustered into spatially homogenous areas or zones. They identified five such zones that were loosely consistent with landscape position within the field: potholes or depressions, toeslopes, footslopes, backslopes, and shoulders. They further showed that these yield zones could be predicted using the first two canonical composites developed from multiple discriminant analysis using selected landscape and soil properties. The properties they found to be significant for differentiating the yield zones were the soil electrical conductivity (EC), elevation (EL), slope, and plan and profile curvature, with an overall prediction accuracy of 78%. Thus, landscape and EC properties could be used to delineate zones with yield patterns similar to those found by Jaynes et al. (2003) in other fields that lacked long-term yield information.

In this study, we wished to use the method developed from the Baker field to delineate management zones in other fields in the surrounding area using the canonical composites developed from the Baker field multiple discriminant analysis and landscape and EC values specific to each field. To identify management zones in other fields, EL would not be particularly helpful because it has meaning only within the context of an individual field. Thus, EL was dropped from the multiple discriminant analysis used in Jaynes et al. (2003). Instead, another landscape property—depression depth (DP)—was used. This property delineates the areas where water may pond and was found by Kaspar et al. (2004) to be an important predictor of areas with lower yields

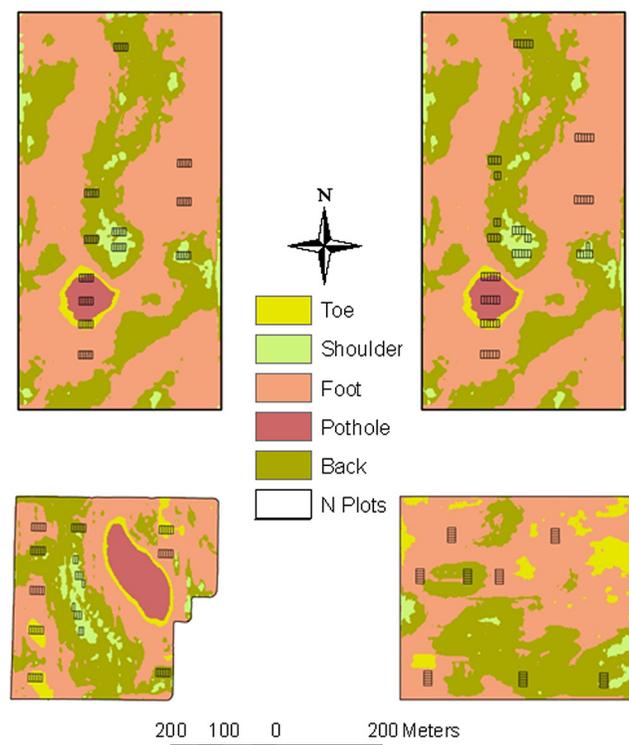


Fig. 1. Location of management zones and N-rate plots in the Baker field in 2001 and 2003 (top left), the Baker field in 2005 (top right), the Tower field (bottom left), and the Coffman field (bottom right). The pothole and toe zones were combined for this study.

during wet years. Rerunning the multiple discriminant analysis of Jaynes et al. (2003) with DP substituted for EL actually improved the prediction accuracy of the multiple discriminant analysis to 80% and greatly improved the identification of pothole and toeslope yield zones. The predicted management zones for the Baker field using the revised multiple discriminant analysis are shown in Fig. 1 and greatly resemble the zones mapped in Jaynes et al. (2003, Fig. 6) except for the pothole and footslope (Clusters 1 and 2) zones, which are now confined to a single pothole in the southern half of the field and more accurately reflect the yield zones found by Jaynes et al. (2003, Fig. 2).

The other two fields used in the study were approximately 15 km from the Baker field and had soils in the Clarion–Niccollet–Webster association. The Tower field was approximately 13.7 ha in size while the Coffman field was 17.5 ha. In these two fields, EC was measured inductively using an EM38 (Geonics Ltd., Mississauga, ON, Canada) and elevation was measured using a kinematic global positioning system (GPS) along transects that were approximately 6 m apart using the methods described in Jaynes et al. (2003). Electrical conductivity and EL were interpolated on a 2- by 2-m grid using the interpolation methods described in Jaynes et al. (2003). The landscape properties slope and plan and profile curvature were computed from the 2-m EL grid using the Arc/Info geographic information system software's CURVATURE command (Arc/Info, ESRI, Redlands, CA). The value of DP was computed from the EL grid by first running the DEMfill script in the Spatial Analyst extension of ArcView to fill the depressions in the EL grid and then subtracting the original EL grid from the filled grid to find the depth of the depressions. Maximum depression depths were 0.3 m in the

Table 1. Corn hybrid and selected dates of field operations.

Field-year	Hybrid	Planting date	N application date	Harvest date
Baker 2001	DeKalb 595BtY	29 Apr.	6 June	2–3 Oct.
Tower 2002	Pioneer 34G13	4 May	28 May	10 Oct.
Baker 2003	DeKalb 5332 Bt	27 Apr.	21 May	7 Oct.
Tower 2005	Pioneer 35Y67	25 Apr.	24 May	5 Oct.
Baker 2005	DeKalb 5878 Bt	8 Apr.	20 May	22 Sept.
Coffman 2006	Pioneer 35Y67	21 Apr.	17 May	5 Oct.

Baker field, 0.47 m in the Tower field, and 0.19 m in the Coffman field. The yield zones for these fields predicted using the first two canonical composites developed from the multiple discriminant analysis of the Baker field yield zones and the field-specific EC and landscape properties are shown in Fig. 1. As in the Baker field, the multiple discriminant analysis predicted yield zones in the two fields that were spatially contiguous, with a Moran's *I* statistic of 0.23 for the Tower field and 0.32 for the Coffman field, where a value of 1 indicates perfect spatial clustering and a -1 perfect spatial dispersion (Moran, 1950).

Fertilizer Treatments

The corn yield in response to N rate was measured in the Baker field in 2001, 2003, and 2005, in the Tower field in 2002 and 2005, and in the Coffman field in 2006. Corn hybrids and important crop management dates are listed for each field-year in Table 1. Soybean was grown in each field the year before corn. Two years separated the corn crops in the Tower field because the corn planted in 2004 was destroyed by a hailstorm in early June and the farmer replanted to soybean. To determine the corn yield response to N fertilizer within the different management zones, small plots were located within each of the predicted management zones in the three fields. We attempted to replicate the N-response plots three times in each management zone for each field. Due to the limited extent and the similarity in yield response found for the pothole and toeslope zones in the Baker field (Jaynes et al., 2003), however, the two zones were combined into a single toeslope zone. For the Tower field, we were only able to fit two N plots into the shoulder zone. In the Coffman field, only two management zones had sufficient spatial extent to have N plots established, and three plots were located in backslope zones and five in the footslope zone. Six N rates were applied to subplots within each N rate plot. Each N rate subplot was 15.1 m long by six rows wide (4.5 m), the width of the harvesting equipment used by each farmer. Where possible, the six N rate subplots that composed an N plot were spatially contiguous across 36 rows; however, for the shoulder management zone in the Baker and Tower fields, the subplots had to be spatially shifted to fit within the zones. In 2005, the farmer of the Baker field switched to a 16-row planter and eight-row combine head and we had to shift the location of the N rate plots slightly to accommodate this spacing. The locations of the N rate plots are shown in Fig. 1 for each field.

Six N rates were randomly assigned to the subplots within each N plot. Nitrogen fertilizer rates were 0, 45, 90, 135, 179, and 224 kg ha⁻¹ and were applied after corn emergence (Table 1). Liquid urea–NH₄NO₃ (UAN) at 28 or 32% N was applied to the subplots using a BLU-JET sidedress applicator (Thurston Manufacturing Co., Thurston, NE) following emergence of

the corn. The applicator was a tow-type toolbar with a supply tank mounted on the toolbar. Liquid fertilizer was applied with coulter injection units mounted on the toolbar. An orifice nozzle sprayed a stream of UAN into a narrow slot created by a smooth, straight coulter. The application rate was controlled by a Raven spray controller (Raven Industries, Sioux Falls, SD) coupled with a Capstan adjustable nozzle system (Capstan Ag Systems, Topeka, KS). The Capstan system adjusted the nozzle output by cycling the nozzles on and off with variable durations (pulse width). Prescription maps were created using ArcView or Arc-Map. The maps were used with either the application software Farm Works Site Mate or Ag Leader's InSight monitor–controller and GPS to send prescription information to the Raven controller. When the Raven controller received a rate change, the Capstan unit adjusted the output by changing the pulse width of the on–off cycle of the nozzles, thus varying the application rate. The remainder of each field had N applied at the farmer's preferred rate, which was 157 kg ha⁻¹ for the Baker and Coffman fields and 146 kg ha⁻¹ for the Tower field. The actual rates applied to each subplot were measured with the same equipment and the measured rates were used when computing N_e .

We measured corn yields from each N rate subplot with a plot combine modified with a weigh tank mounted inside the combine grain storage box (Colvin, 1990). Grain yields from three center rows of each 15.1-m-long N subplot were harvested and weighed and the moisture content measured and used to adjust the grain yields to 155 g kg⁻¹ water content. In addition, end-of-season corn stalk testing was conducted following Binford et al. (1992) by harvesting the portion of the stalk 15 to 35 cm above the ground after black layer. Ten plants from a row adjacent to the harvested rows of each N treatment plot were harvested, dried, and analyzed for NO₃.

Fitting the Yield Response Curve

The economically optimal N rate, N_e , was computed for each management zone of a field by fitting the exponential response curve to all measured yield vs. N rate data from the replicated sites. The exponential function can take many equivalent forms through reparameterization (Ratkowsky, 1983), but is often expressed as

$$Y = Y_{\max} \{1 - \exp[-C(N_r + N_{\text{soil}})]\} \quad [1]$$

where Y is the yield (Mg ha⁻¹) at N rate N_r (kg ha⁻¹), Y_{\max} (Mg ha⁻¹) is the maximum yield obtainable, N_{soil} (kg ha⁻¹) is the residual or mineralized N available to the crop from the soil, and C (ha kg⁻¹) is a proportionality constant. This form of the exponential equation is often referred to as Mitscherlich's equation because the equation can be derived from the mathematical expression proposed by E.A. Mitscherlich to describe the decreasing response in yield with increases in a single limiting nutrient (Tisdale and Nelson, 1975):

$$\frac{\partial Y}{\partial N} = (Y_{\max} - Y)C \quad [2]$$

Advantages to the exponential function are that its parameters can be interpreted as having physical meaning. Disadvantages of the exponential function include that maximum yield is only approached as the N rate approaches infinity and that most experiments fail to measure yields at very high N rates to help

establish this behavior. Not having a good measurement of yield at a high N rate sometimes leads to nonconvergence of the nonlinear least squares fitting of the function (Jaynes, 2011). The function is easily differentiated, however, to determine N_c and for most fertilizer/grain price ratios and the computed N_c is usually within typical N rates used by farmers.

The economically optimal N rate, N_c , is defined as the N rate where the income from the last increment of yield increase just equals the cost of the added unit of N fertilizer. The value of N_c (kg ha⁻¹) is found by equating the first derivative of Eq. [1] with the ratio of N fertilizer cost to corn grain price, R (US\$ US\$⁻¹) and solving for N_c :

$$N_c = \frac{-\ln[R/(Y_{\max}C)]}{C} - N_{\text{soil}} \quad [3]$$

Substituting Eq. [3] into [1] and rearranging gives

$$Y = Y_{\max} - \frac{R}{C} \exp[-C(N_r - N_c)] \quad [4]$$

Thus, N_c can be solved for directly. The Proc NLIN nonlinear regression routine in SAS (SAS Institute, Cary, NC) was used to solve Eq. [4] for each field-year management zone data set. For all computations, a fertilizer cost (US\$0.96 kg⁻¹ N)/grain price (US\$0.143 kg⁻¹) ratio of 6.7 was used. To test whether the value of C is unique for the corn yield response to N as asserted by Mitscherlich, Eq. [4] was first fit to the data from each management zone within a field-year. Equation [4] was then fit a second time to the data from all management zones within a field-year using a common value for C in all management zones while the other parameters were allowed to vary by management zone. An F -test of the resulting residual mean squares of the differences from these two regressions was used to test whether C was unique for each management zone or if a single C could be used for all management zones within a field-year (Daniel and Wood, 1980; Ratkowsky, 1983). A final F -test using the residual mean squares was conducted to see if a common C for all field-years could be assumed by fitting Eq. [4] to all field-year and management zone data with a common C while Y_{\max} and N_{soil} were allowed to vary by management zone.

Nonlinear regression will give a set of values for N_c , Y_{\max} , and C ; however, as pointed out by Jaynes (2011), estimates of these coefficients are usually quite imprecise due to scatter in the yield data, and thus some measure of uncertainty in the coefficients should always be reported. While NLIN provides the variance of the estimated parameters, Eq. [4] is highly nonlinear (Ratkowsky, 1983) and its fitted coefficients can be highly skewed and thus the computed confidence bands for the coefficients would not be accurate. To overcome this problem, Jaynes (2011) suggested using Hougaard's skew statistic in the NLIN procedure (Hougaard, 1985) and transforming the unknown parameters in Eq. [1] or [4] before solving:

$$Y = Y_{\max}^{\prime\alpha} \left\{ 1 - \exp \left[-C^{\prime\beta} \left(N_r + N_{\text{soil}}^{\prime\gamma} \right) \right] \right\} \quad [5]$$

$$Y = Y_{\max}^{\prime\alpha} - \frac{R}{C^{\prime\beta}} \exp \left[-C^{\prime\beta} \left(N_r - N_c^{\prime\delta} \right) \right] \quad [6]$$

where $Y_{\max}^{\prime\alpha} = Y_{\max}$, $C^{\prime\beta} = C$, $N_{\text{soil}}^{\prime\gamma} = N_{\text{soil}}$, and $N_c^{\prime\delta} = N_c$. Solving Eq. [5] or [6] with NLIN while minimizing

Hougaard's skew statistic for each of the transformed parameters by manually adjusting α , β , γ , or δ results in accurate estimates of the uncertainty of each parameter. Note that solving Eq. [1] or [5] and [4] or [6] gives the same values for the coefficients when appropriately transformed, but the computed confidence limits for the coefficients in Eq. [5] and [6] are correct while those for Eq. [1] and [4] are not due to the nonlinearity of the exponential response function. For this study, Hougaard's skew was considered sufficiently small, and thus the fitted parameters unbiased, when its absolute magnitude was below 0.1 (Hougaard, 1985). Thus, a final set of regressions were run using Eq. [6] in NLIN and solving for Y_{\max}^{\prime} , C^{\prime} , and N_c^{\prime} by manually adjusting the α , β , and δ exponents and then back-transforming to give accurate estimates of the coefficients and their confidence bands. Similarly, a final regression was run using Eq. [5] and the previously fitted Y_{\max}^{\prime} and C^{\prime} coefficients to find accurate confidence bands for N_{soil} after back-transformation.

RESULTS

The growing-season precipitation (May–August) measured in Ames, IA, was about 90 mm below the 40-yr average in 2001 and 2006. In particular, 2001 was dry, with monthly precipitation below average in June, July, and August. Conversely, the growing-season precipitation was >50 mm greater than average in 2005 and >100 mm greater in 2002. While just about equal to the 40-yr average, the precipitation in 2003 was greater than average in both May and June when soil N is most susceptible to leaching due to limited uptake by the young crop. Degree days (0°C base) during May to August were computed from the daily minimum and maximum temperatures and were less in the years of this study than the 40-yr average. This was primarily due to May being cooler than normal in every year, while July was warmer than average in all years. A cool spring–warm summer pattern is typically not conducive to large corn yields—the desired pattern is for a warm spring followed by a cooler than average July and August (Yang et al., 2006). Nevertheless, average corn yields in 2005 for Story and Boone counties, where the fields were located, were the second highest during the past 10 yr, following 2004 (National Agricultural Statistics Service, 2010). The year 2001 had the lowest two-county average corn yield in the past 10 yr, presumably because of the lower than average precipitation in June to August of that year.

Grain yields from the Baker field are shown in Fig. 2 and for the Tower and Coffman fields in Fig. 3. Yields are plotted vs. the measured N applied and not the target rate. In a few cases, there were substantial differences between the two rates due to errors in the programming of the applicator or errors in the GPS data. There were measureable yield differences among the three replications at each N rate within a management zone, often spanning several megagrams per hectare, which is typical for small-plot yield data (Cerrato and Blackmer, 1990; Kwaw-Mensah and Al-Kaisi, 2006; Kyveryga et al., 2007; Williams et al., 2010). Yield variation was particularly great for the shoulder management zone in the Baker field, where yields differed by as much as 6 Mg ha⁻¹ within a single N rate target.

In several field-years, the toeslope management zone showed little yield response to added N. These included the toeslope management zone in the Baker field for 2001 and the Baker and

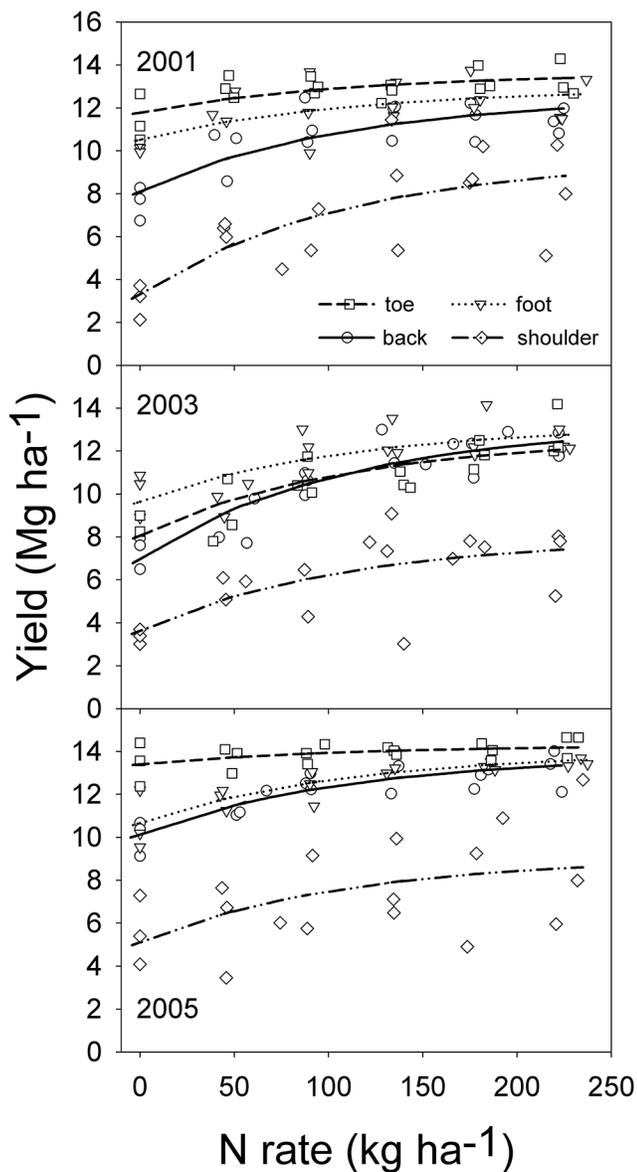


Fig. 2. Measured corn yield vs. N rate and fitted yield response curves for management zones in the Baker field in 2001, 2003, and 2005.

Tower fields in 2005. Otherwise, the average yield for each N rate increased by 2 to $>5.5 \text{ Mg ha}^{-1}$ as the N rate increased from 0 to 224 kg ha^{-1} . Yield at the highest N rate was always greater (analysis not shown for unrestricted LSD with $P = 0.05$) for the toeslope zone than for the shoulder zone in the Baker and Tower fields and the footslope zone yields were greater than shoulder yields in the 2005 Tower field, but there were no significant differences among the other toeslope, footslope, and backslope zones. For the Coffman field in 2006, the footslope zone had higher yields than the backslope zone at the 224 kg ha^{-1} N rate.

The nonlinear yield response equation, Eq. [4], fitted to the yield data within each management zone for every field-year converged to stable solutions and gave a RMSE of $\approx 1.05 \text{ Mg ha}^{-1}$ when averaged across all management zones. The residuals from fitting all management zones were normally distributed ($P < 0.01$), as indicated by the Kolmogorov–Smirnov statistic and a linear relationship between the residuals and the cumulative frequency (not shown). Fitting the exponential equation to yields

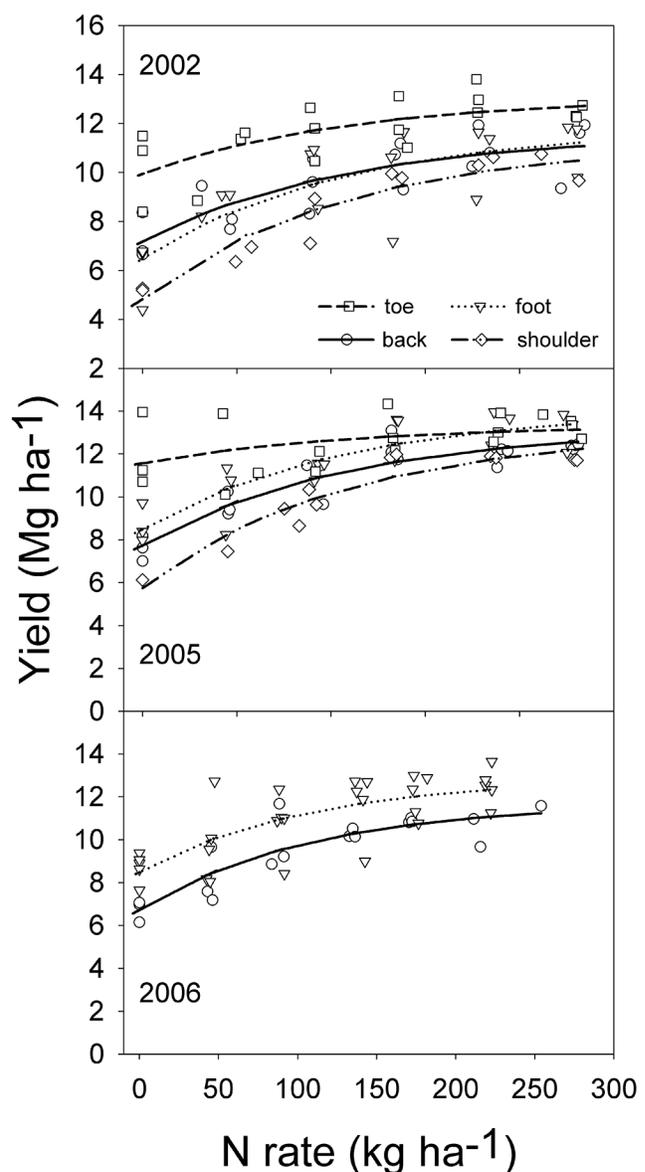


Fig. 3. Measured corn yield vs. N rate and fitted yield response curves for management zones in the Tower field in 2002 and 2005 and the Coffman field in 2006.

from each zone within a field-year using a single C but different Y_{\max} and N_{soil} values for each management zone also converged successfully, with a RMSE of 1.15 Mg ha^{-1} averaged across all field-years. The F -test of the differences in the resulting residual mean squares from fitting each management zone separately vs. fitting the management zones within a field-year with a single C term are given in Table 2. For all six field-years, using an independent estimate of C for each management zone did not improve the fit significantly compared with a single fitted C for all zones within a field-year. Thus, there was no advantage to using a separate estimate of C for each management zone. The global regression was also conducted where Y_{\max} and N_{soil} were fitted to each management zone individually but a single C was fitted for all management zones, fields, and years. Again, the F -test indicated no significant improvement by using separate estimates of C for each field-year (Table 2, and thus all management zones were fitted with a single estimate of C across all field-years.

Table 2. Statistical comparison of using of a common proportionality parameter C for all management zones vs. using a separate C for each management zone when fitting the exponential model to the yield data as a function of N rate for six field-years and overall for all field-years.

Field-year	Source	df	Residual sum of squares	Residual mean square	F of residual mean square of difference	Prob. > F
Baker 2001	Common C'	63	83.65			
	Sum individual C' values	60	81.70			
	Difference	3	1.96	0.65	0.479	0.698
Baker 2003	Common C'	63	71.60			
	Sum individual C' values	60	69.72			
	Difference	3	1.88	0.63	0.539	0.657
Baker 2005	Common C'	63	92.68			
	Sum individual C' values	60	90.74			
	Difference	3	1.94	0.65	0.427	0.734
Tower 2002	Common C'	57	65.73			
	Sum individual C' values	54	63.14			
	Difference	3	2.60	0.87	0.741	0.532
Tower 2005	Common C'	57	48.69			
	Sum individual C' values	54	47.98			
	Difference	3	0.71	0.24	0.265	0.850
Coffman 2006	Common C'	43	53.05			
	Sum individual C' values	42	52.33			
	Difference	1	0.72	0.72	0.581	0.450
All field-years	Common C'	387	420.83			
	Sum individual C' values	382	415.40			
	Difference	5	5.43	1.09	0.999	0.418

A final regression was made using Eq. [5] to solve for N_e' directly using a single C' for all management zones. The transformation coefficients used for all parameters are shown in Table 3. The single C' value fitted to all data was 0.0407 with an exponent value of $\beta = 1.457$ to reduce the skew in C'. Transforming back gives a value of $C = 0.0094 \text{ ha kg}^{-1}$, with a

68% confidence band of 0.0080 to 0.0109. Exponents for the other coefficients were successfully adjusted to bring the absolute value of Hougaard's skew to below 0.1 and thus removing much of the skew from the parameter distribution. To remove the skew, the δ exponent for N_e' in Eq. [6] was adjusted from -20 to 5.15 among the different management zones. For the

Table 3. Parameter exponents and resulting Hougaard's skew values for the transformed economically optimal N rate (N_e'), maximum yield (Y_{max}'), and N available in the soil (N_{soil}') using the fitted global value of the proportionality constant $C' = 0.0407$ with an exponent $\beta = 1.457$.

Field-year	Management zone	N_e'		Y_{max}'		N_{soil}'	
		Parameter exponent'	Hougaard's skew	Parameter exponent'	Hougaard's skew	Parameter exponent'	Hougaard's skew
Baker 2001	toeslope	0.52	0.060	-6.0	-0.018	-1.5	-0.052
	footslope	0.52	0.041	-2.5	-0.022	-1.5	0.088
	backslope	3.80	-0.017	-0.83	-0.039	-3.0	0.057
	shoulder	-1.90	-0.021	-0.98	-0.046	4.5	0.008
Baker 2003	toeslope	5.15	-0.085	-0.80	-0.025	-3.0	0.053
	footslope	0.95	-0.014	-1.03	-0.034	-2.0	0.092
	backslope	-2.7	-0.065	-0.63	-0.085	-4.0	0.082
	shoulder	2.2	0.032	-2.80	-0.053	20	-0.003
Baker 2005	toeslope	0.83	-0.232	2.0	0.019	-0.7	0.007
	footslope	0.82	-0.039	-0.97	-0.019	-2.0	0.069
	backslope	0.98	0.017	-0.93	-0.052	-2.0	0.088
	shoulder	1.36	-0.041	-2.50	-0.038	-4.0	0.080
Tower 2002	toeslope	0.69	0.005	-1.37	-0.033	-2.0	0.061
	footslope	-3.4	0.000	-0.76	-0.040	-4.0	0.081
	backslope	4.0	-0.021	-0.91	-0.023	-3.0	0.081
	shoulder	-1.75	0.097	-0.70	0.070	8.0	0.034
Tower 2005	toeslope	0.54	-0.014	-6.0	-0.009	-1.5	-0.050
	footslope	-3.0	-0.011	-0.53	-0.036	-4.0	0.031
	backslope	-3.0	0.006	-0.59	-0.030	-4.0	0.047
	shoulder	-1.90	0.095	-0.57	0.062	20	-0.063
Coffman 2006	footslope	-2.30	0.094	-0.27	0.091	-3.0	0.037
	backslope	-20	-0.005	-0.82	-0.036	-4.0	0.062

Table 4. Fitted economically optimal N rate, N_c , and its 68% confidence limits using a common proportionality parameter, C.

Zone	Baker 2001	Baker 2003	Baker 2005	kg ha ⁻¹		
				Tower 2002	Tower 2005	Coffman 2006
Toeslope	101†	190	23†	156	96†	
	22–142	164–220	0–107	123–187	16–138	
Footslope	129†	170	163	214	218	193
	82–163	140–200	132–192	189–243	193–248	170–220
Backslope	194	230	174	193	216	206
	167–223	204–261	144–203	167–222	191–246	180–236
Shoulder	231	191	178	233	247	
	205–262	164–220	150–207	205–266	218–282	

† Not significantly different than zero at $P = 0.05$; all others significant at $P < 0.001$.

Y_{\max} parameter, the exponent α was adjusted from -6.0 to 2.0 to remove the skew in the coefficient. For N_{soil} , using Eq. [5] and the C' and Y_{\max} values already found, the γ coefficient had to be adjusted from -4.0 to 20 to reduce the skew satisfactorily. The fitted yield response curves for each management zone are shown in Fig. 2. In all cases, the exponential response curve was a good representation of the measured yield response.

The computed back-transformed N_c values and their 68% confidence bands are listed in Table 4. All N_c estimates were significantly different than 0 as indicated by the Student's t statistic except for the toeslope management zone in Baker 2001, Baker 2005, and Tower 2005 and the footslope management zone in Baker 2001. These zones within field-years tended to have small responses to N rate and the computed N_c values were <130 kg ha⁻¹. For all of the management zone field-years, N_c ranged from 23 to 247 kg ha⁻¹. Variations in N_c among years were most variable for the toeslope management zone in the Baker field, varying from 23 kg ha⁻¹ in 2005 to 190 kg ha⁻¹ in 2003. This management zone was also quite variable for N_c for the 2 yr measured at the Tower field. Formal tests of significant differences among N_c values, such as the unrestricted LSD (Saville and Rowarth, 2008), are not applicable to this data because of the non-normal distributions. Instead, the overlap of the 68% confidence bands ($\pm 1\sigma$) was used to judge significance (Schenker and Gentleman, 2001). Within a field-year, the typical pattern was for N_c to increase in the order toeslope \leq footslope \leq backslope \leq shoulder management zones. The most dramatic exception to this pattern was in Baker 2003, where the N_c for the toeslope management zone was as high as the shoulder management zone.

The maximum yield potential, Y_{\max} , varied from 7.92 to 14.28 Mg ha⁻¹ (Table 5). There were few significant differences in Y_{\max} among the toeslope, footslope, and backslope management zones, but the Y_{\max} of the shoulder management zone was

significantly less than the other zones in the Baker field. There was also a significant difference between the shoulder and toeslope zones in the Tower 2002 field and between the two management zones in the Coffman 2006 field. The 68% confidence bands for Y_{\max} spanned <1.2 Mg ha⁻¹ for all management zones and had a CV of 0.18 when the standard deviation was replaced with one-half of the 68% confidence bands in the computation of the CV. The CV for N_c was at 1.18, indicating much greater certainty for Y_{\max} estimates than for N_c . The 68% confidence bands for N_{soil} typically spanned >50 kg ha⁻¹ (Table 6) with an average CV of 0.96, again much greater than for Y_{\max} . The N provided by the soil, N_{soil} , varied from 45 to 295 kg ha⁻¹ among all the management zones. The toeslope management zone had the highest N_{soil} in four out of five field-years, while the shoulder management zone always had the lowest. The N_{soil} value for the footslope management zone was greater than the backslope management zone in all field-years.

DISCUSSION

Both yield at the highest N rate and fitted Y_{\max} typically followed the pattern of toeslope \geq footslope \geq backslope \gg shoulder among the management zones. This pattern is similar to the yield pattern observed in 1995 and 1997 in the Baker field (Jaynes et al., 2003) and was deemed characteristic of yield patterns when the growing-season precipitation (May–August) was less than average (Jaynes et al., 2003; Kaspar et al., 2003, Table 2). In this study, the exceptions for this yield pattern were met in the Baker field in 2003 and the Tower field in 2005, when the yield at the highest N rate for the toeslope zone was less than that for the footslope zone. In the earlier study, lower toeslope than footslope zone yields occurred in years when May to August precipitation was greater than average, and indeed that was what was observed in this study for the Baker 2003 and Tower 2005 fields

Table 5. Fitted maximum yield, Y_{\max} , and its 68% confidence limits using a common proportionality parameter, C.

Zone	Baker 2001	Baker 2003	Baker 2005	Mg ha ⁻¹		
				Tower 2002	Tower 2005	Coffman 2006
Toeslope	13.60†	12.40	14.28	13.04	13.32	
	13.14–14.08	11.9–12.96	13.82–14.74	12.57–13.55	12.88–13.78	
Footslope	12.89	13.16	13.95	11.80	14.02	12.85
	12.42–13.39	12.67–13.69	13.47–14.46	11.27–12.39	13.48–14.61	12.44–13.34
Backslope	12.49	13.18	13.78	11.54	13.17	11.67
	11.98–13.05	12.6–13.83	13.29–14.31	11.04–12.1	12.64–13.76	11.15–12.26
Shoulder	9.56	7.92	8.96	11.18	13.02	
	8.99–10.2	7.41–8.47	8.46–9.49	10.54–11.92	12.34–13.8	

† All significantly different than zero at $P < 0.001$.

Table 6. Fitted soil N, N_{soil} , and its 68% confidence limits using a common proportionality parameter, C.

Zone	Baker 2001	Baker 2003	Baker 2005	Tower 2002	Tower 2005	Coffman 2006
	kg ha ⁻¹					
Toeslope	212 167–281	113 92–139	295† 220–494	152 124–191	215 169–285	
Footslope	179 144–230	139 114–174	153 126–190	84 69–104	98 82–119	114 96–137
Backslope	110 91–136	80 66–97	140 115–174	103 84–128	94 77–114	91 74–112
Shoulder	45 34–58	64 48–86	91 71–118	60 46–77	61 46–81	

† Not significantly different than zero at $P = 0.05$; all others significant at $P < 0.001$.

(Table 7). Tower 2002 and Baker 2005, however, were also wetter than average in May to August but did not have toeslope zone yields less than footslope zone yields. One probable cause for this discrepancy is that in the original study N fertilizer was applied to the Baker field in the fall, whereas in this study it was sidedressed in mid- to late May. Thus, excess precipitation in early May would have contributed to increased leaching and denitrification in the original study but not here. When sidedressing N in mid-May, it seems that greater than average precipitation in June is a better predictor of relatively lower toeslope zone yields (Table 7) because June precipitation was much greater than average in 2003 but only slightly greater in 2005 (3 mm) and drier than average in 2002. Variations in local precipitation may also have contributed to the differences observed in 2005 for the Baker and Tower fields because the precipitation measured 15 km from either field was only slightly greater than average in June and rainfall events during this month can vary considerably across these distances (Hatfield et al., 1999).

The pattern of toeslope \geq footslope \geq backslope \gg shoulder was the same pattern observed for N_{soil} in all years except 2003, indicating that the zones with the higher yields also were areas where more soil N was available to the crop. Fitted N_{soil} values ranged from a low of 45 kg ha⁻¹ in the shoulder zone to >200 kg ha⁻¹ in the toeslope zone in some years. While the upper end of the fitted N_{soil} values seems high, they easily fall within the N mineralization ranges found by Cambardella et al. (1994) for similar soils near these fields. In addition, N_{soil} includes not only N mineralized before and during the growing season but also any residual mineral N in the soil and N in wet and dry deposition. These sources of N were sufficient, at least for the Baker 2005 toeslope zone, to supply the entire N required for the corn crop. The fitted values for N_{soil} for the toeslope zone were lowest in 2003, which had not only the wettest June but also the lowest cumulative degree days of all field-years, possibly leading to less mineralization as well (Honeycutt et al., 1988). A lack of soil N available to the crop in the toeslope zone in 2003 is also indicated by the end-of-year stalk test results (not shown), where nine of the 18 N rate plots in the toeslope zone were below the detection limit for NO₃ (<0.02 mg L⁻¹), whereas in all other years no more than two of the 18 N rate plots were below detection in this zone.

The economically optimal N rate was markedly different for several of the management zones in 2001 and, in most years, much lower for the toeslope zone than the other zones despite the toeslope zone typically having the highest yield. Conversely, the shoulder zone typically had the greatest N_c despite having the lowest yield of any zone every year. The only exception again occurred in 2003,

when N_c was the same for the toeslope and shoulder zones, probably in response to the wetter June and lower cumulative degree days for this year resulting in less soil N. Overall, it is apparent from these results that yield at the high N rate is a poor indicator of the optimal N rate in these fields, similar to what has been observed in other studies (Vanotti and Bundy, 1994; Fox and Piekielek, 1995; Kachanoski et al., 1996; Lory and Scharf, 2003) and impugning the yield goal approach toward N fertilizer management.

Using the grain price/fertilizer cost ratio, R , of 6.7 and the fitted yield response for each zone and field-year, we computed the single optimal N rate for each management zone across all years. For the toeslope zone this was 128 kg ha⁻¹, while for the footslope, backslope, and shoulder zones the rates were 185, 204, and 219 kg ha⁻¹, respectively. For a strategy using fixed N rates each year, these rates would have optimized the economic return when applied to all field-years of this study. If these rates had been used, the economic benefit with $R = 6.7$ would have been US\$9.19 ha⁻¹ above the N rate used by the collaborating farmers (157 kg ha⁻¹ for the Baker and Coffman fields and 146 kg ha⁻¹ for the Tower field), but this benefit does not include costs associated with management zone delineation or the acquisition of variable-rate application equipment. Note that if R changes, the multiyear N_c values would also change for each zone but the differences between the rates would remain constant, e.g., 57 kg ha⁻¹ difference between the toeslope and footslope zone rates, as the differences do not depend on R .

Table 7. Monthly and May–August total precipitation and degree days for the years of the study and the 40-yr average recorded within 15 km of the study fields.

Year	May	June	July	Aug.	May–Aug.
Precipitation, mm					
2001	190	50	48	74	362
2002	130	81	150	209	569
2003	122	150	168	25	465
2005	111	124	104	172	511
2006	55	21	141	156	373
40-yr avg.†	116	121	110	114	460
Degree days, °C‡					
2001	370	605	808	738	2521
2002	287	701	830	661	2479
2003	302	560	737	748	2346
2005	333	703	785	677	2497
2006	396	654	806	702	2557
40-yr avg.†	507	641	725	685	2558

† Average for 1968–2007.

‡ Calculated as $[(T_{\text{max}} - T_{\text{min}})/2]$, where the maximum temperature T_{max} and the minimum temperature $T_{\text{min}} \geq 0^\circ\text{C}$.

Obviously, economic optimization could be improved if N rates were adjusted each year. But based on these results, it would only be possible to adjust the yearly N rate for the toeslope zone based on June precipitation being greater than average. This would require that the N be sidedressed at the end of June, which is several weeks past the typical time for sidedressing N in central Iowa (Jaynes et al., 2004). Delaying too long after the end of June to sidedress N would result in lower yields and more NO₃ leached to tile drains (Jaynes and Colvin, 2006). Thus, this strategy would not benefit the farmer or the environment. This result also illustrates that much more N fertilizer needs to be applied to the shoulder zone than can possibly be removed in the grain yield. The difference in N applied vs. removed for this zone probably contributes to both water and air quality problems. Thus, the simple economic constraints represented by N_c may not always lead to the most environmentally friendly N management scheme.

Mitscherlich (1947) proposed that the proportionality coefficient, C , should be a constant for the response of all crops to N. It has been convincingly shown, however, that this is not the case even with the data originally used by Mitscherlich (Van Der Paauw, 1952; Black et al., 1955). Vavra and Bray (1959), however, showed that C can be constant for the response of wheat yield to P, but that C would be expected to vary by: (i) the kind of plant, (ii) the form of nutrient, (iii) the fertility pattern, and (iv) the planting rate and pattern. In this study, using unique C values for different management zones or field-years did not improve the fit of the data, therefore we used a single value of C in fitting all yield response curves. A constant C may have been a result of meeting the criteria of Vavra and Bray (1959)—by examining only the corn yield after soybean, applying UAN as a sidedress treatment, and having similar planting rates. The fields also had similar soil and landscape properties because all were located in the Clarion–Nicolet–Webster soil association. Undoubtedly, the yield variation measured among the replicate samples also contributed to the use of a single C value (Van Der Paauw, 1952) because it may have helped mask small real differences in C among the management zones. Yield measurements per N treatment plot were based on a spatial support area of 11.4 m² (three rows by 15 m). The yield variability we measured among replicates was similar to other studies using similar support areas (Kyveryga et al., 2007; Scharf et al., 2005). Yield variability might be reduced by taking measurements across a larger support area, but this would have been problematic in this study given the limited contiguous areal extent of some of the management zones (Fig. 1). Thus, it may be difficult to obtain less variable data to more thoroughly test the constancy of C by management zone in this landscape.

CONCLUSIONS

Yields at the highest N rate tended to decrease from zones in lower landscape positions to zones in higher landscape positions. Conversely, the economically optimal N rate increased along this toposequence. The inference from the fitted N_{soil} values is that considerable N was provided by the soil for each management zone, but typically more was provided in the toeslope zone where soil organic contents are probably higher, leading to lower economically optimal N rates for this zone. The exception to these patterns occurred in 2003, which experienced much greater than normal June rainfall that may have leached or denitrified much of the soil N in the lower toeslope

management zones. Averaging across the six field-years, the economically optimal N rate decreased from shoulder to backslope to footslope to toeslope management zones.

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